Title: Lecture - Quantum Theory (Core), PHYS 605

Speakers: Dan Wohns

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Subject: Particle Physics, Quantum Fields and Strings, Quantum Information

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Quantum Theory Part II

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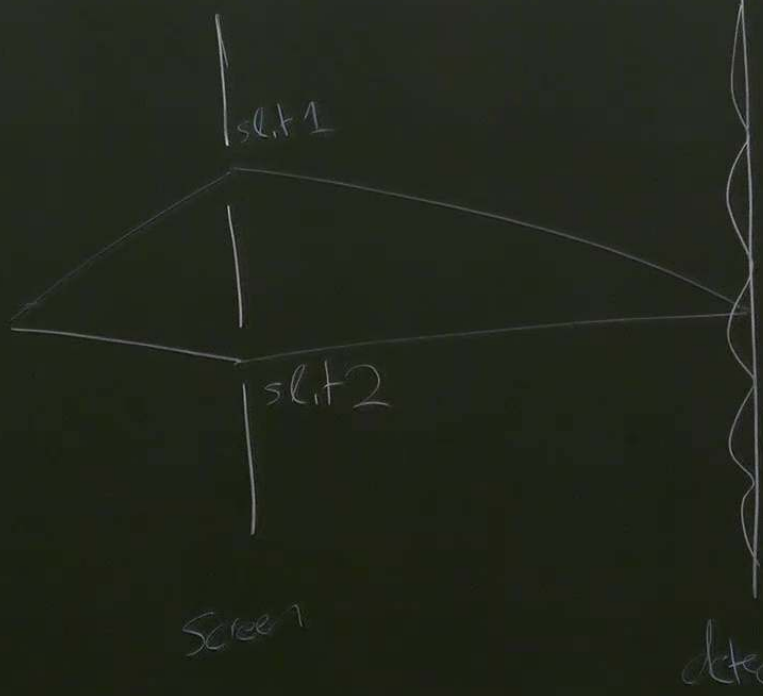
Outline: Path Integrals - today

Remainder: relativistic quantum theory (QFT)

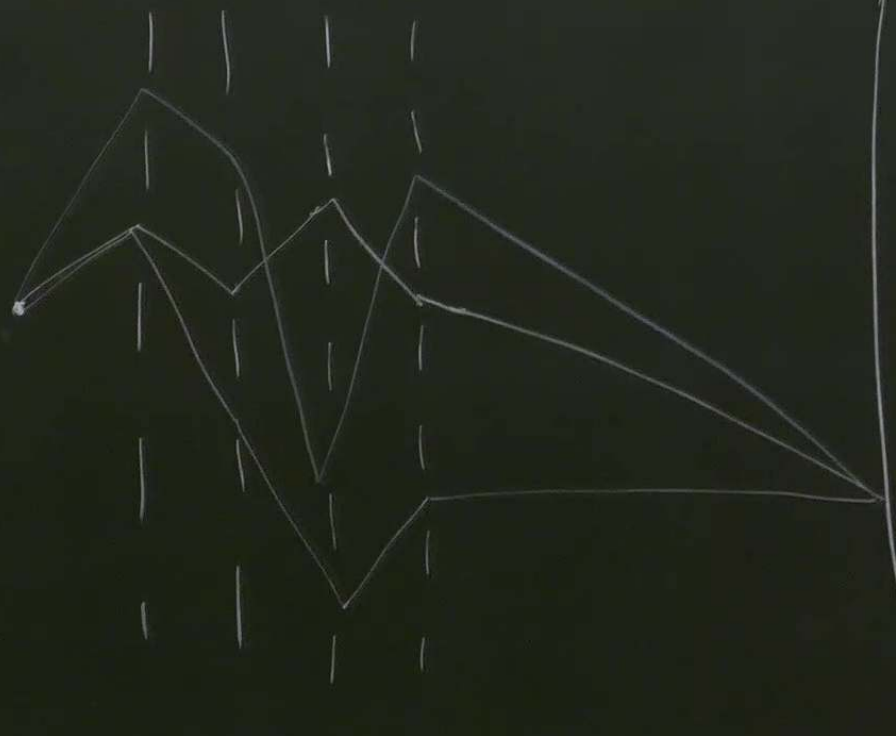
Double-slit experiment



$$A = A_1 + A_2$$

more screens with more slits?

