Title: Theory of Gravitational Waves

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Subject: Particle Physics

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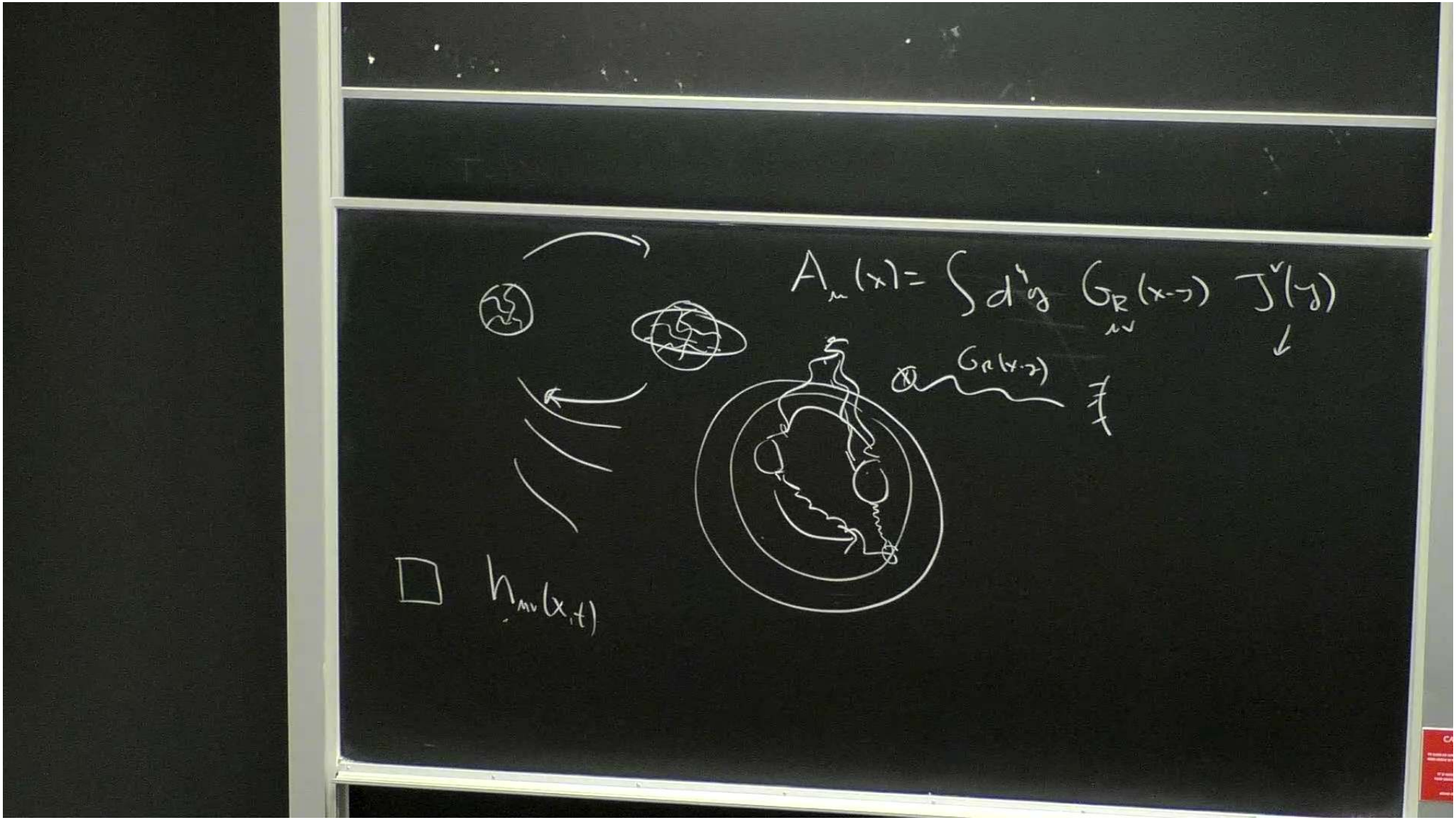
