Title: Lecture - Quantum Theory (Core), PHYS 605

Speakers: David Schmid

Collection/Series: Quantum Theory (Core), PHYS 605, September 9 - October 23, 2026

Subject: Particle Physics, Quantum Fields and Strings, Quantum Information

Date: September 11, 2026 - 1:00 PM

URL: https://pirsa.org/26090011

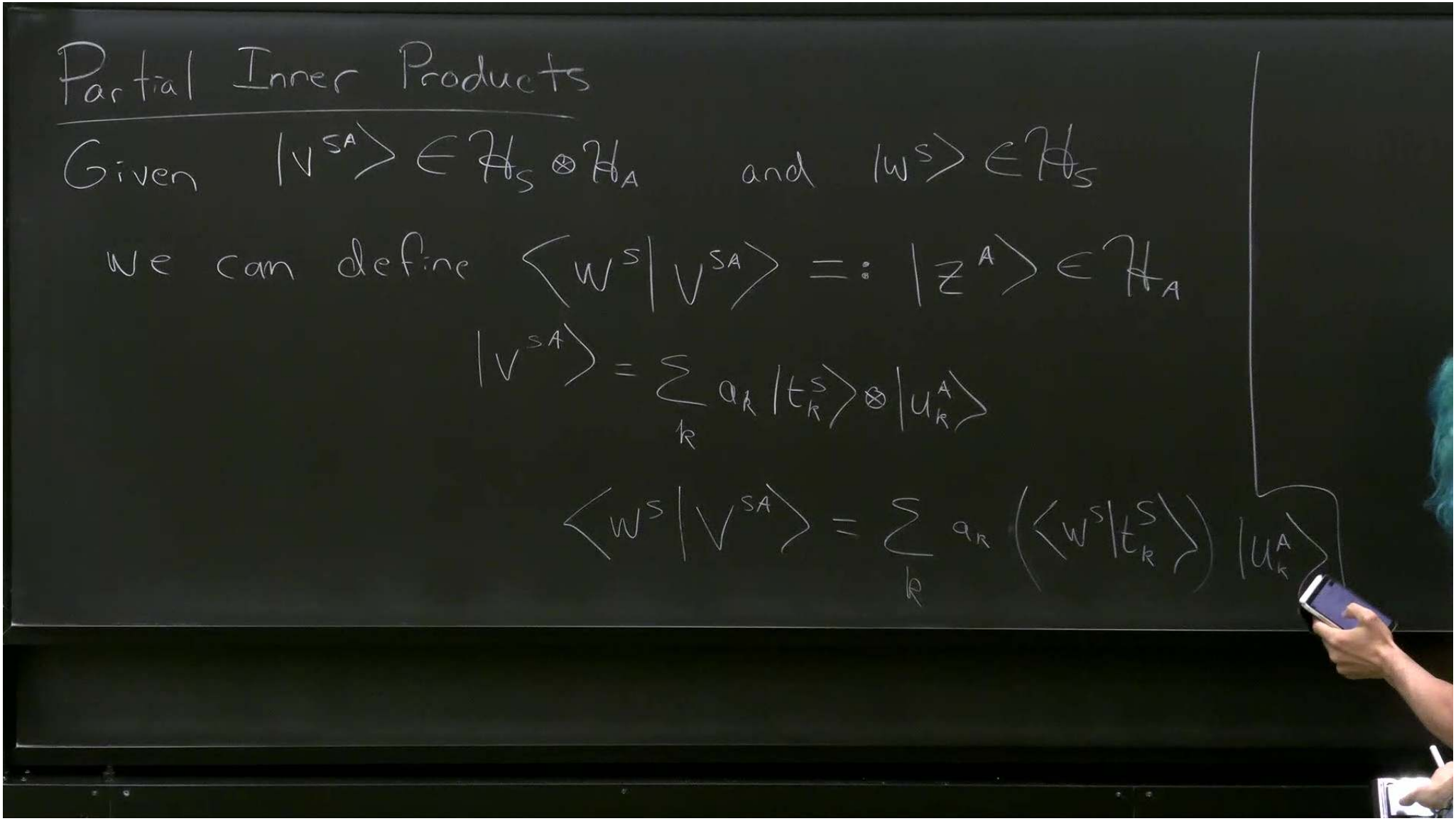
Partial Inner Products

Given $|v^{SA}\rangle \in \mathcal{H}_S \otimes \mathcal{H}_A$ and $|w^S\rangle \in \mathcal{H}_S$

we can define $\langle w^S | v^{SA} \rangle =: |z^A\rangle \in \mathcal{H}_A$

$$|v^{SA}\rangle = \sum_k a_k |t_k^S\rangle \otimes |u_k^A\rangle$$

$$\langle w^S | v^{SA} \rangle = \sum_k a_k \left(\langle w^S | t_k^S \rangle \right) |u_k^A\rangle$$



Partial Trace

$$\mathrm{Tr}_S\left[B^{SA}\right] := \sum_k \langle k^S| B^{SA} |k^S\rangle \quad \text{where } \{|k^S\rangle\}_k \text{ is an ON basis for } S$$

