Title: Dark Matter Theory

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Collection/Series: TRISEP 2026

Subject: Particle Physics

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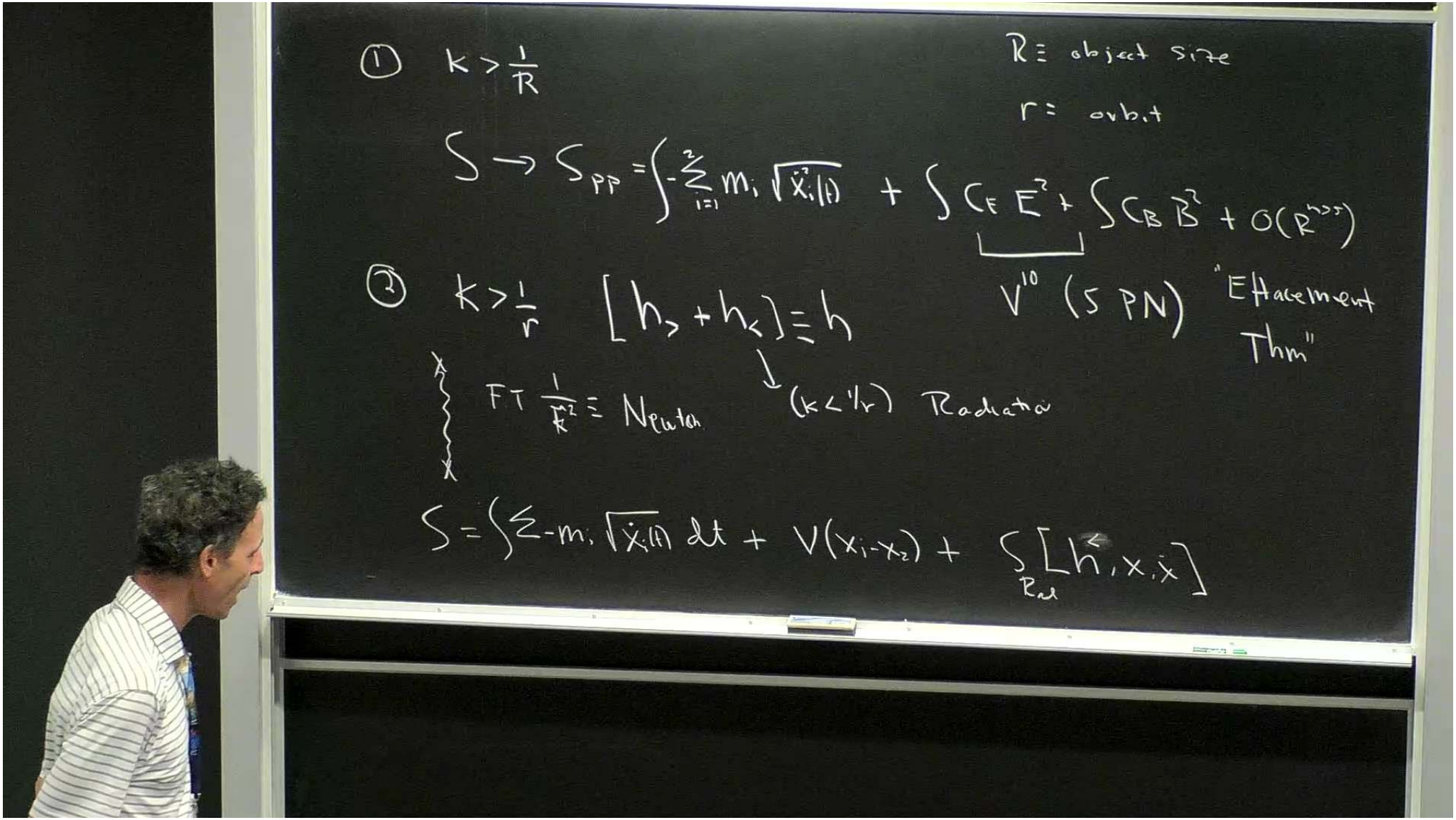

① $k > \frac{1}{R}$
R ≡ object size
r ≡ orbit
$S \to S_{PP} = \int -\sum_{i=1}^{2} m_i \sqrt{\dot{x}_i^2(t)} + \int C_E E^2 + \int C_B B^2 + O(R^{n>7})$
V^{10} (5 PN)
"Effacement Thm"
② $k > \frac{1}{r}$ $[h_> + h_<] \equiv h$
$FT \frac{1}{k^2} \equiv$ Newton
$(k < 1/r)$ Radiation
$S = \int \sum -m_i \sqrt{\dot{x}_i(t)}\, dt + V(x_1 - x_2) + \int_{Rad} [\vec{h}, x, \dot{x}]$

$$S = \int \sum -m_i \sqrt{\dot{x}_i(t)}\, dt + V(x_1 - x_2) + S_{Rad}[h^<, x, \dot{x}]$$

S_{Rad} $\quad k < \frac{1}{r}$ $\quad r$

$x \gg r$ $\quad [\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}]^{1/2} \to [1 + \frac{h_{00}}{M_{Pl}}]^{1/2}$

Rest from

(MASS)

QUADRAPOLE

$$S_{Rad} = -M \int \sqrt{\dot{x}^2}\, dt + \int E_{ij} Q_{ij}(x_1, x_2)\, dt$$

TOTAL MASS

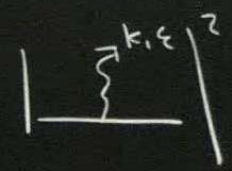
$\cancel{R}$, $\cancel{V_\mu V_\nu R^{\mu\nu}}$, $E^{\mu\nu}, B^{\mu\nu}$ $\quad (V \cdot E = V \cdot B = 0)$