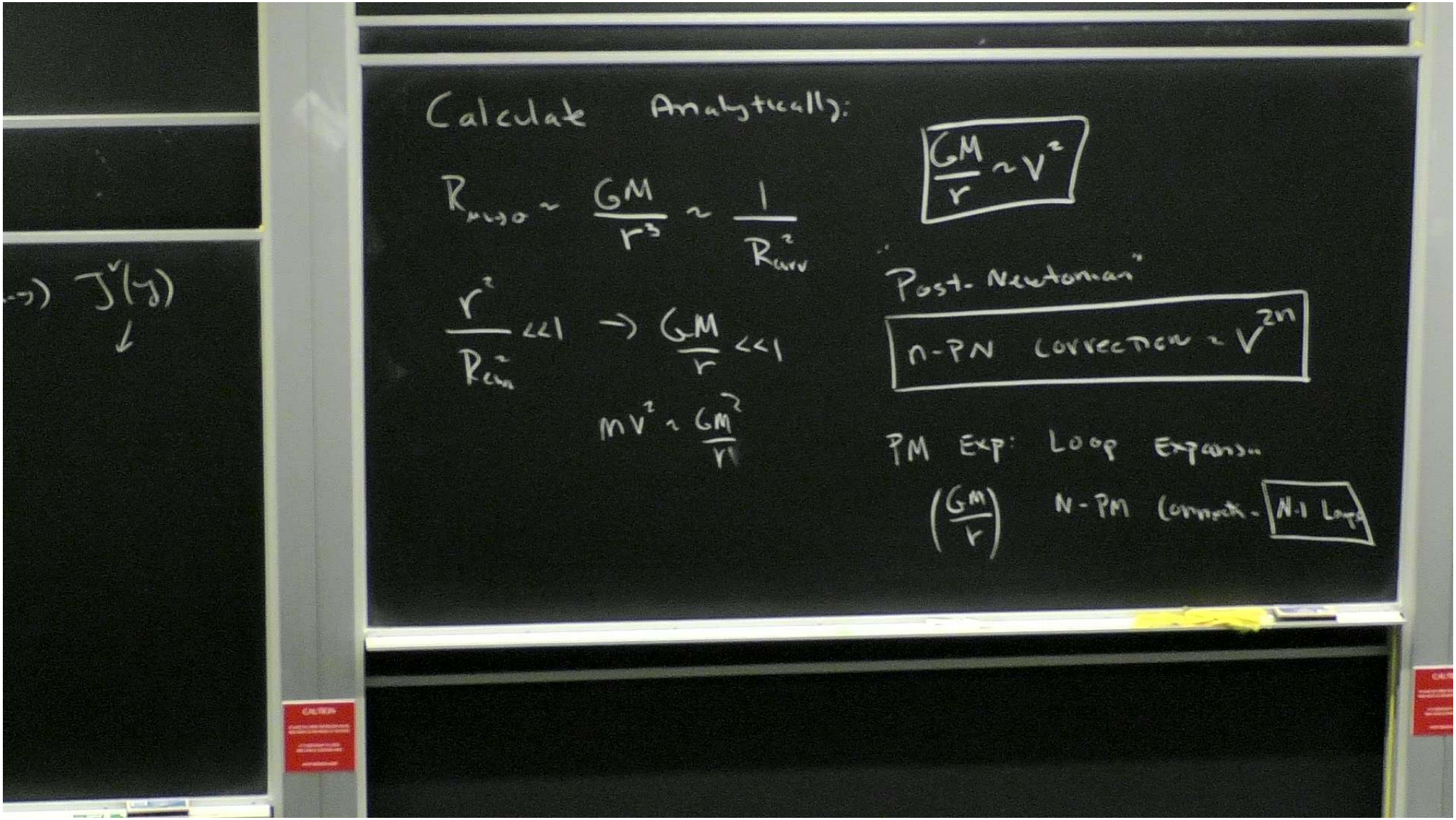
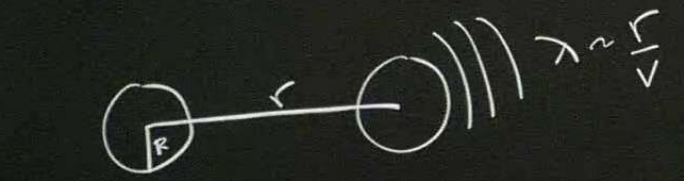
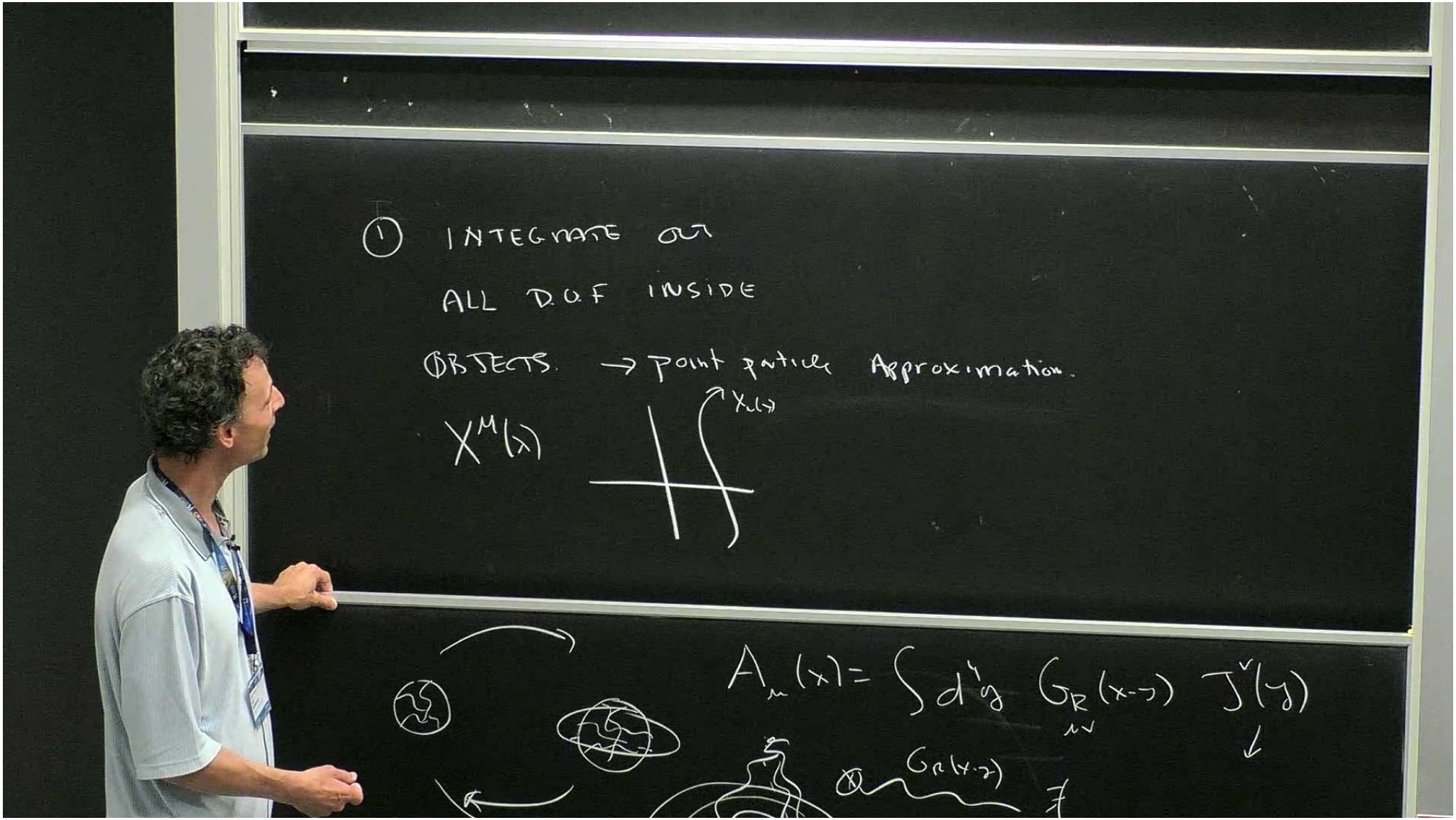

