Title: Lecture - Quantum Theory (Core), PHYS 605

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Subject: Particle Physics, Quantum Fields and Strings, Quantum Information

Date: September 25, 2026 - 1:00 PM

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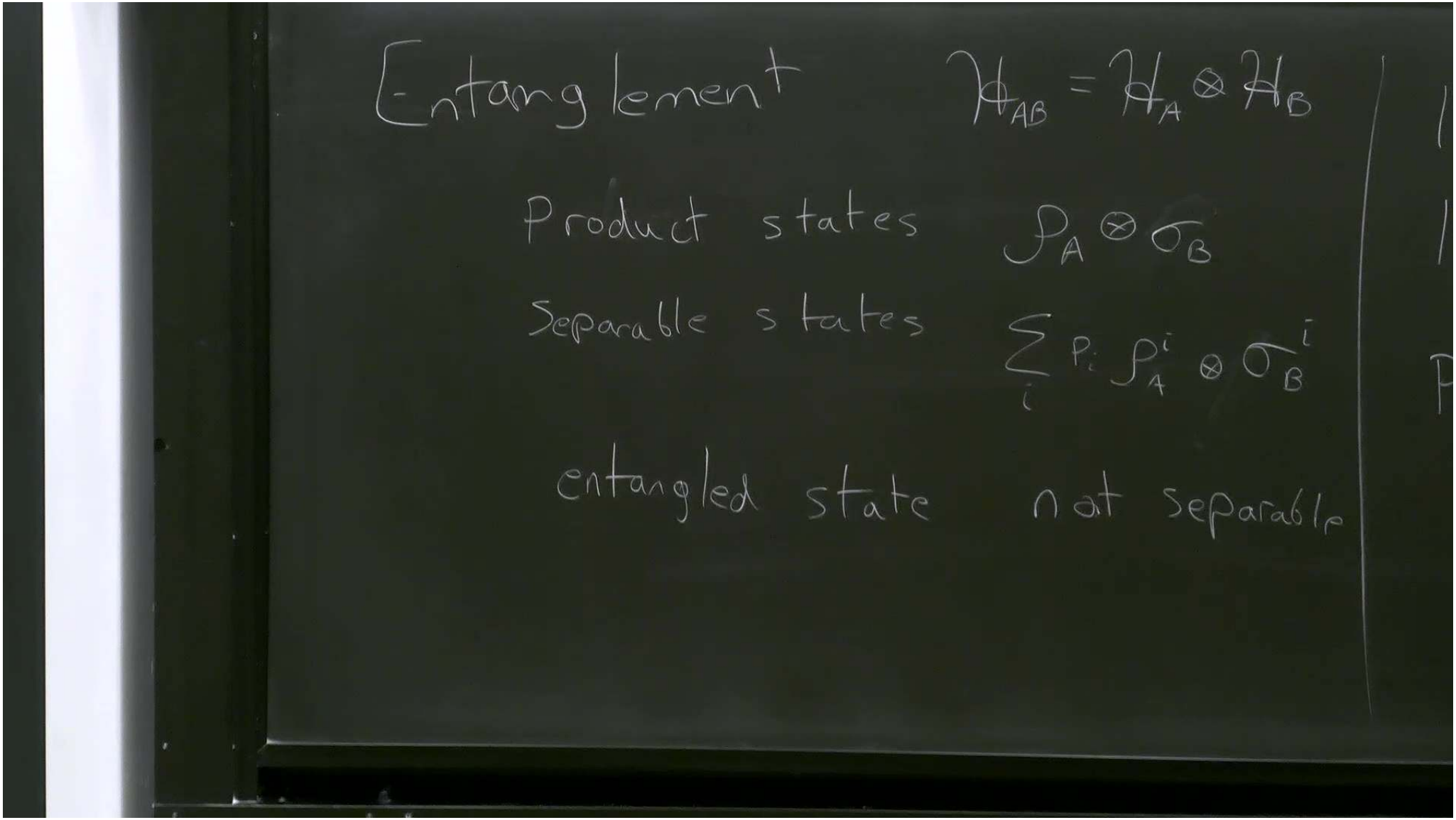

Entanglement
$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$
Product states
$\rho_A \otimes \sigma_B$
Separable states
$\sum_i p_i \rho_A^i \otimes \sigma_B^i$
entangled state
not separable

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

ON basis for $\mathbb{C}^2 \otimes \mathbb{C}^2$

Pure states: $|\psi\rangle\langle\psi|$ is separable iff it has Schmidt rank 1

Given ρ_{AB}, how do you describe system A?