$V^\mu V^\nu E_{\mu\nu}$

Calculate Power Loss:

$$iM = \langle 0 | \int Q(t) E \, dt \, | k \rangle$$

$$P \sim \sum_{p'} \int \frac{d^3k}{2k} |M|^2 k$$



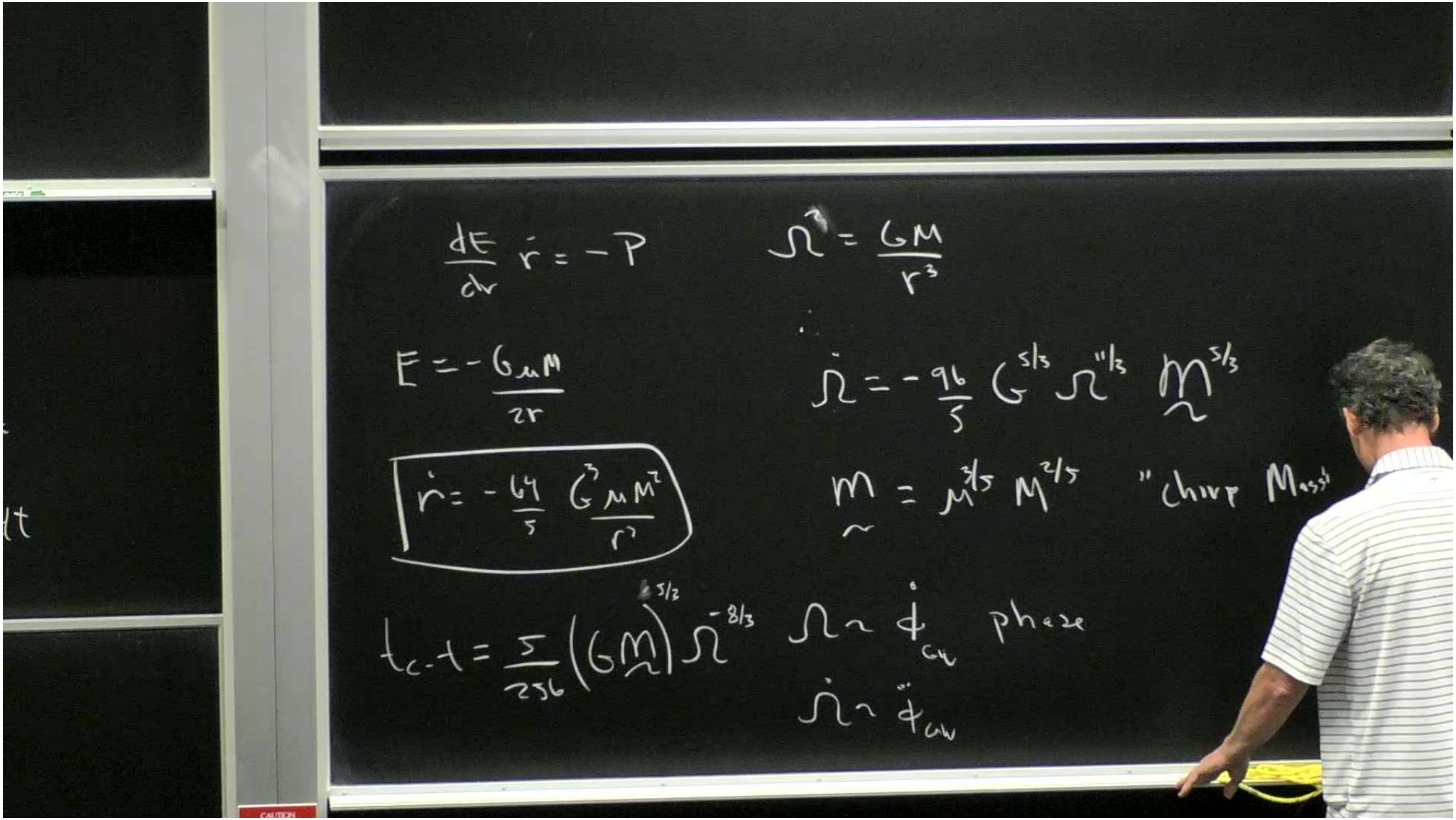
$$P = \frac{G}{5} \dddot{Q}_{ij} \dddot{Q}_{ij}$$ "Burke-Thorne"