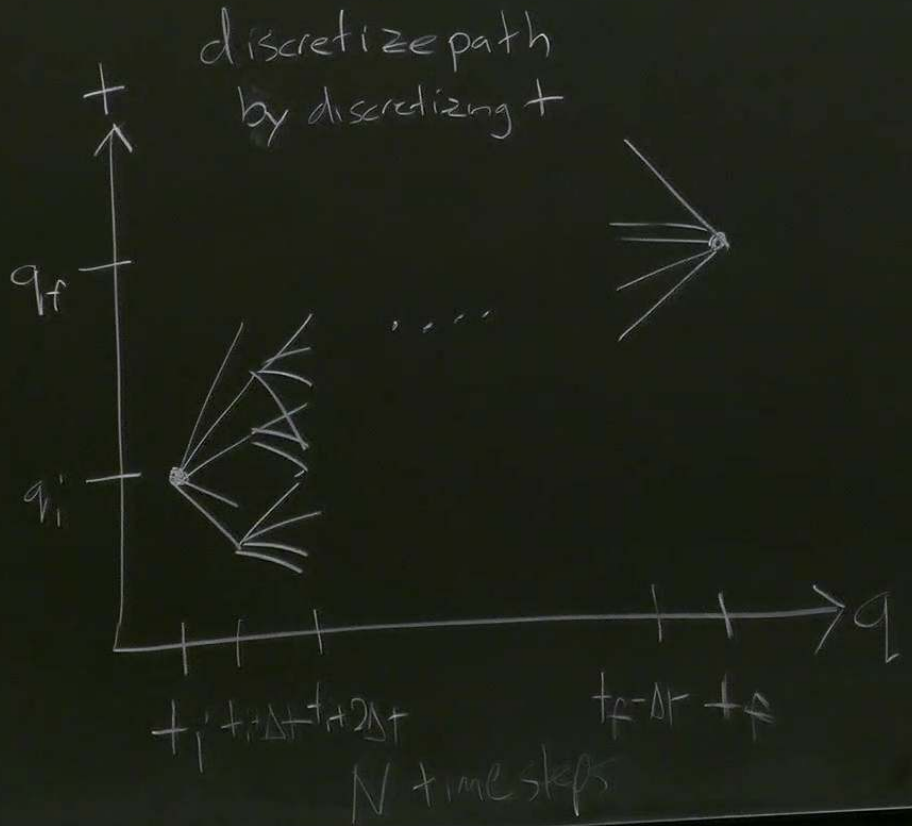
A_2



$$A = \sum_{\text{paths } \gamma} A[\gamma]$$

Propagator $K(q_f, t_f; q_i, t_i) = \langle q_f | e^{-iH(t_f - t_i)/\hbar} | q$

$= \langle q_f, t_f | q_i, t_i \rangle_{\text{Heisenberg}}$



$K = \langle q_f | (e^{-$

$= \langle q_f | e^{-iH$

$$K(q_f,t_f;q_i,t_i) = \langle q_f | e^{-iH(t_f-t_i)/\hbar} | q_i \rangle_{\text{Schrödinger}} \quad \text{with} \quad H = \frac{p^2}{2m} + V_0$$

$$= \langle q_f,t_f | q_i,t_i \rangle_{\text{Heisenberg}}$$

e path
lizing t

...

$$K = \langle q_f | \left(e^{-iH(\overbrace{t_f-t_i}^{T})/N\hbar} \right)^N | q_i \rangle$$

$$= \langle q_f | e^{-iHT/N\hbar} \underline{1} \cdots \underline{1} e^{-iHT/N\hbar} | q_i \rangle$$

$$\underline{1} = \int dq_{N-1} |q_{N-1}\rangle\langle q_{N-1}|$$

$\longrightarrow q$

$t_f - \Delta t \quad t_f$

time steps

$$K = \int dq_1 \cdots dq_{N-1} \underbrace{\langle q_N | e^{-iHT/N\hbar} | q_{N-1} \rangle} \cdots \langle q_1 | e^{-iHT/N\hbar} | q_0 \rangle$$

$K_{N,N-1}$ infinitesimal propagator

$$K_{N,N-1} = \langle q_N | \left(1 - iHT/N\hbar \cdots\right) | q_{N-1} \rangle$$