Schmidt Decomposition:
any (length-one) $|v\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$
can be written $\sum_i \sqrt{p_i}\,|i\rangle_A \otimes |i\rangle_B$

where $\{|i\rangle_A\}_i$, $\{|i\rangle_B\}_i$ are ON bases

and $\{p_i\}_i$ is a probability distribution

$|U_k^A\rangle$

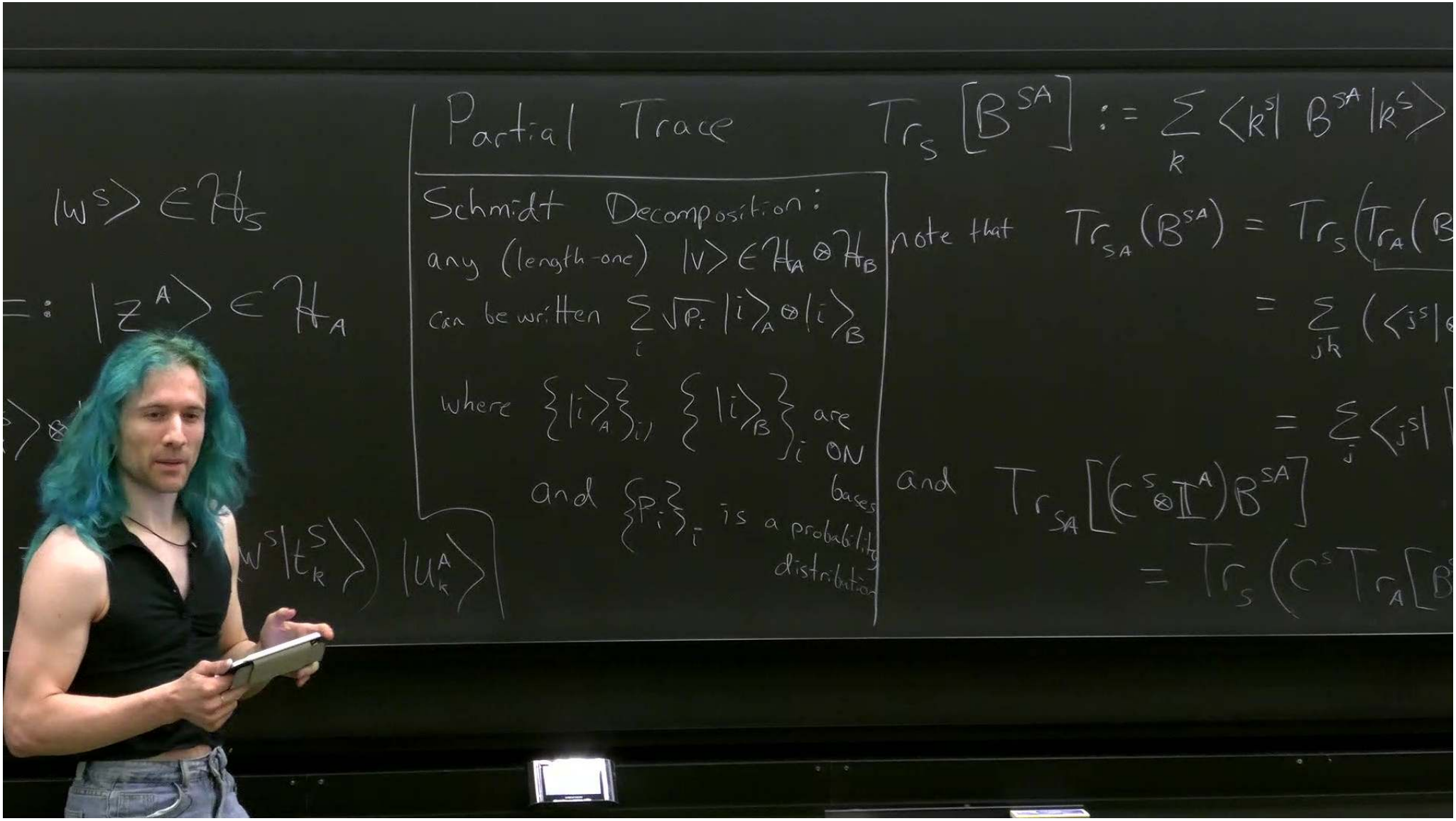
note that $\mathrm{Tr}_{SA}(B^{SA}) = \mathrm{Tr}_S\left(\mathrm{Tr}_A(B^{SA})\right)$

$$= \sum_{jk} \left(\langle j^S| \otimes \langle k^A|\right) B^{SA} \left(|j^S\rangle \otimes |k^A\rangle\right)$$

$$= \sum_j \langle j^S| \left[\sum_k \langle k^A| B^{SA} |k^A\rangle\right] |j^S\rangle$$

and $\mathrm{Tr}_{SA}\left[\left(C^S \otimes \mathbb{I}^A\right) B^{SA}\right]$

$$= \mathrm{Tr}_S\left(C^S \mathrm{Tr}_A\left[B^{SA}\right]\right)$$


Partial Trace
$\mathrm{Tr}_S[B^{SA}] := \sum_k \langle k^S| B^{SA} |k^S\rangle$
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$= \sum_{jk} (\langle j^S|$
$= \sum_j \langle j^S|$
and $\mathrm{Tr}_{SA}[(C^S \otimes \mathbb{I}^A) B^{SA}]$
$= \mathrm{Tr}_S(C^S \mathrm{Tr}_A[B$
$|w^S\rangle \in \mathcal{H}_S$
$=: |z^A\rangle \in \mathcal{H}_A$
$|w^S|t_k^S\rangle) |u_k^A\rangle$

Textbook QT Postulates

State Space Postulate: Associated to each system is a Hilbert space $\mathcal{H}$, The "state" of the system is described by its state vector $|\psi\rangle$, a unit vector in $\mathcal{H}$

- states differing by a global phase are physically equivalent
- in most settings, any unit vector in $\mathcal{H}$ corresponds to a physical state

ce $\mathcal{H}$,

unit vector in $\mathcal{H}$

state

Evolution Postulate: Closed system evolutions are described by unitary operators on $\mathcal{H}$. $|\psi(t)\rangle = U(t)|\psi(0)\rangle$

- For continuous time evolution, $i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$ where H is Hermitian

$$|\psi(t_2)\rangle = e^{-iH(t_2-t_1)/\hbar}|\psi(t_1)\rangle$$

Textbook QT Postulates

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$\frac{1+7i}{N}$ $\frac{23+0.5^{-}}{N'}$ (X, P)

State Space Postulate: A quantum system is a Hilbert space $\mathcal{H}$.