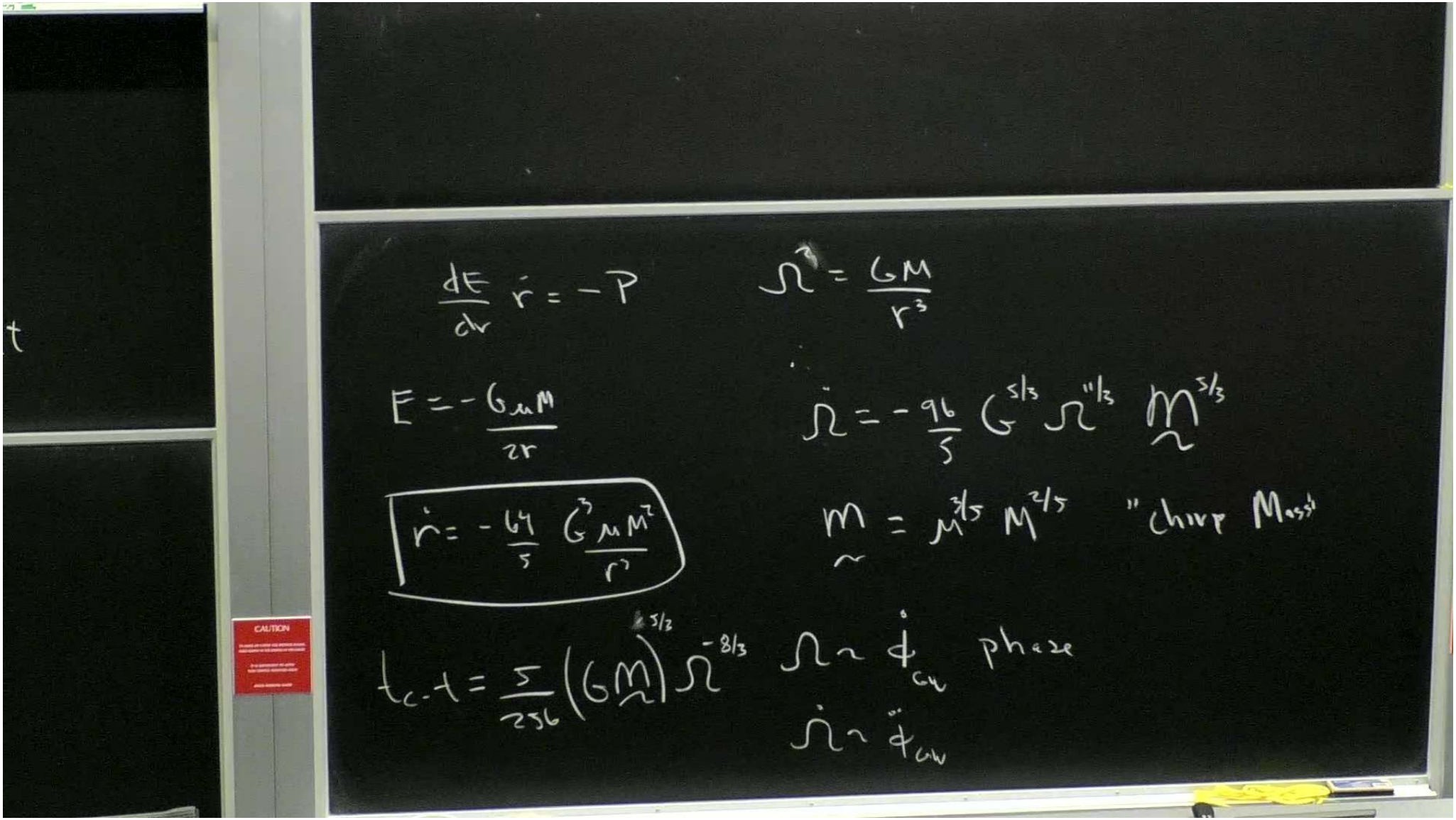
For circular orbit

$$P_{GW} = -\frac{32}{5} \frac{G^4 \mu^2 M^3}{r^5}$$

$$\mu = \frac{m_1 m_2}{M}$$

$$M = m_1 + m_2$$


$\frac{dE}{dr}\dot{r} = -P$
$\Omega^2 = \frac{GM}{r^3}$
$E = -\frac{G\mu M}{2r}$
$\dot{\Omega} = -\frac{96}{5} G^{5/3} \Omega^{11/3} \mathcal{M}^{5/3}$
$\dot{r} = -\frac{64}{5} \frac{G^3 \mu M^2}{r^3}$
$\mathcal{M} = \mu^{3/5} M^{2/5}$ "Chirp Mass"
$t_c - t = \frac{5}{256} (G\mathcal{M})^{-5/3} \Omega^{-8/3}$
$\Omega \sim \dot{\phi}_{GW}$ phase
$\dot{\Omega} \sim \ddot{\phi}_{GW}$