On order i: $\langle q_N | q_{N-1} \rangle = \delta(q_N - q_{N-1})$

$$= \int \frac{dp_{N-1}}{2\pi} e^{i p_{N-1}(q_N - q_{N-1})/\hbar}$$

$$K_{j,j-1} = \langle q_j | e^{-iHT/N\hbar} \int \frac{dp_{j-1}}{2\pi} | p_{j-1} \rangle \langle p_{j-1} | q_{j-1} \rangle$$

$$K_{j,j-1} = \int \frac{dp_{j-1}}{2\pi\hbar}\, e^{-ip_{j-1}(q_j - q_{j-1})/\hbar}\, e^{-i(p_{j-1}^2/2m + V_0)T/N\hbar}$$

Gaussian integral

discretized

$$K = \lim_{N\to\infty} \left(\frac{m}{2\pi\hbar\,\Delta t}\right)^{N/2} \int \left(\prod_{j=1}^{N-1} dq_j\right) \exp\left[\frac{i}{\hbar}\Delta t \sum_{j=1}^{N} \left(\left(\frac{q_j - q_j}{2m}\right.\right.\right.$$

$$K = \int_{q(t_1)=q_1}^{q(t_f)=q_f} \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$$

$$\int \frac{dp_{j-1}}{2\pi\hbar}\, e^{-ip_{j-1}(q_j - q_{j-1})/\hbar}\, e^{-i(p_{j-1}^2/2m - V_0)T/N\hbar}$$