$\mathbb{C}^2$

(classically $(x_1, x_2, p_1, p_2) \longrightarrow (x_1, p_1)$

answer: partial trace

$\rho_A := \mathrm{Tr}_B(\rho_{AB}) \in \mathcal{L}(\mathcal{H}_A)$ $\quad \mathrm{Tr}_A(E_k^A \rho^A) = \mathrm{Tr}_{AB}(E_k^A \otimes \mathbb{I}^B \rho^{AB})$

marginal / reduced

$\sum_a p(a,b) = p(b)$

$$\rho_A = Tr_B\left[|\phi^+\rangle\langle\phi^+|_{AB}\right] = Tr_B\left[\frac{1}{2}\left(|00\rangle + |11\rangle\right)\left(\langle 00| + \langle 11|\right)\right]$$

$$= \frac{1}{2} Tr_B\left[|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right]$$

$$\rho_B = \mathbb{I}_B/2$$

$$= \frac{1}{2}|0\rangle\langle 0|_A + \frac{1}{2}|1\rangle\langle 1|_A = \mathbb{I}_A/2$$

$$Tr_B\left[|0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B\right] \equiv \sum_i |0\rangle\langle 0|_A \otimes \langle i|0\rangle\langle 0|i\rangle_B$$

$$= \sum_i |0\rangle\langle 0|_A \otimes \langle 0|i\rangle\langle i|0\rangle = |0\rangle\langle 0|_A$$

$$\sum_a p(a,b) = p(b)$$

(X_1, P_1), (X_2, P_2) $\xrightarrow{\text{Cartesian Product}}$ (X_1, P_1, X_2, P_2)

$$\rho_A \otimes \rho_B = \mathbb{I}_{A/2} \otimes \frac{\mathbb{I}_B}{2} = \mathbb{I}_{AB/4} = \frac{1}{4}\begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \neq |\phi^+\rangle\langle\phi^+| = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

(pure state; 00 01 10 11)

...ically, say you have the state (of knowledge)

$$P(c_1, c_2) \quad \frac{1}{2}\delta_{c_1,H}\delta_{c_2,H} + \frac{1}{2}\delta_{c_1,T}\delta_{c_2,T}$$

$$P(c_1) = \sum_{c_2} P(c_1, c_2) = \frac{1}{2}$$

$$P(c_2) = \tfrac{1}{2}$$

$$\sum_a p(a,b) = p(b)$$

$$\rho_A \otimes \rho_B = \frac{\mathbb{I}_A}{2} \otimes \frac{\mathbb{I}_B}{2} = \frac{\mathbb{I}_{AB}}{4} = \frac{1}{4}\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \neq |\phi^+\rangle\langle\phi^+| = \frac{1}{2}\begin{bmatrix} 1 & & & 1 \\ & 0 & & \\ & & 0 & \\ 1 & & & 1 \end{bmatrix}$$

(00 01 10 11)

pure state

Classically, say you have the state (of knowledge)

$$P(c_1,c_2) \quad \frac{1}{2}\delta_{c_1,H}\delta_{c_2,H} + \frac{1}{2}\delta_{c_1,T}\delta_{c_2,T}$$

$$P(c_1) := \sum_{c_2} P(c_1,c_2) = \frac{1}{2}$$

$$P(c_2) = \frac{1}{2}$$

Useful as a resource. e.g. Superdense coding

$|\phi^+\rangle\langle\phi^+|_{AB}$

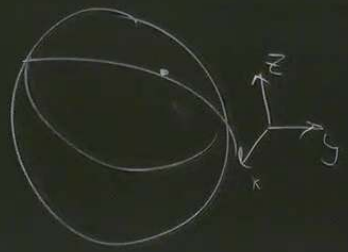
$A \quad \mathbb{C}^2 \longrightarrow B$

Alice does $\sigma_{b_1,b_2} \in \{\mathbb{I}, Z, X, Y\}$ on her half of $|\phi^+\rangle\langle\phi^+|_{AB}$

want to send two bits (b_1, b_2)

Quantum tomography: How do you experimentally characterize an unknown quantum state?
(given many copies of density operator)

$$\rho = \frac{1}{2}\left(\mathbb{I} + s_1 X + s_2 Y + s_3 Z\right) \qquad s_i = \overbrace{Tr\left[\rho \sigma_i\right]}^{\text{directly observable}}$$