$\frac{dE}{dt}\dot{r} = -P$
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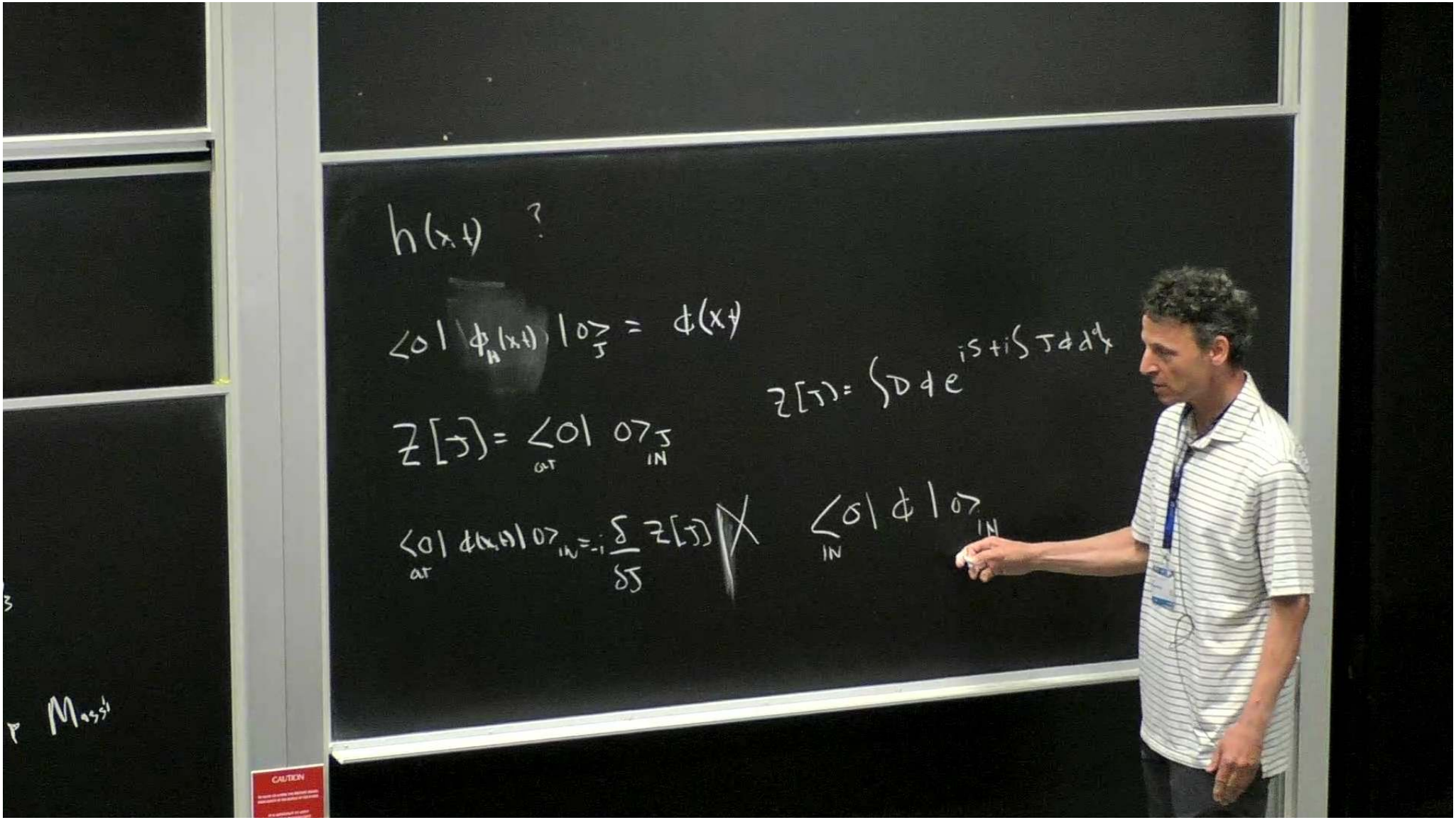
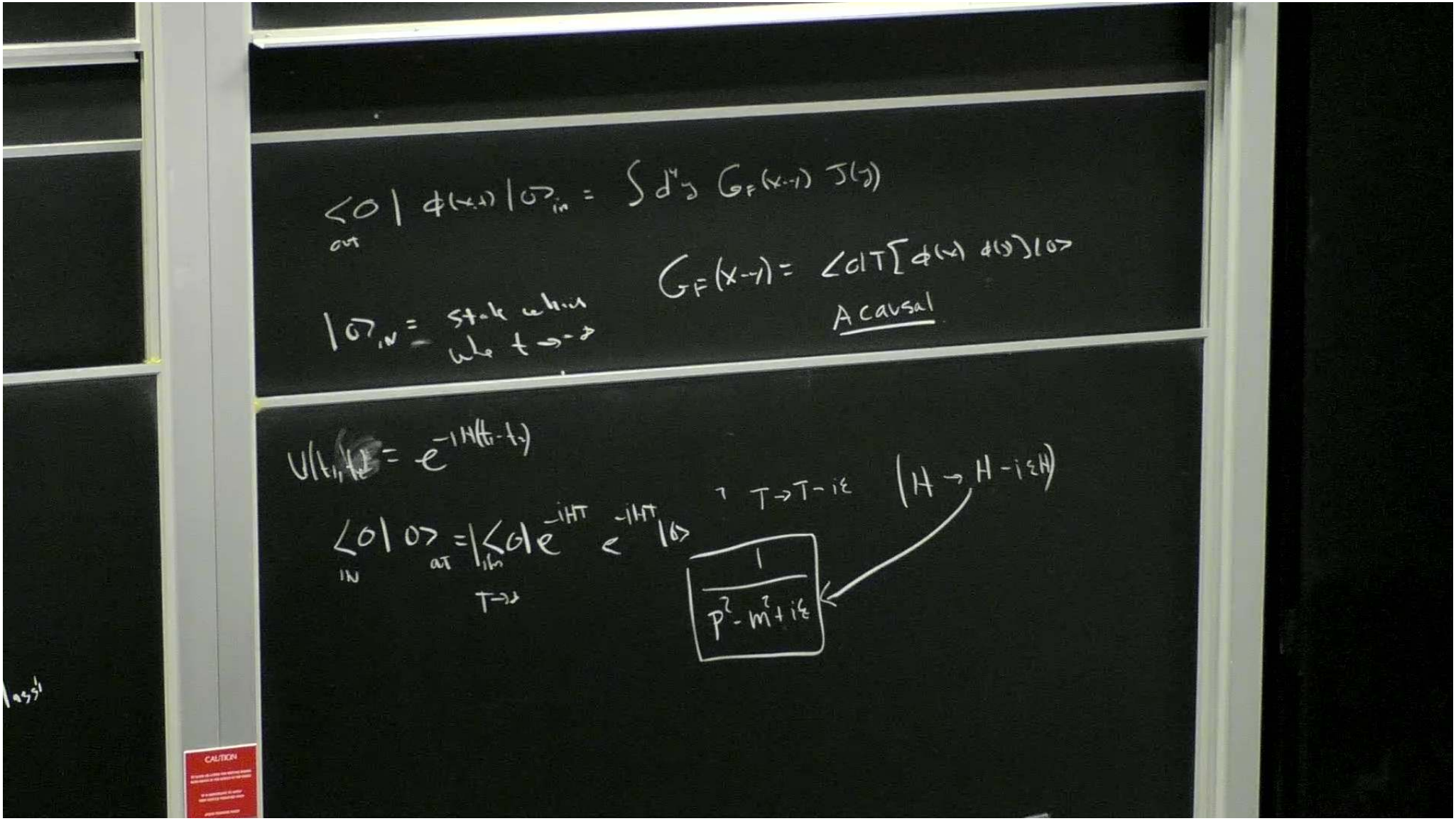