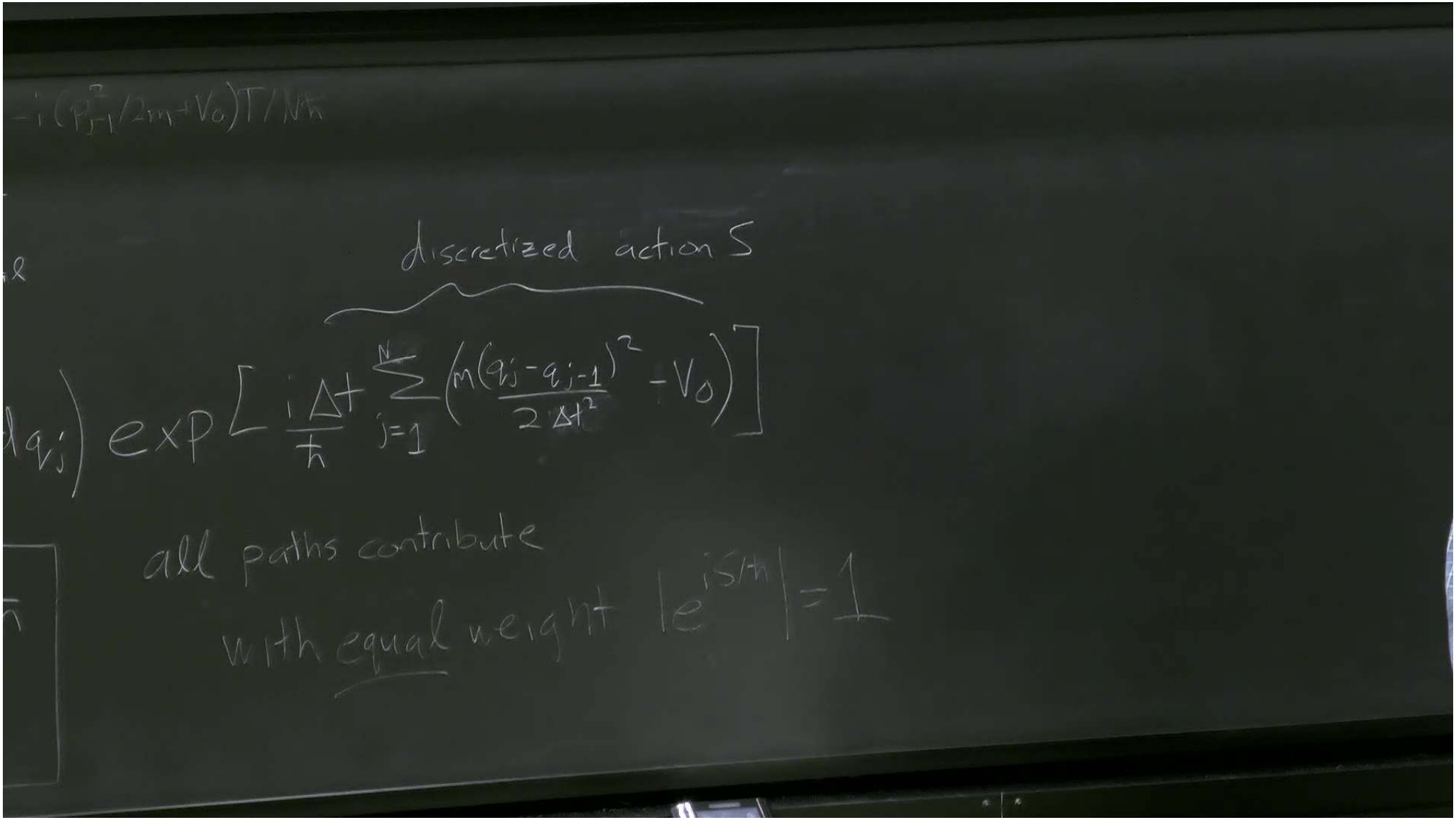
Gaussian integral

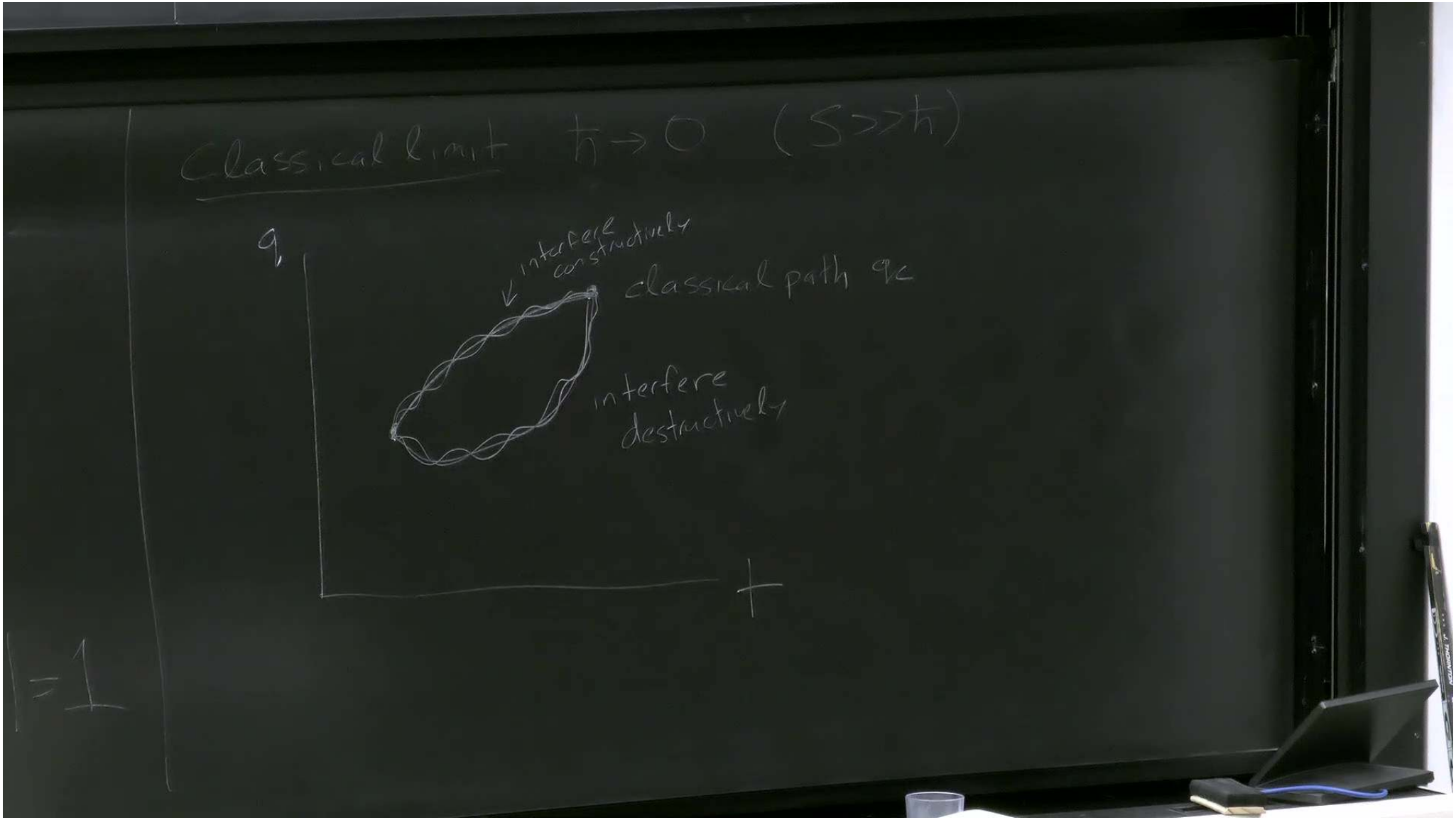
$$\lim_{N\to\infty} \left(\frac{m}{2\pi i\hbar\,\Delta t}\right)^{N/2} \int \left(\prod_{j=1}^{N-1} dq_j\right) \exp\left[\frac{i\Delta t}{\hbar}\sum_{j=1}^{N}\left(\frac{(q_j - q_{j-1})^2}{2m\Delta t^2} - V_0\right)\right]$$