The "state" of the system is described by its state vector $|\psi\rangle$, a unit vector in $\mathcal{H}$

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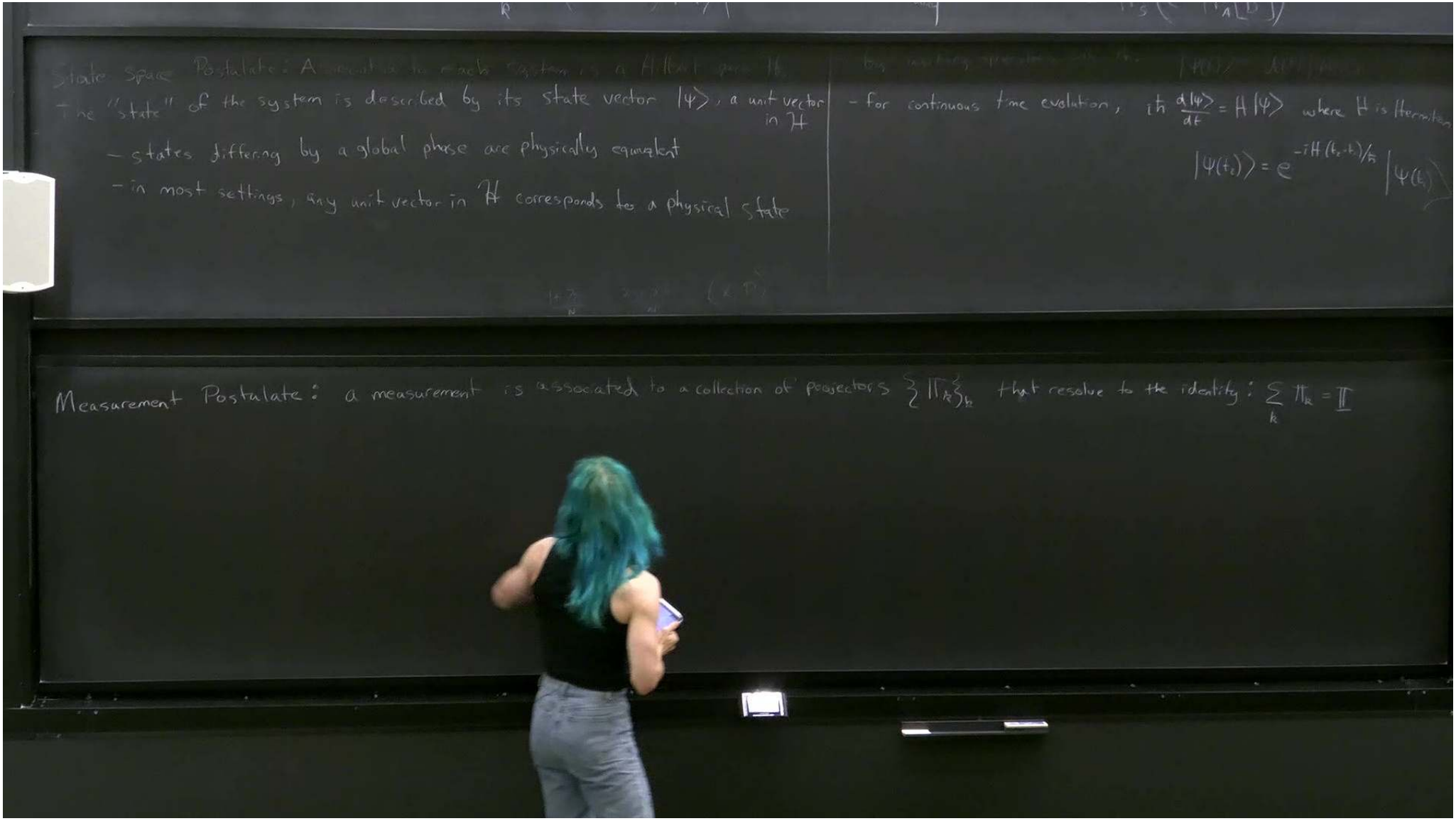
- for continuous time evolution, $i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$ where H is Hermitian

$$|\psi(t_2)\rangle = e^{-iH(t_2-t)/\hbar} |\psi(t)\rangle$$

Measurement Postulate: a measurement is associated to a collection of projectors $\{\Pi_k\}_k$ that resolve to the identity: $\sum_k \Pi_k = \mathbb{I}$



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by unitary operators ... $|\psi(t)\rangle$

- for continuous time evolution, $i\hbar \frac{d|\psi\rangle}{dt} =$

$|\psi($

$\frac{1+\mathbb{Z}}{N}$ $\frac{2\pi\mathbb{Q}}{N}$ (X, P)

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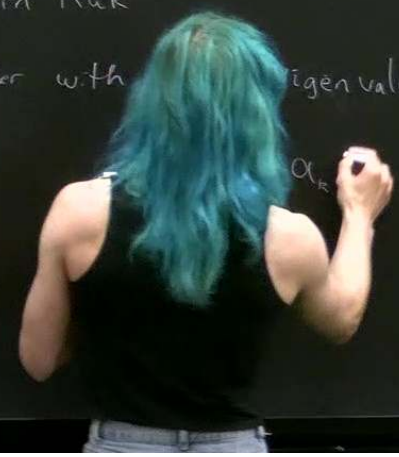
$$P(k|\psi) = Tr[|\psi\rangle\langle\psi| \Pi_k] = Tr[\langle\psi|\Pi_k|\psi\rangle] = \langle\psi|\Pi_k|\psi\rangle$$ Born Ruk