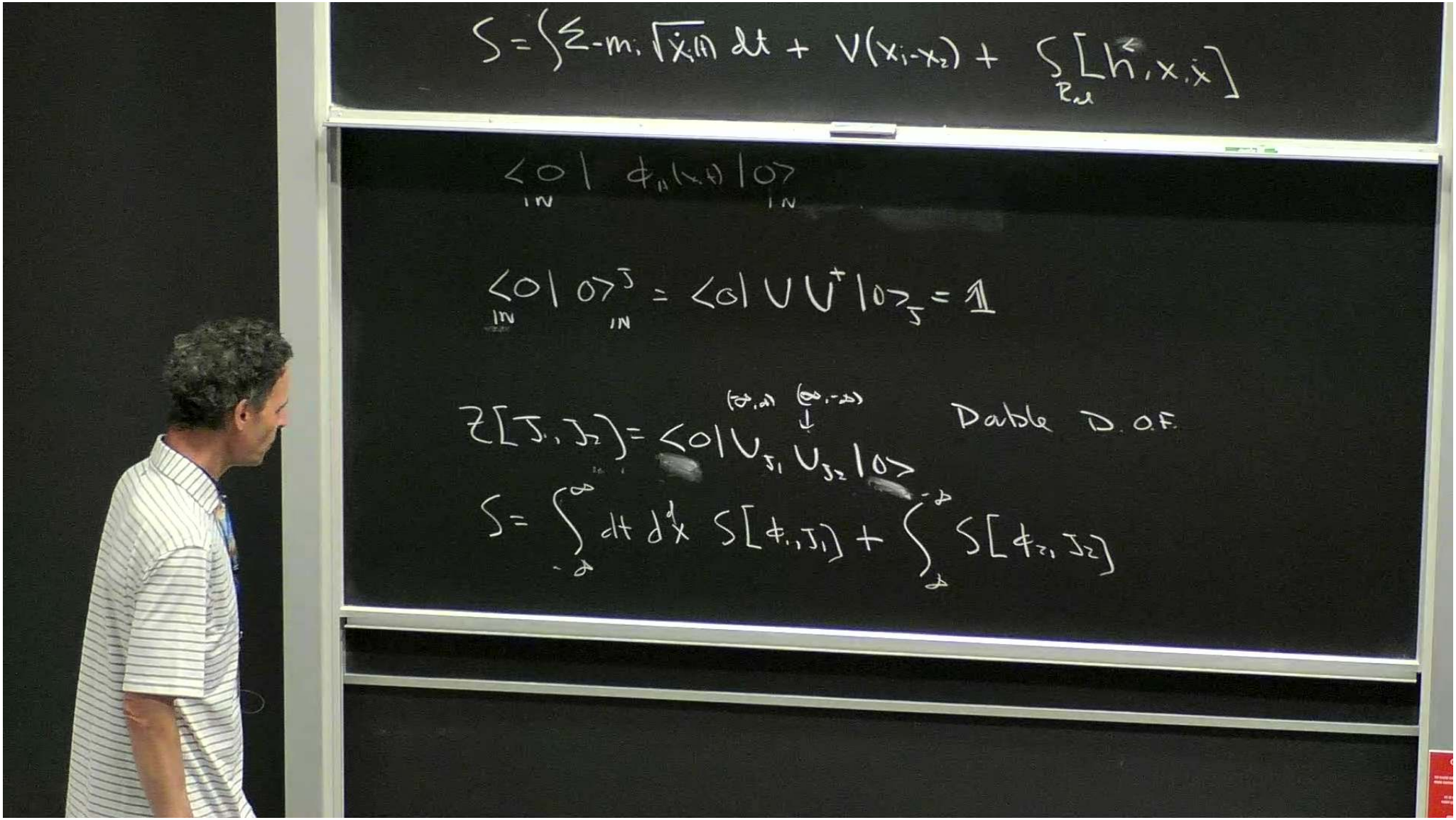
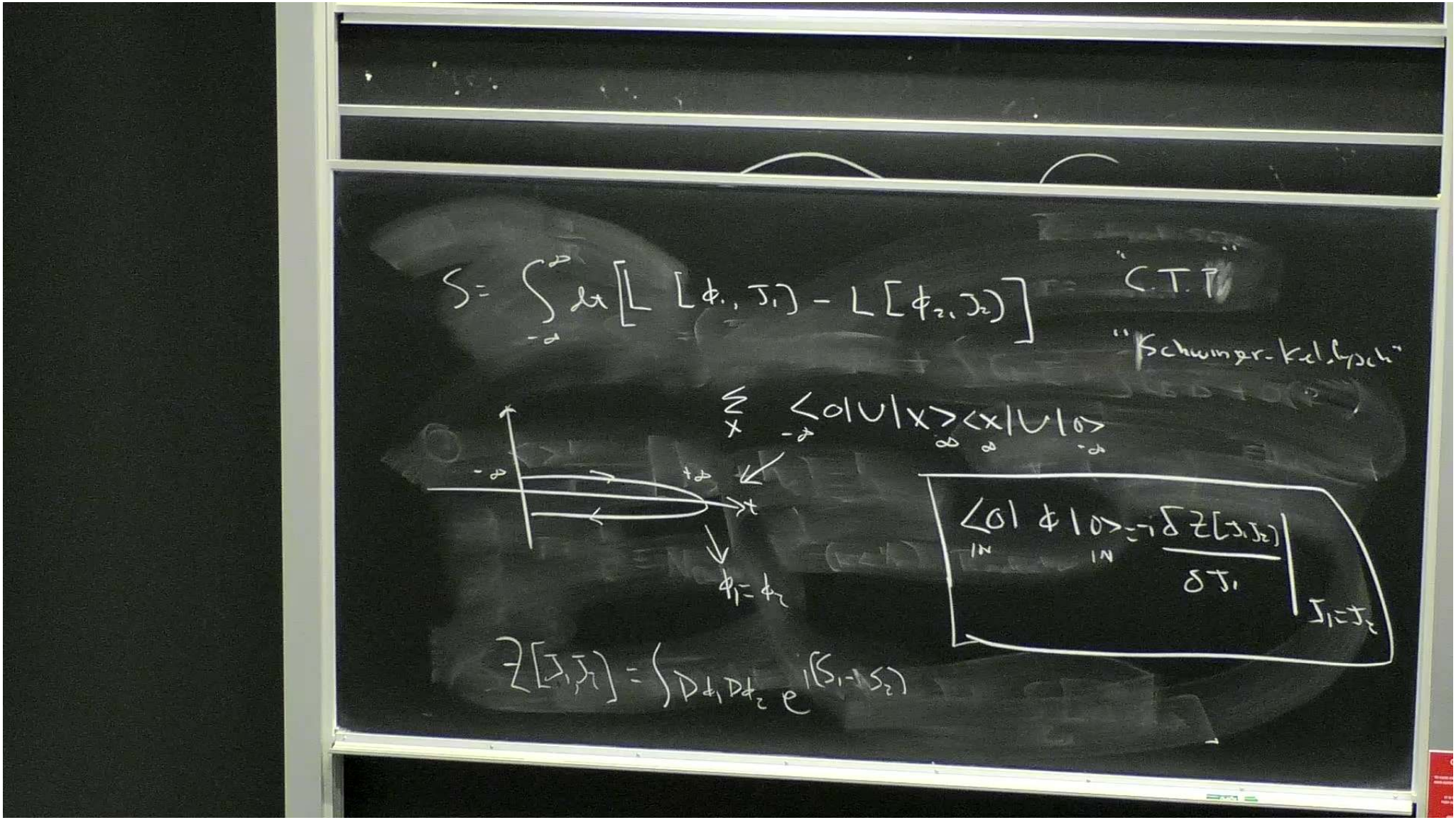
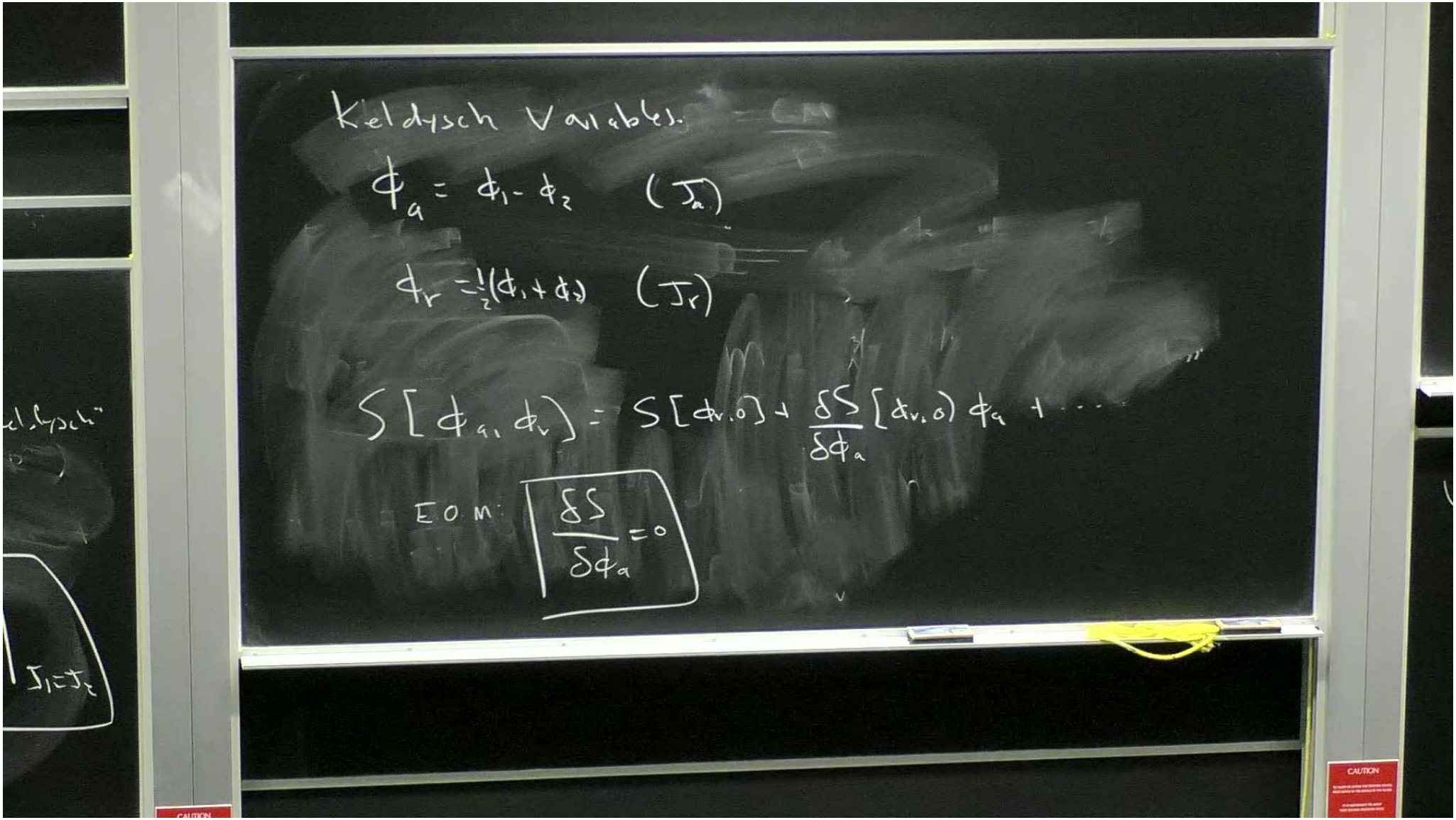
CAUTION