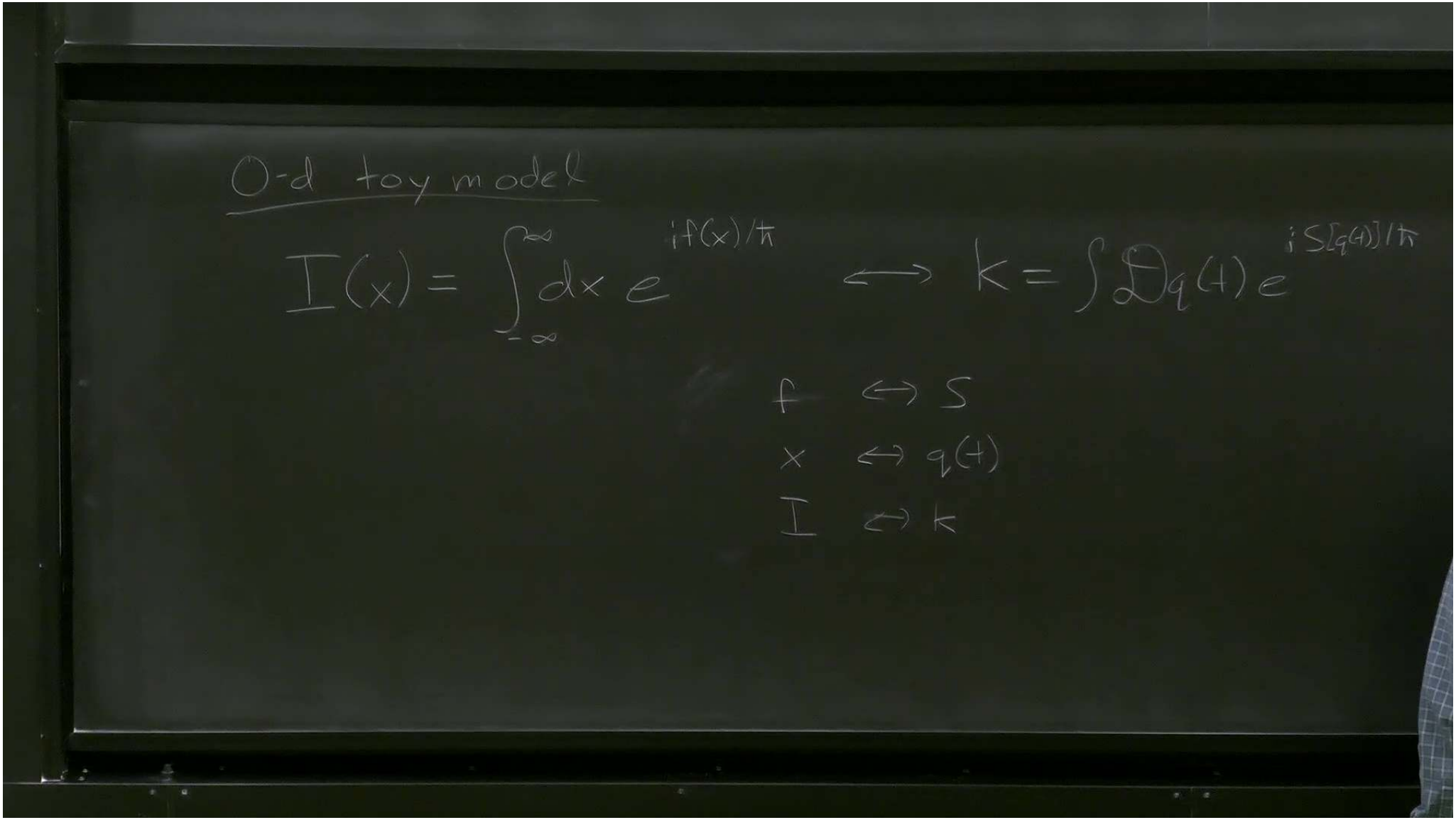
discretized action S

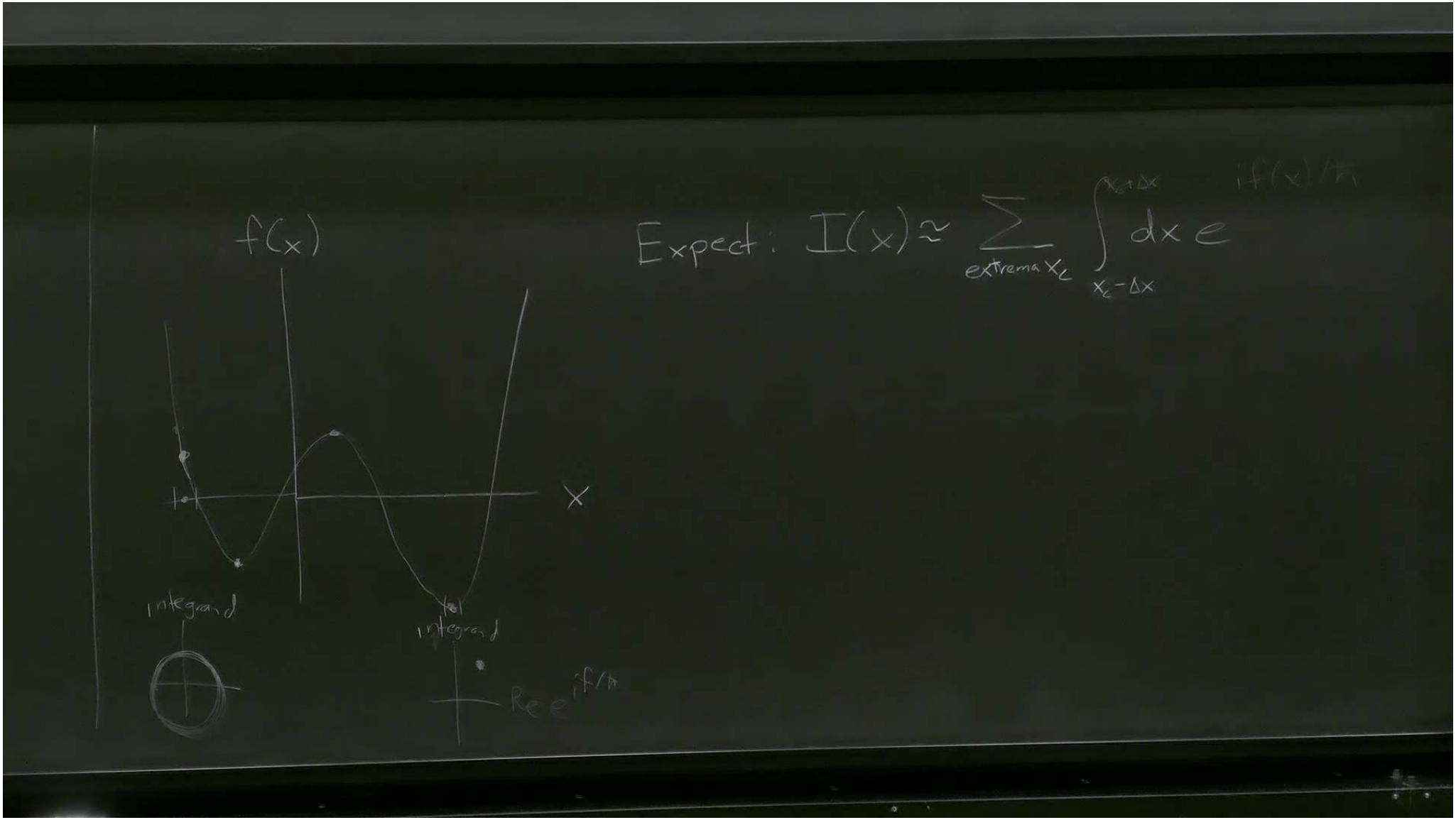
all paths contribute

$$\int_{q(t_i)=q_i}^{q(t_f)=q_f} \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$$


$-i(p_{j-1}^2/2m+V_0)T/N\hbar$
discretized action S
$dq_j)\exp\left[\frac{i}{\hbar}\Delta t\sum_{j=1}^{N}\left(m\frac{(q_j-q_{j-1})^2}{2\Delta t^2}-V_0\right)\right]$
all paths contribute
with equal weight $|e^{iS/\hbar}|=1$


Classical limit $\hbar \to 0$ $(S \gg \hbar)$
q
interfere constructively
classical path q_c
interfere destructively
t
$|=1$


0-d toy model
$I(x) = \int_{-\infty}^{\infty} dx\, e^{if(x)/\hbar}$
$\longleftrightarrow$
$k = \int \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$
$f \leftrightarrow S$
$x \leftrightarrow q(t)$
$I \leftrightarrow k$


f(x)
Expect: $I(x) \approx \sum_{\text{extrema } x_c} \int_{x_c-\Delta x}^{x_c+\Delta x} dx\, e^{if(x)/\hbar}$
X
integrand
integrand
Re $e^{if/\hbar}$

Perturbation Theory (Semi-classical limit)

$$I(x) \approx \sum_{x_c} \int_{x_c-\Delta x}^{x_c+\Delta x} dx\, e^{i\left(f(x_c) + \frac{1}{2}(x-x_c)^2 f''(x_c) + \cdots\right.}$$