Side note: any "observable" ≡ Hermitian operator is equivalent to a set of projectors together with ... igenvalues

a_k



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$\varphi\rangle] = \langle\psi|\Pi_k|\varphi\rangle$ Born Ruk

set of projectors together with some eigenvalues

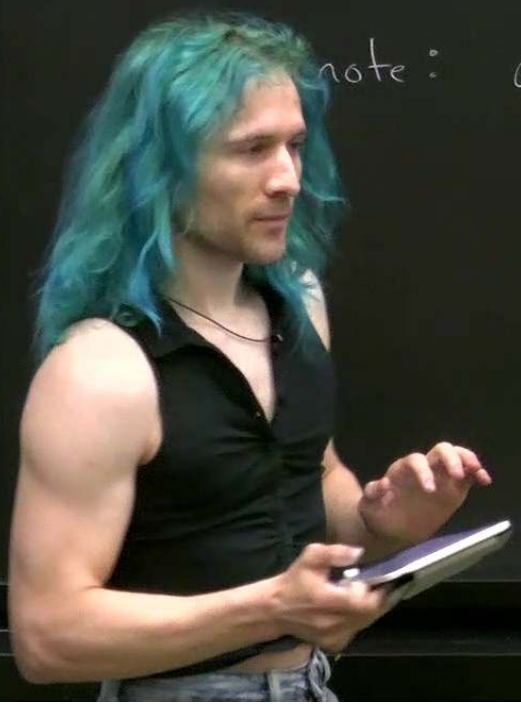
$$A = \sum_\alpha \alpha_k \Pi_k$$

conventional

Measurement Postulate: a measurement is associated to a collection

$$P(k|\psi) = Tr\left[|\psi\rangle\langle\psi| \Pi_k\right] = Tr\left[\langle\psi|\Pi_k|\psi\rangle\right]$$

note: any "observable" ≡ ~~Hermitian~~ Normal operator is equivalent to a set



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$\varphi\rangle] = \langle\varphi|\Pi_k|\varphi\rangle$ Born Rule

t of projectors together with some eigenvalues

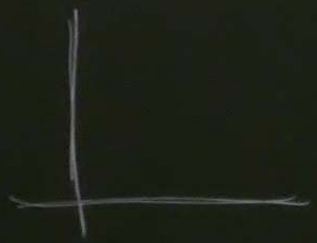
$$A = \sum_\alpha \alpha_k \Pi_k$$

conventional

to a collection of projectors $\{\Pi_k\}_k$ that resolve to the identity: $\sum_k \Pi_k = \mathbb{I}$

$\mathrm{Tr}[\langle\psi|\Pi_k|\psi\rangle] = \langle\psi|\Pi_k|\psi\rangle$ Born Rule

a set of projectors together with some eigenvalues



$$A = \sum_a a_k \Pi_k$$

(conventional)

$\frac{1+7i}{N}$ $\frac{23+0.6^{-}}{N'}$ (X, P)

a measurement is associated to a collection of projectors $\{\Pi_k\}_k$ that

$$P(k|\psi) = Tr[|\psi\rangle\langle\psi|\,\Pi_k] = Tr[\langle\psi|\Pi_k|\psi\rangle] = \langle\psi|\Pi_k|\psi\rangle \quad \text{Born Rule}$$

Normal

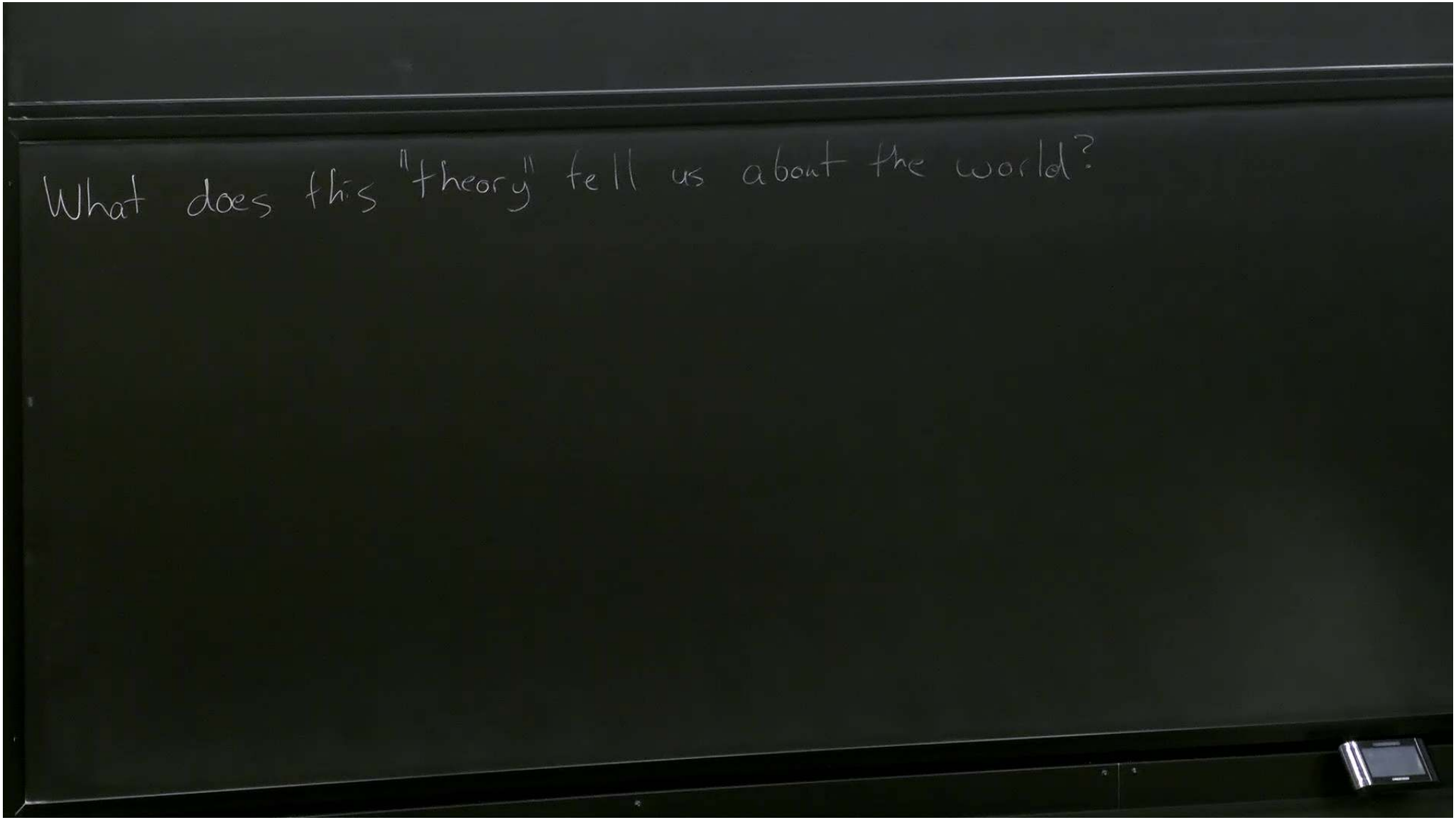
able" ≡ ~~Hermitian~~ operator is equivalent to a set of projectors together with so