h(x,t) ?
$\langle 0|\, \phi_H(x,t)\, |0\rangle_J = \phi(x,t)$
$Z[J] = \langle 0|\, 0\rangle_J$
out IN
$Z[J] = \int D\phi\, e^{iS + i\int J\phi\, d^4x}$
$\langle 0|\, \phi(x,t)\, |0\rangle_{IN} = -i \frac{\delta}{\delta J} Z[J]$
$\langle 0|\, \phi\, |0\rangle_{IN}$
CAUTION


$\langle 0 | \phi(x,t) | 0 \rangle_{in} = \int d^4y \, G_F(x-y) \, J(y)$
out
$G_F(x-y) = \langle 0 | T[\phi(x) \, \phi(y)] | 0 \rangle$
$|0\rangle_{IN}$ = state which
when $t \to -\infty$
A causal
$U(t_1, t_2) = e^{-iH(t_1 - t_2)}$
$T \to T - i\varepsilon$
$(H \to H - i\varepsilon H)$
$\langle 0 | 0 \rangle_{out} = \lim_{T \to \infty} \langle 0 | e^{-iHT} e^{-iHT} | 0 \rangle$
IN
$\frac{1}{p^2 - m^2 + i\varepsilon}$
CAUTION

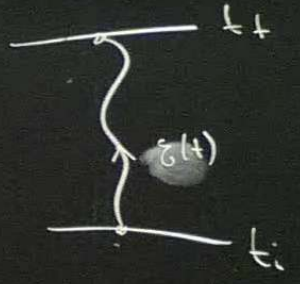

$S = \int \sum -m_i \sqrt{\dot{x}_i(t)}\, dt + V(x_1 - x_2) + S_{Rel}[h^{\leftarrow}, x, \dot{x}]$
$\langle 0 |_{IN} \phi_{IN}(x,t) | 0 \rangle_{IN}$
$\langle 0 | 0 \rangle^{J}_{IN} = \langle 0 | U U^{\dagger} | 0 \rangle_J = \mathbb{1}$
$Z[J_1, J_2] = \langle 0 | U_{J_1} U_{J_2} | 0 \rangle$
$(-\infty, \infty)$
$(\infty, -\infty)$
Double D.O.F.
$S = \int_{-\infty}^{\infty} dt\, d^d x\, S[\phi_1, J_1] + \int_{\infty}^{-\infty} S[\phi_2, J_2]$


$S = \int_{-\infty}^{\infty} dt \left[L[\phi_1, J_1] - L[\phi_2, J_2] \right]$
"C.T.P."
"Schwinger-Keldysh"
$\sum_x \langle 0|U|x\rangle\langle x|U|0\rangle$
$-\infty$
$+\infty$
$-\infty$
$\phi_1 = \phi_2$
$\langle 0|\phi|0\rangle_{IN} = -i \frac{\delta Z[J_1,J_2]}{\delta J_1}\Big|_{J_1=J_2}$
$Z[J_1,J_2] = \int D\phi_1 D\phi_2 \, e^{i(S_1 - S_2)}$


Keldysch Variables
$\phi_a = \phi_1 - \phi_2$ (J_a)
$\phi_r = \frac{1}{2}(\phi_1 + \phi_2)$ (J_r)
$S[\phi_a, \phi_r) = S[\phi_{r,0}] + \frac{\delta S}{\delta \phi_a}[\phi_{r,0}]\phi_a + \ldots$
EOM: $\frac{\delta S}{\delta \phi_a} = 0$
$J_1 = J_2$
CAUTION
CAUTION

Hamilton:

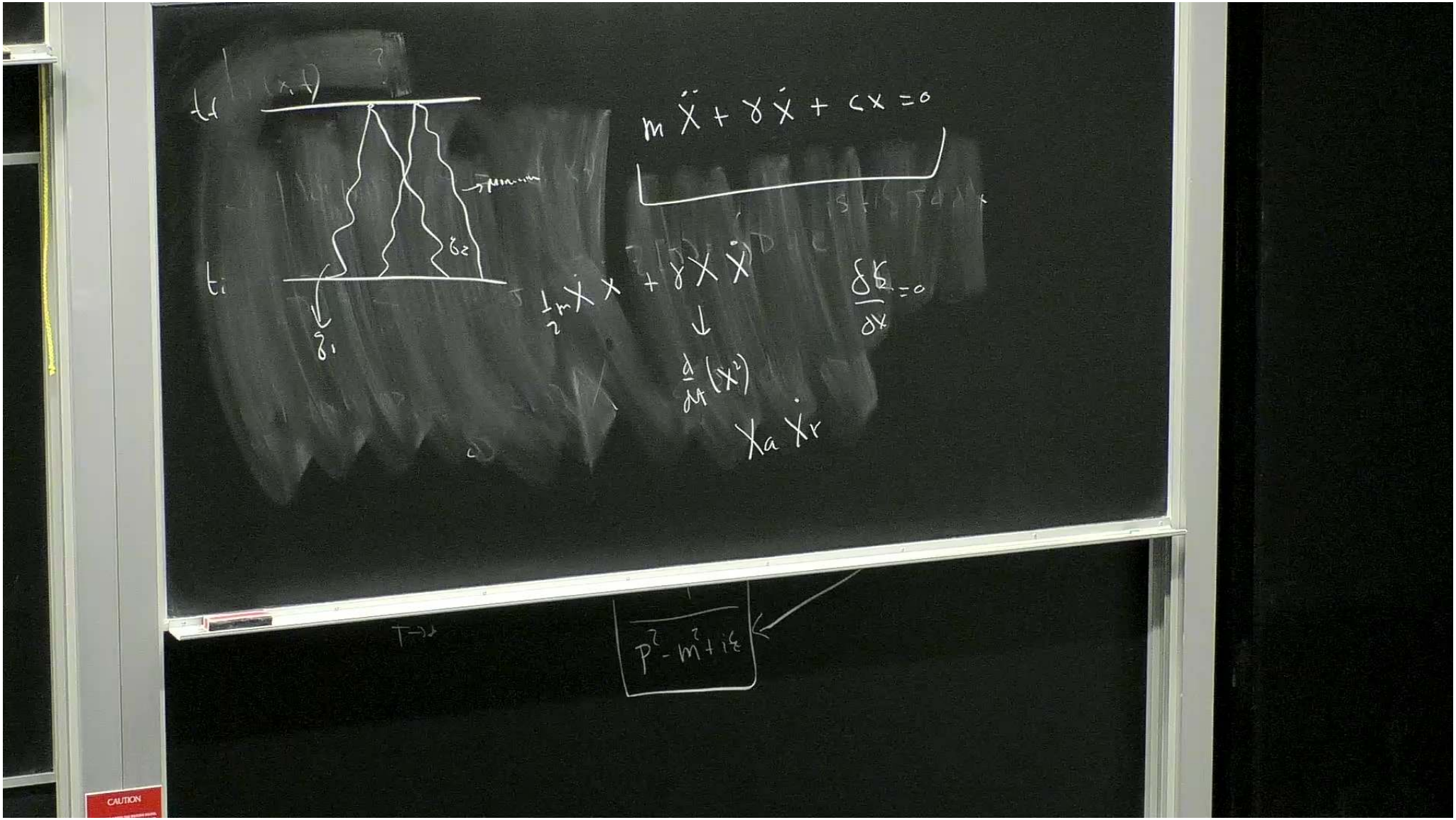
$$S = \int dt\, L(q, \dot{q})$$

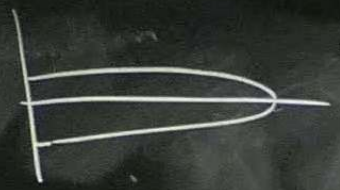


$$\delta S = \frac{\partial L}{\partial \dot{q}} \delta \dot{q} + \frac{\partial L}{\partial q} \delta q$$

$$\delta q \left[\frac{\partial L}{\partial \dot{q}} \right] - \left[\delta q \frac{\partial L}{\partial \dot{q}} \right] \frac{dp}{dt}$$

$$\delta q \Big|_{t_i, t_f} = 0$$


$m\ddot{x} + \gamma\dot{x} + cx = 0$
$\frac{1}{2}m\dot{x}x + \gamma x\dot{x}$
$\frac{d}{dt}(x^2)$
$x_a \dot{x}_r$
$\frac{\delta S}{\delta x} = 0$
$\frac{1}{p^2 - m^2 + i\epsilon}$
t_f
t_i
q_1
q_2
CAUTION



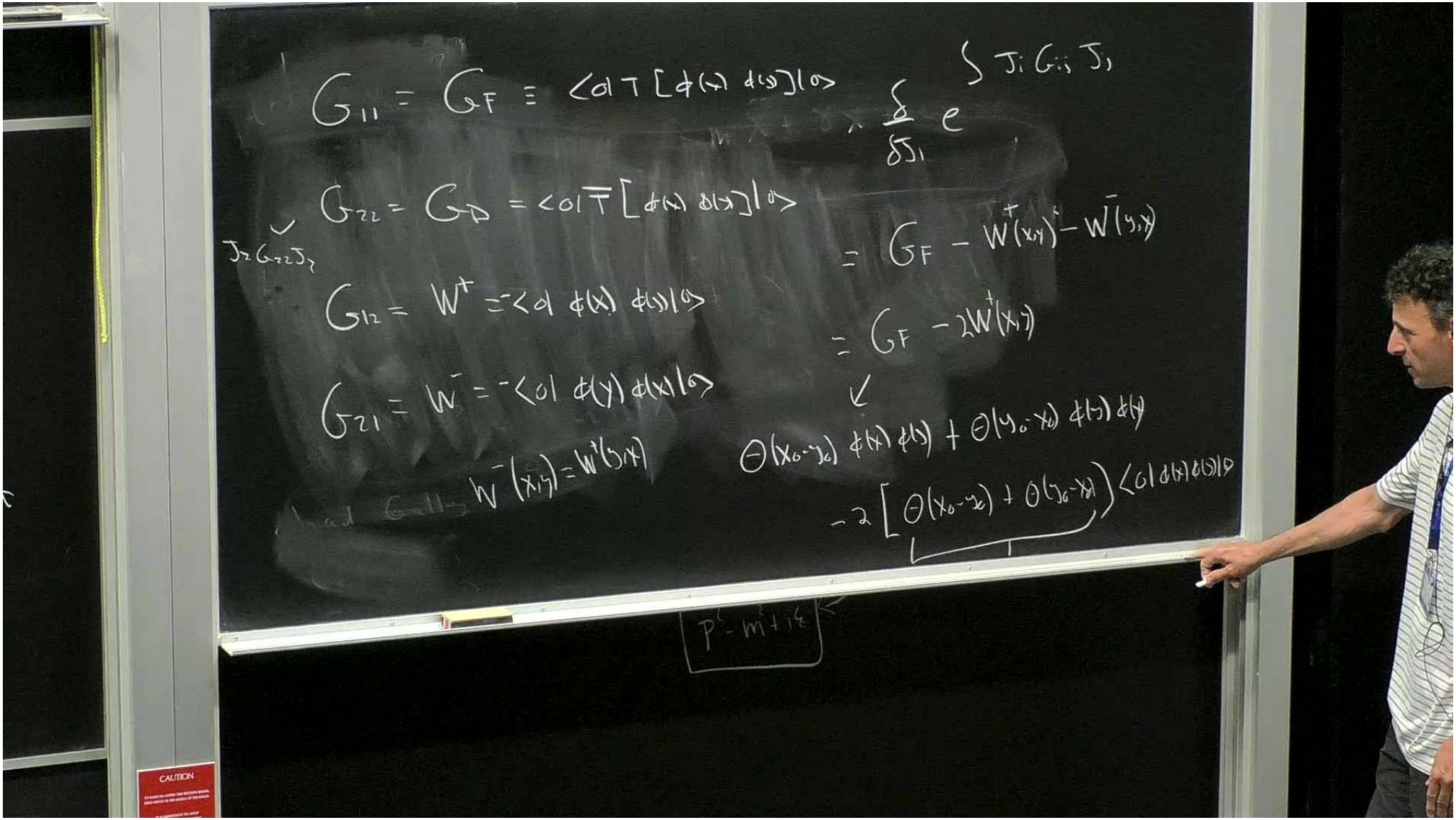
$$\left.\frac{-i\delta}{\delta J_1(x,t)} Z[J_1, J_2]\right|_{J_1=J_2} = {}_{IN}\langle 0 | \phi(x,t) | 0 \rangle$$