1) measure X
2) measure Y
3) " Z

Two qubit

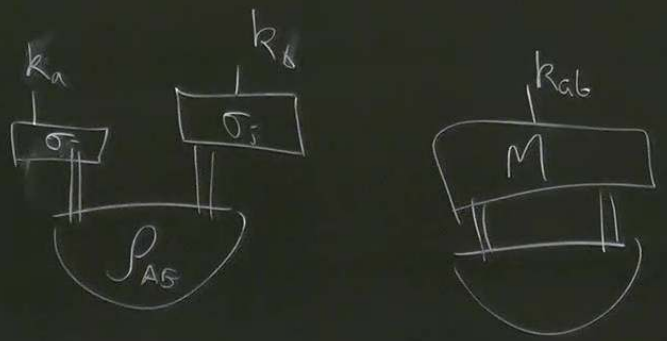
want to send two bits (b_1, b_2)

Two qubits, $\{\sigma_i \otimes \sigma_j\}_{i,j=0,1,2,3}$

$$\rho_{AB} = \frac{1}{4} \sum_{i,j=0}^{3} t_{ij}\, \sigma_i \otimes \sigma_j$$

$$t_{ij} := \overline{Tr\left[\rho_{AB}\, \sigma_i \otimes \sigma_j\right]}$$

measurable... even by local/independent measurements!

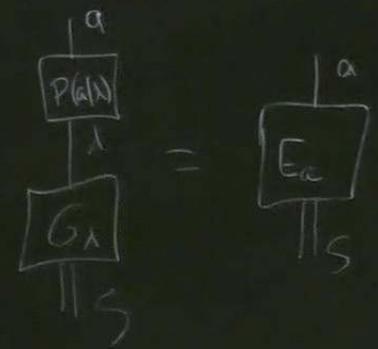


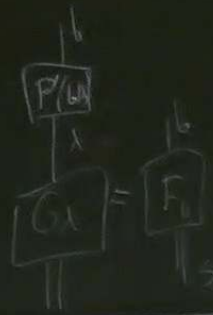
Incompatibility / joint measurability

Defn Two POVMs $\{E_a\}_a$ and $\{F_b\}_b$ are jointly measurable (or compatible) if there exists a POVM $\{G_\lambda\}_\lambda$ such that