A

Postulate: after a mmt, the evolution is not unitary, but rather

$$|\psi\rangle \longrightarrow \frac{\Pi_k|\psi\rangle}{\sqrt{P(k)}} \qquad \text{e.g. if } \Pi_k = |k\rangle\langle k|, \quad |\psi_{initial}\rangle \longrightarrow |k\rangle$$

"Collapse rule"


What does this "theory" tell us about the world?

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$\rho\rangle] = \langle\psi|\Pi_k|\psi\rangle$ Born Rule

set of projectors together with some eigenvalues

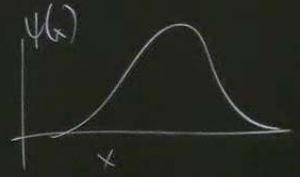
$$A = \sum_a a_k \Pi_k$$ (conventional)

not unitary, but rather

if $\Pi_k = |k\rangle\langle k|$, $|\psi_{initial}\rangle \longrightarrow |k\rangle$

"collapse rule"

$|X=4\rangle$



What does this "theory" tell us about the world?

e.g.

Orthodox/Textbook Interpretation

1) Projective measurements $\longleftrightarrow$ Properties e.g. $\{|x\rangle\langle x|\}_x$ $\longleftrightarrow$ Positions

2) eigenstate-eigenvalue link: a state has a definite value for a property iff it is an eigenstate of the associated projector

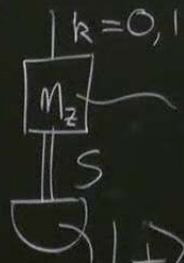
e.g. $|X=0\rangle$

ns

projector

A (version of) the measurement problem

Model 1)



$\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$

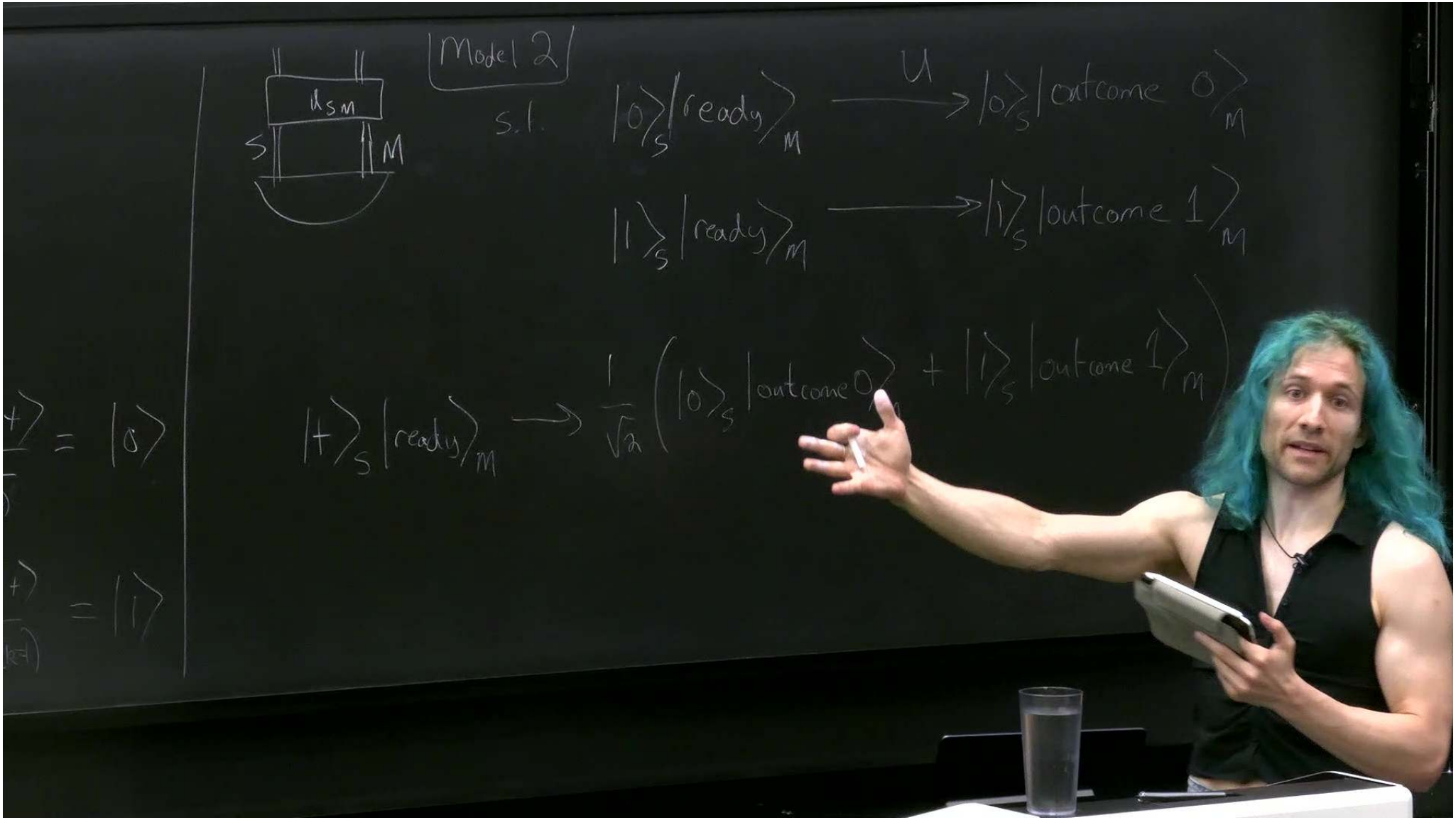
$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$k=0 \quad P(k=0) = \langle +|0\rangle\langle 0|+\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$|+\rangle \xrightarrow{k=0 \text{ mnt}} \frac{|0\rangle\langle 0|+\rangle}{\sqrt{P(0)}} = |0\rangle$