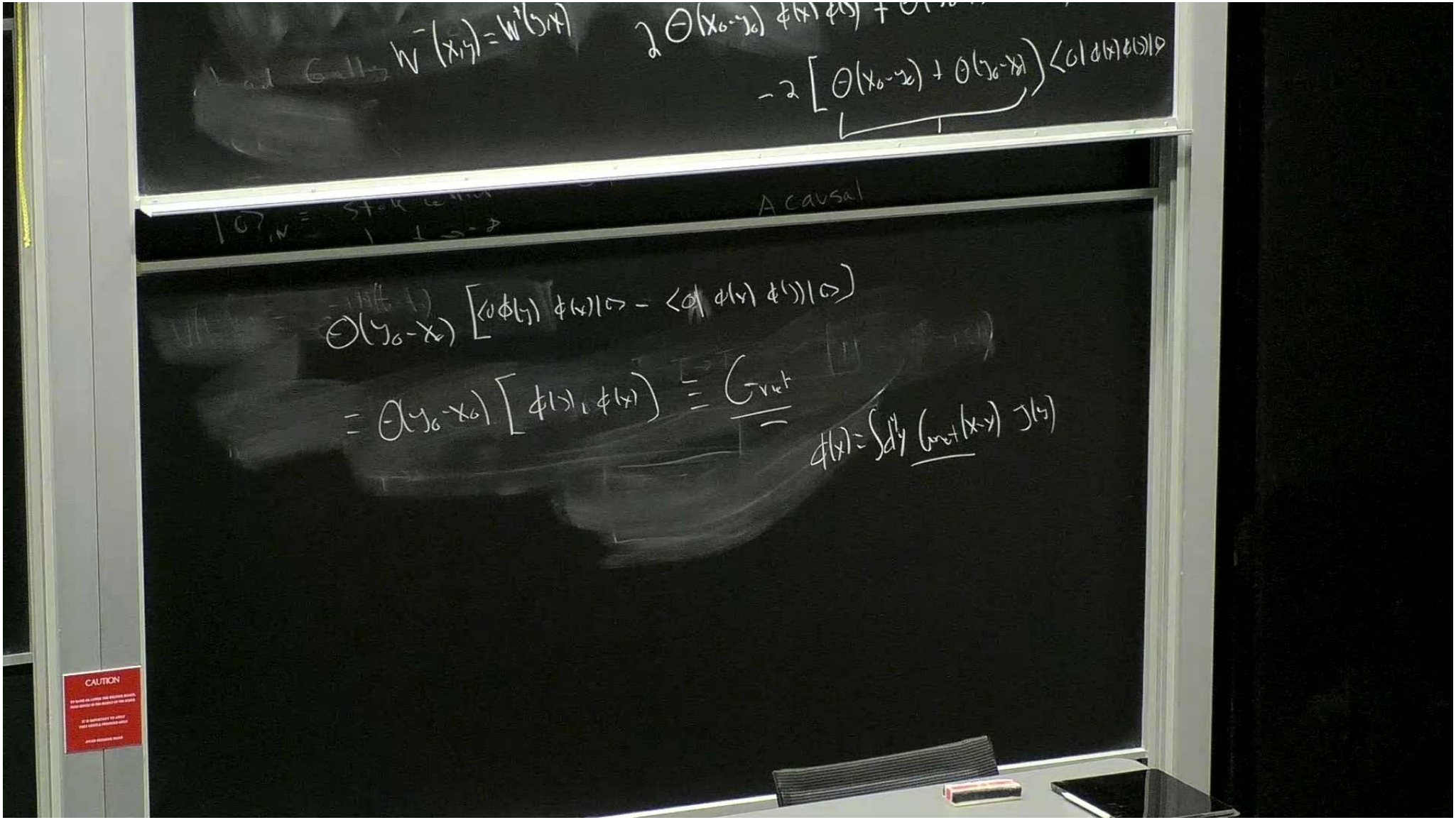
NO INTERACTIONS.

$$S = \int_{-\infty}^{\infty} dt\, d^3x \left[\phi_1 \Box \phi_1 + J_1 \phi_1 - \phi_2 \Box \phi_2 - J_2 \phi_2 \right]$$

$\phi_1 = \phi_2$ at $t = \infty$

perform P.I OVER ϕ_1, ϕ_2. $\sum_{i,j=1}^{2} \int d^4p\, \phi_i^*(p)\, G_{ij}(p)\, \phi_j(p)$


$G_{11} = G_F \equiv \langle 0|T[\phi(x)\phi(y)]|0\rangle$
$\int J_i G_{ij} J_j$
$\frac{\delta}{\delta J_1}$ e
$G_{22} = G_D = \langle 0|\bar{T}[\phi(x)\phi(y)]|0\rangle$
$J_2 G_{22} J_2$
$= G_F - W^+(x,y)^* - W^-(y,x)$
$G_{12} = W^+ = -\langle 0|\phi(x)\phi(y)|0\rangle$
$= G_F - 2W^+(x,y)$
$G_{21} = W^- = -\langle 0|\phi(y)\phi(x)|0\rangle$
$W^-(x,y) = W^+(y,x)$
$\theta(x_0-y_0)\phi(x)\phi(y) + \theta(y_0-x_0)\phi(y)\phi(x)$
$-2\left[\theta(x_0-y_0) + \theta(y_0-x_0)\right]\langle 0|\phi(x)\phi(y)|0\rangle$
$p^2 - m^2 + i\varepsilon$
CAUTION


$W^-(x,y) = W^+(y,x)$
$2\theta(x_0-y_0)\,\phi(x)\phi(y) + \cdots$
$-2\left[\theta(x_0-y_0) + \theta(y_0-x_0)\right]\langle 0|\phi(x)\phi(y)|0\rangle$
Acausal
$\theta(y_0-x_0)\left[\langle 0|\phi(y)\,\phi(x)|0\rangle - \langle 0|\phi(x)\,\phi(y)|0\rangle\right]$
$= \theta(y_0-x_0)\left[\phi(y), \phi(x)\right] = G_{ret}$
$\phi(x) = \int d^4y\, G_{ret}(x-y)\, J(y)$
CAUTION