$$\approx \sum_{x_c} \int_{x_c-\Delta x}^{x_c+\Delta x} dx\, e^{i\left(f(x_c) + \frac{1}{2}(x-x_c)^2 f''(x_c)\right)} \left(1 + \frac{1}{3}\right.$$

$$\approx \sum_{x_c} \int_{-\infty}^{\infty} dx\, e^{i f(x_c) + \frac{1}{2}(x-x_c)^2 f''(x_c)} \left(1 + \frac{1}{4!}(x-x\right.$$

(Semi-classical limit)

$$i\left(f(x_c) + \tfrac{1}{2}(x-x_c)^2 f''(x_c) + \cdots\right)/\hbar$$

$$\int dx\, e$$

$$e^{(\cdots)} = 1 + (\cdots) + \tfrac{1}{2}(\cdots)^2 + \cdots$$

Cubic + higher

$$\times e^{i\left(f(x_c) + \frac{1}{2}(x-x_c)^2 f''(x_c)\right)}\left(1 + \tfrac{1}{3!}(x-x_c)^3 f'''(x_c) + \tfrac{1}{4!}(x-x_c)^4 f''''(x_c) + \cdots\right)$$

$$i f(x_c) + \tfrac{1}{2}(x-x_c)^2 f''(x_c)\left(1 + \tfrac{1}{4!}(x-x_c)^4 f''''(x_c) + \cdots\right)$$

$$f(x) \to f(\vec{x})$$

$$(x-x_c)^2 f''(x_c) \to (\vec{x}-\vec{x}_c)\cdot f''(x_c)\cdot(\vec{x}-\vec{x}_c)$$

$$(x-x_c)_i \; \frac{\partial^2 f(x_c)}{\partial x_i \partial x_j} (x-x_c)_j$$

$\cdots)^2 + \cdots$

$(x-x_c)^4 f''''(x_c) + \cdots \Big)$

$$K \approx \sum_{q_c} \int \mathcal{D}q(t)\, e^{i(S[q_c] + \frac{1}{2}(q-q_c)\cdot S''[q_c]\cdot(q-q_c))/\hbar}\,(1$$