$k=1 \quad P(k=1) = \langle +|1\rangle\langle 1|+\rangle = \frac{1}{2}$

$|+\rangle \xrightarrow{k=1} \frac{|1\rangle\langle 1|+\rangle}{P(k=1)} = |1\rangle$


Model 2
S
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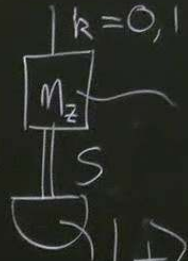
ready
outcome 0
outcome 1
U
ready
outcome 0
outcome 1

A (version of) the measurement problem

Model 1)



$\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$ $\longleftrightarrow$ "0-1" property

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$k=0$ $\quad P(k=0) = \langle +|0\rangle\langle 0|+\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

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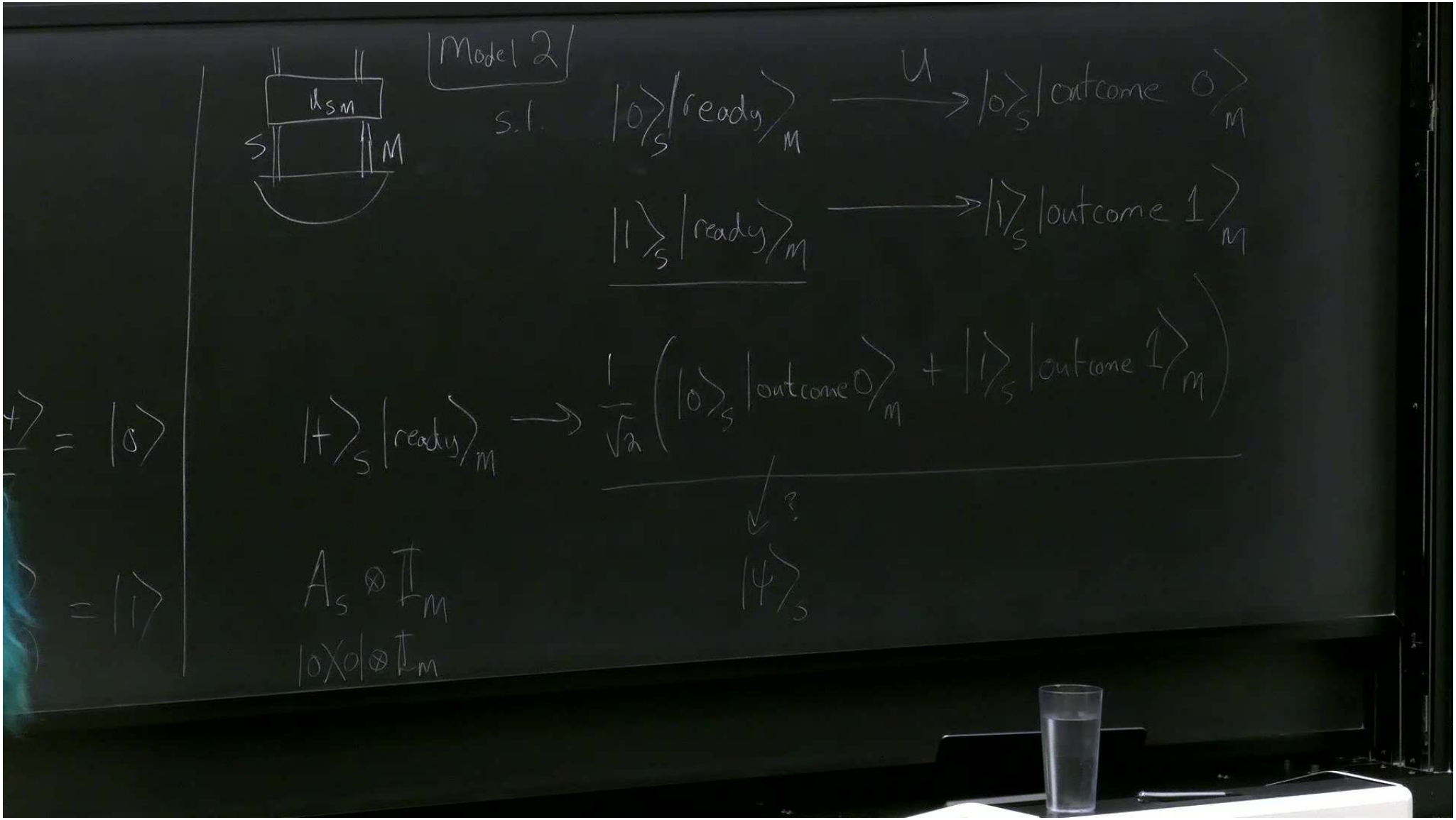
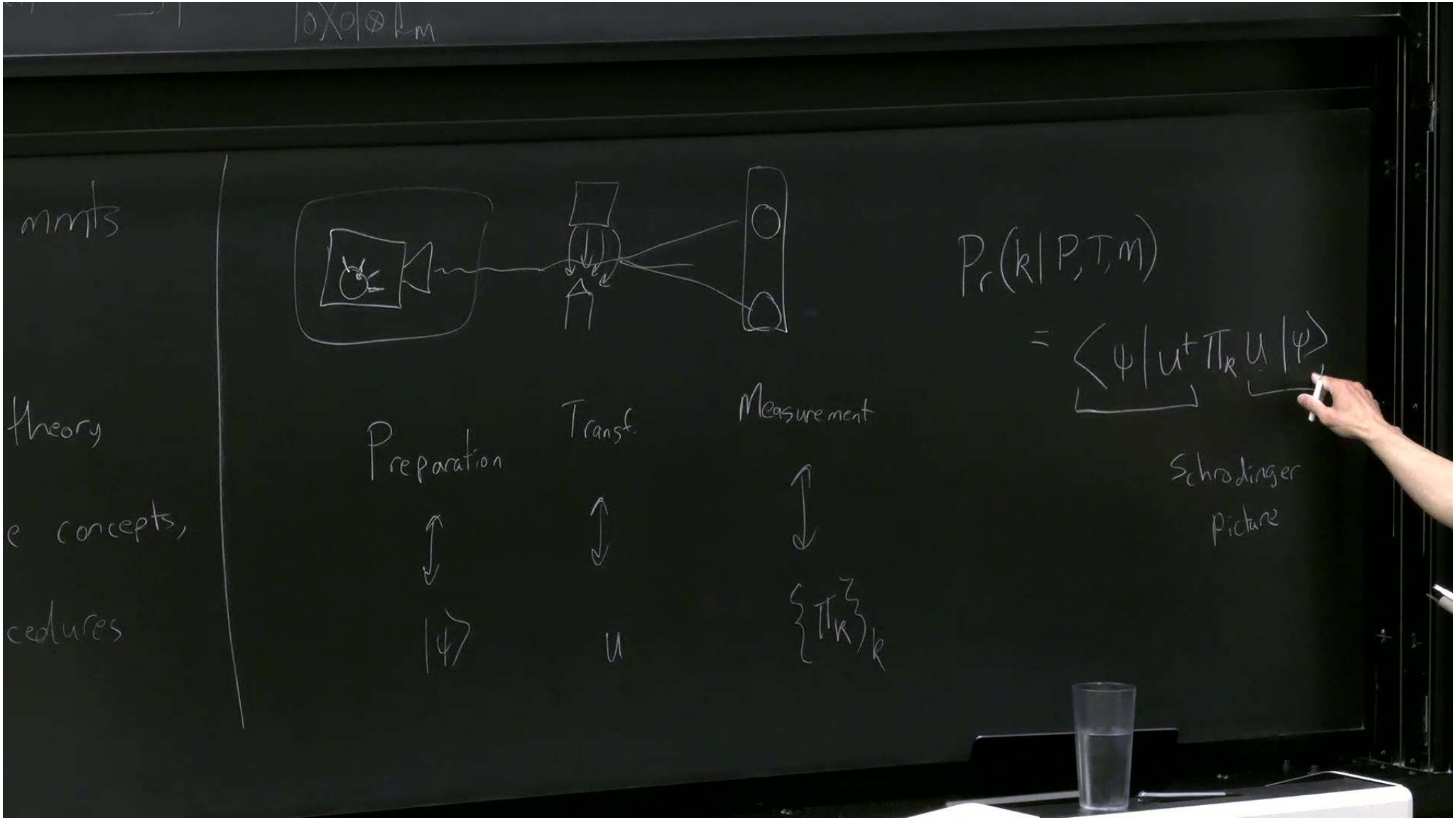

Model 2
U_{SM}
S
M
s.1.
$|0\rangle_S|\text{ready}\rangle_M \xrightarrow{U} |0\rangle_S|\text{outcome }0\rangle_M$
$|1\rangle_S|\text{ready}\rangle_M \longrightarrow |1\rangle_S|\text{outcome }1\rangle_M$
$|+\rangle_S|\text{ready}\rangle_M \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle_S|\text{outcome }0\rangle_M + |1\rangle_S|\text{outcome }1\rangle_M\right)$
$|+\rangle = |0\rangle$
$= |1\rangle$
$A_S \otimes I_M$
$|0\rangle\langle 0| \otimes I_M$
?
$|\psi\rangle_S$

$|\psi\rangle$ → simply a catalogue of possible probabilities for all possible mmts

Π_R → dictionary for reading out the probabilities

Operational QT Strategy: mathematical symbols in the theory refer only to directly observable concepts, in particular, laboratory procedures