$$E_a = \sum_\lambda p(a|\lambda)\, G_\lambda \quad \forall a$$

$$F_b = \sum_\lambda p'(b|\lambda)\, G_\lambda$$

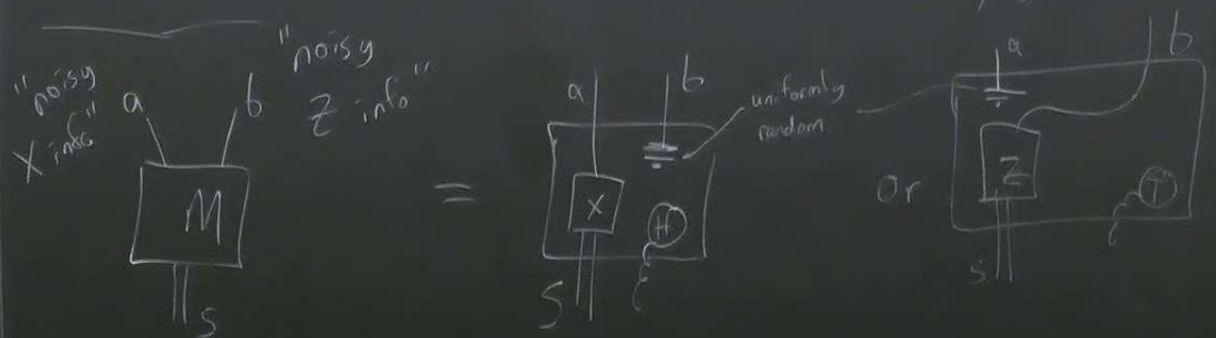




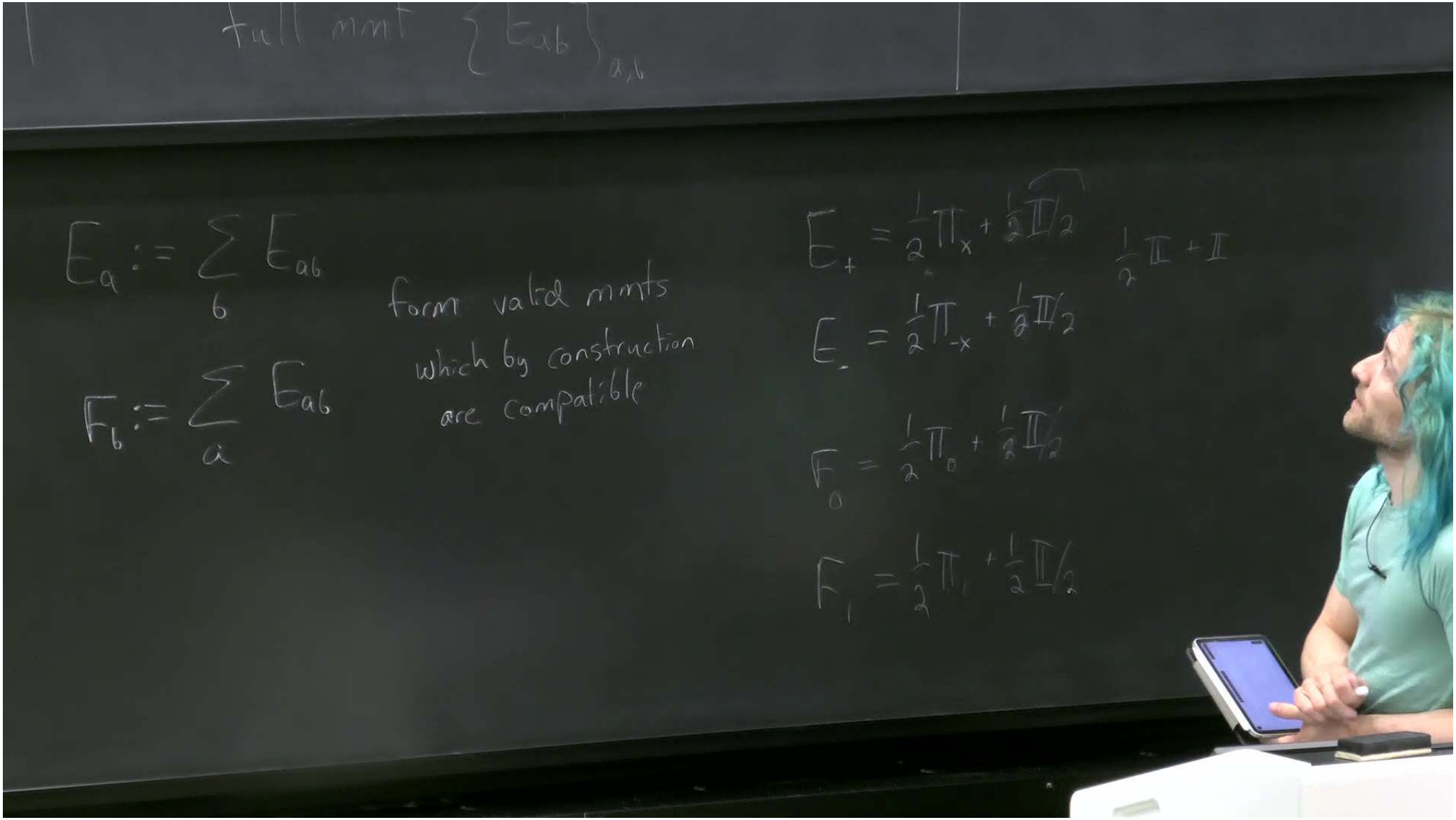
For Projective mmts, compatibility $\iff$ commutativity

For POVMs, compatibility $\Longleftarrow$ commutativity

$\not\Longrightarrow$



full mmt $\{E_{a,b}\}_{a,b}$


full mmt $\{E_{ab}\}_{a,b}$
$E_a := \sum_b E_{ab}$
$F_b := \sum_a E_{ab}$
form valid mmts
which by construction
are compatible
$E_+ = \frac{1}{2}\Pi_x + \frac{1}{2}\mathbb{I}/2$
$\frac{1}{2}\mathbb{I} + \mathbb{I}$
$E_- = \frac{1}{2}\Pi_{-x} + \frac{1}{2}\mathbb{I}/2$
$F_0 = \frac{1}{2}\Pi_0 + \frac{1}{2}\mathbb{I}/2$
$F_1 = \frac{1}{2}\Pi_1 + \frac{1}{2}\mathbb{I}/2$