$$(q-q_c)\cdot S''[q_c]\cdot(q-q_c) = \int dt\, dt' (q(t)-q_c($$

Possibility: $I(x) = e^{-C/\hbar^2}$ ← non-perturbative function

$$\underset{\hbar \to 0}{\approx} 0 + 0\hbar + 0\hbar^2 + 0\hbar^3$$

Taylor series vanishes

$$e^{\frac{i}{\hbar}\left(S[q_c] + \frac{1}{2}(q-q_c)\cdot S''[q_c]\cdot(q-q_c)\right)\left(1+\cdots\right)}$$

$$(q-q_c)\cdot S''[q_c]\cdot(q-q_c) = \int dt\,dt' (q(t)-q_c(t)) \frac{\delta^2 S[q_c]}{\delta q(t)\,\delta q(t')} (q(t')-q_c(t'))$$

$$(x) = e^{-c/\hbar^2}$$ ← non-perturbative function

$$\simeq 0 + 0\hbar + 0\hbar^2 + 0\hbar^3$$

$\hbar \to 0$

Taylor series vanishes

$$\ldots, t_1 > \langle q_1, t_1 | q_1(t) | q_i, t_i \rangle$$

$$= \int_{q(t_1)=q_1}^{q(t_f)=q_f} \mathcal{D}q(t) \int dq_1 \; q_1(t) \, e^{iS[q]/\hbar}$$

$$S[q] = \left(\int_{t_i}^{t_1} + \int_{t_1}^{t_f} \right) L \, dt$$

Matrix Elements

$$\langle q_f, t_f | \hat{Q}(t) | q_i, t_i \rangle \qquad t_i < t < t_f$$

$$= \int dq_1 \langle q_f, t_f | q_1, t \rangle \langle q_1, t | q_1(t) | q_i, t_i \rangle$$

$$\mathbb{1} = \int dq_1 | q_1, t \rangle \langle q_1, t |$$

$$= \int dq_1 \int_{q(t_i)=q_i}^{q(t)=q_1} \mathcal{D}q(t') \int_{q(t)=q_1}^{q(t_f)=q_f} \mathcal{D}q(t') \; \ldots \; q_1(t) \, e$$

$$= \int_{q(t_i)=q_i}^{q(t_f)=q_f} \mathcal{D}q(t) \, e^{\frac{i}{\hbar} S[q]} \, q_1(t)$$

ements

$t_i < t < t_f$

$$|q_i; t_i\rangle = \int dq_1 \langle q_f, t_f | q_1, t\rangle \langle q_1, t | q_1(t) | q_i; t_i\rangle$$

$$\int dq_1 |q_1, t\rangle\langle q_1, t|$$

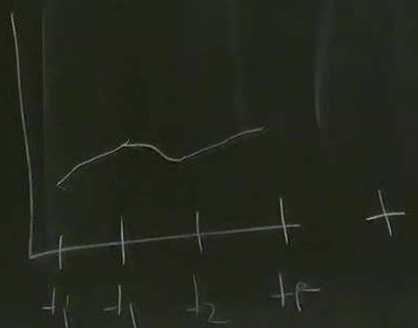
$$= \int dq_1 \int_{q(t_i)=q_i}^{q(t_f)=q_f} \mathcal{D}q(t) \int_{q(t)=q_1} \mathcal{D}q(t') \; q_1(t)\, e^{\frac{i}{\hbar}\left(\int_{t_i}^{t_1} + \int_{t_1}^{t_f}\right) L\, dt}$$

$$= \int_{q(t_i)=q_i}^{q(t_f)=q_f} \mathcal{D}q(t)\, e^{\frac{i}{\hbar} S[q]}\, q_1(t)$$

Works for products of operators

$$\langle q_f, t_f | T\hat{Q}(t_2)\,\hat{Q}(t_1) | q_i, t_i \rangle$$

↑ time-ordering symbol

$$= \int_{q(t_i)=q_i}^{q(t_f)=q_f} \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}\, q(t_2)\, q(t_1)$$

$$T\hat{Q}(t_1)\hat{Q}(t_2) = \begin{cases} \hat{Q}(t_1)\hat{Q}(t_2) & \text{if } t_1 > t_2 \\ \hat{Q}(t_2)\hat{Q}(t_1) & \text{if } t_2 > t_1 \end{cases}$$