mmts
theory
e concepts,
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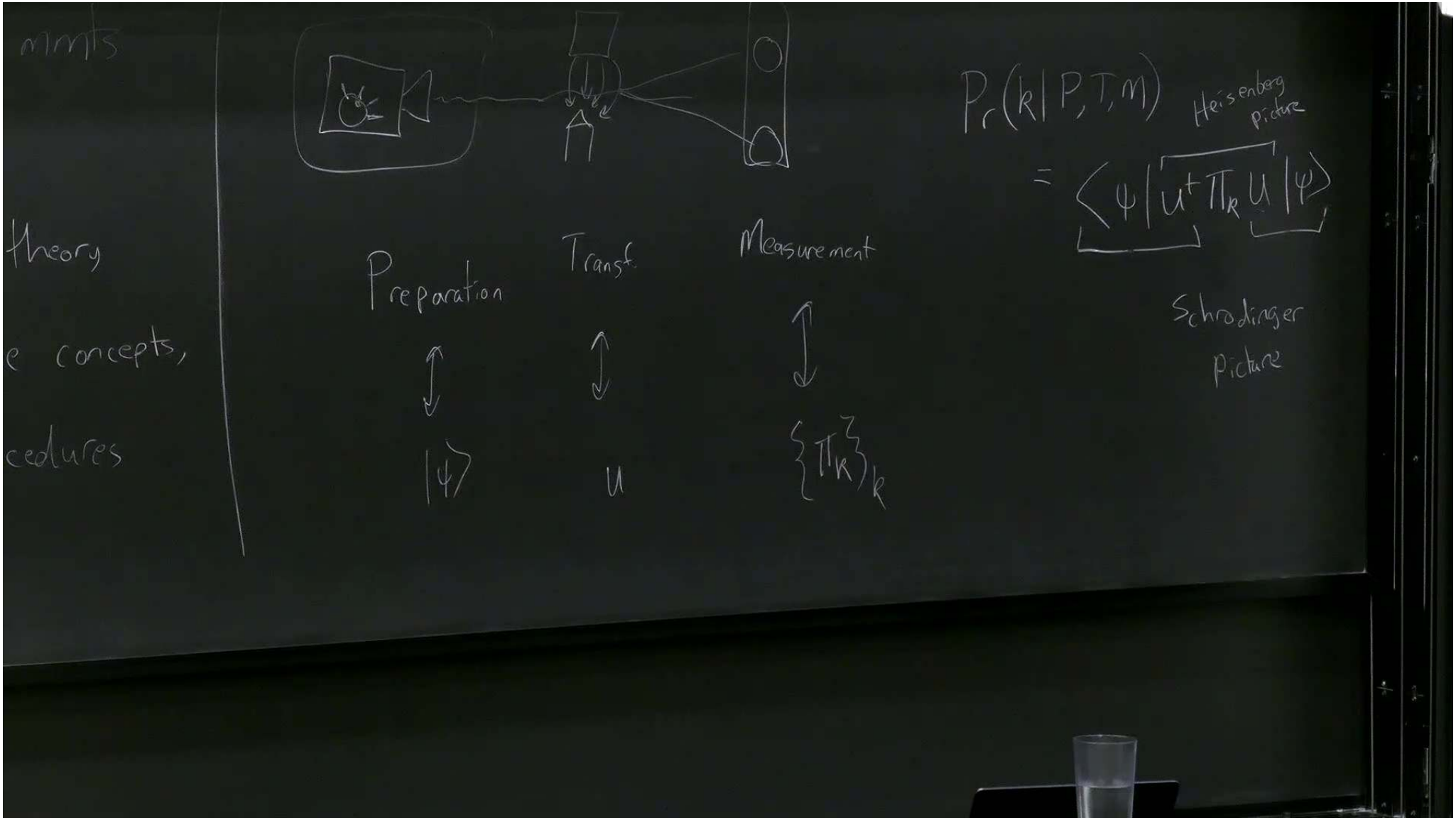
Preparation
Transf.
Measurement
$|\psi\rangle$
U
$\{\Pi_k\}_k$
$P_r(k|P,T,m)$
$= \langle\psi|U^\dagger \Pi_k U|\psi\rangle$
Schrodinger picture


nments
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Transf.
Measurement
$|\psi\rangle$
U
$\{\Pi_k\}_k$
$P_r(k|P,T,m)$
Heisenberg picture
$= \langle\psi|U^\dagger \Pi_k U|\psi\rangle$
Schrodinger picture

Probability of k given $M = \{\Pi_k\}_k$

We take $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ to be our

$$P_r(k|\vec{p}, M) = \sum_i P_r(\psi_i) P_r(k|P_{\psi_i}, M)$$

$$= \sum_i P_r(\psi_i) K(\psi_i | \Pi_k | \psi_i)$$

$$= \sum_i P_r(\psi_i) \, \mathrm{Tr}\left[|\psi_i\rangle\langle\psi_i| \, \Pi_k\right]$$

$$= \mathrm{Tr}\left[\underbrace{\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)}_{\rho} \Pi_k\right]$$

$\rightarrow \frac{\Pi_k |\psi\rangle}{\sqrt{P(k)}}$ e.g. if $\Pi_k = |k\rangle\langle k|$, $|\psi_{initial}\rangle \rightarrow |k\rangle$

"collapse rule"

Probability of k given $M = \{\Pi_k\}_k$

$$P_r(k|\rho, M) = \sum_i P_r(\psi_i) P_r(k|\rho_{\psi_i}, M)$$

$$= \sum_i P_r(\psi_i) \langle\psi_i|\Pi_k|\psi_i\rangle$$

$$= \sum_i P_r(\psi_i)\, Tr\left[|\psi_i\rangle\langle\psi_i|\,\Pi_k\right]$$

$$= Tr\left[\underbrace{\left(\sum_i P_i |\psi_i\rangle\langle\psi_i|\right)}_{\rho}\Pi_k\right]$$

We take $\rho = \sum_i P_i |\psi_i\rangle\langle\psi_i|$ to be our catalogue of predictions

$$Tr\left[|\psi\rangle\langle\psi|\,\Pi_k\right] \longrightarrow Tr\left[\rho\,\Pi_k\right]$$ Born rule

- $Tr\,\rho = Tr\left(\sum_i P_i |\psi_i\rangle\langle\psi_i|\right)$

$$= \sum_i P_i\, Tr\left[|\psi_i\rangle\langle\psi_i|\right] = \sum_i P_i\, Tr\left[\langle\psi_i|\psi_i\rangle\right] = 1$$

- $\langle\phi|\rho|\phi\rangle = \sum_i P_i \langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle = \sum_i P_i |c_i|^2 \geq 0$