$A_\mu(x) = \int d^4y \; G_{R\,\mu\nu}(x-y) \; J^\nu(y)$
$G_R(x-y)$
$\Box \; h_{\mu\nu}(x,t)$

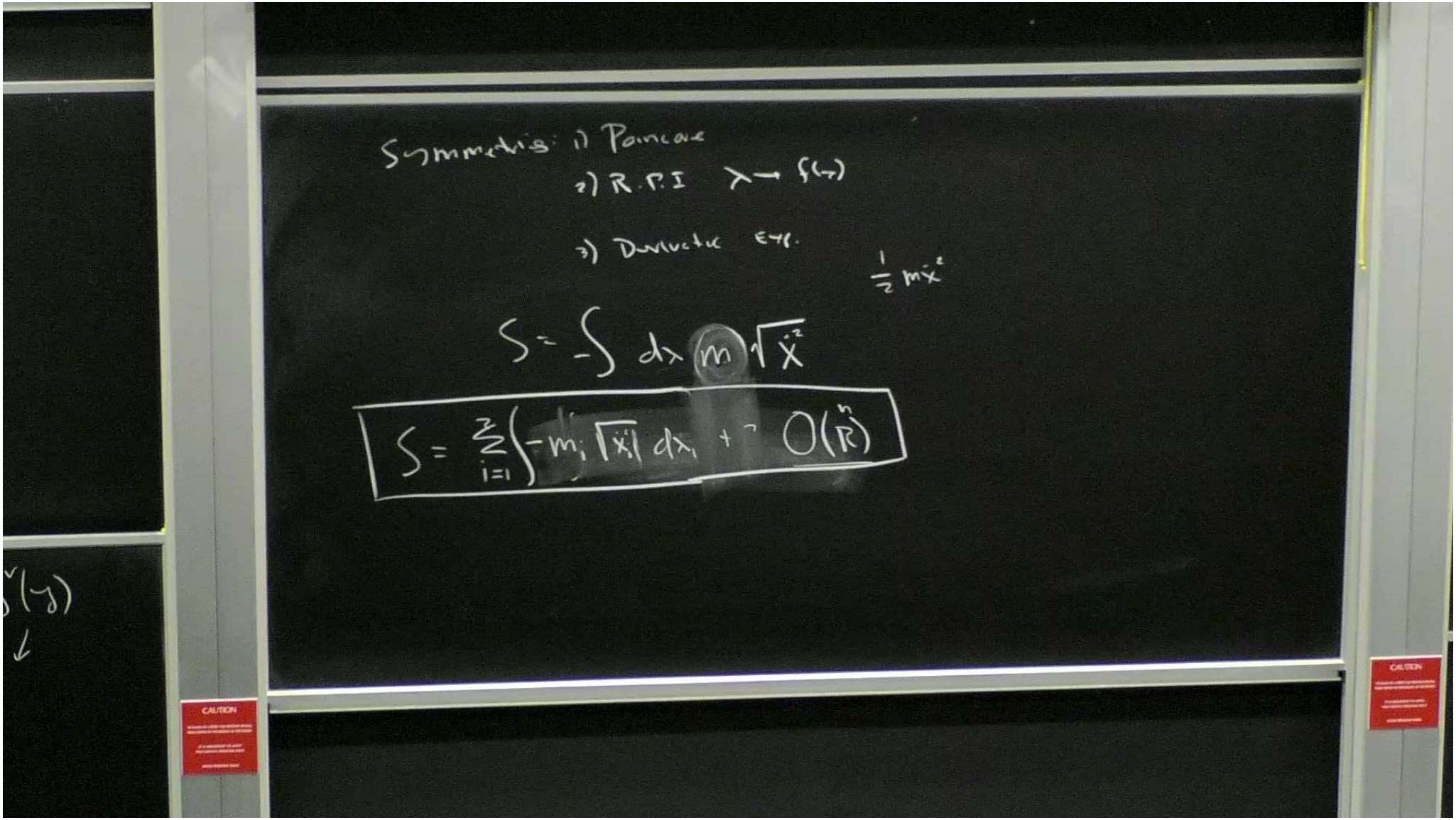

Calculate Analytically:
$R_{\mu\nu\rho\sigma} \sim \frac{GM}{r^3} \sim \frac{1}{R_{curv}^2}$
$\frac{r^2}{R_{curv}^2} \ll 1 \rightarrow \frac{GM}{r} \ll 1$
$mv^2 \sim \frac{GM^2}{r}$
$\boxed{\frac{GM}{r} \sim v^2}$
"Post-Newtonian"
$\boxed{\text{n-PN correction} \sim v^{2n}}$
PM Exp: Loop Expansion
$\left(\frac{GM}{r}\right)$ N-PM Correction - $\boxed{\text{N-1 Loops}}$
$J^\nu(y)$

Goal: Calculate $h_{\mu\nu}(x,t)$ $\vec{x} \to \infty$ in PN-EXPANSION



$R \ll r \ll \frac{r}{v}$


① INTEGRATE OUT
ALL D.O.F INSIDE
OBJECTS → point particle Approximation.
$X^\mu(\tau)$
$X_\mu(\tau)$
$A_\mu(x) = \int d^4y \; G_R(x-y) \; J^\nu(y)$
$G_R(x-z)$


Symmetries: i) Poincare
ii) R.P.I $x \to f(x)$
iii) Derivative Eqn.
$\frac{1}{2} m\dot{x}^2$
$S = -\int dx\, m\sqrt{\dot{x}^2}$
$S = \sum_{i=1}^{3}\left(-m_i \sqrt{\dot{x}_i^2}\, dx_i + \dots\right) O(R^n)$

- Remove all terms which vanish on-shell.
("EQUIVALENCE Theorem")

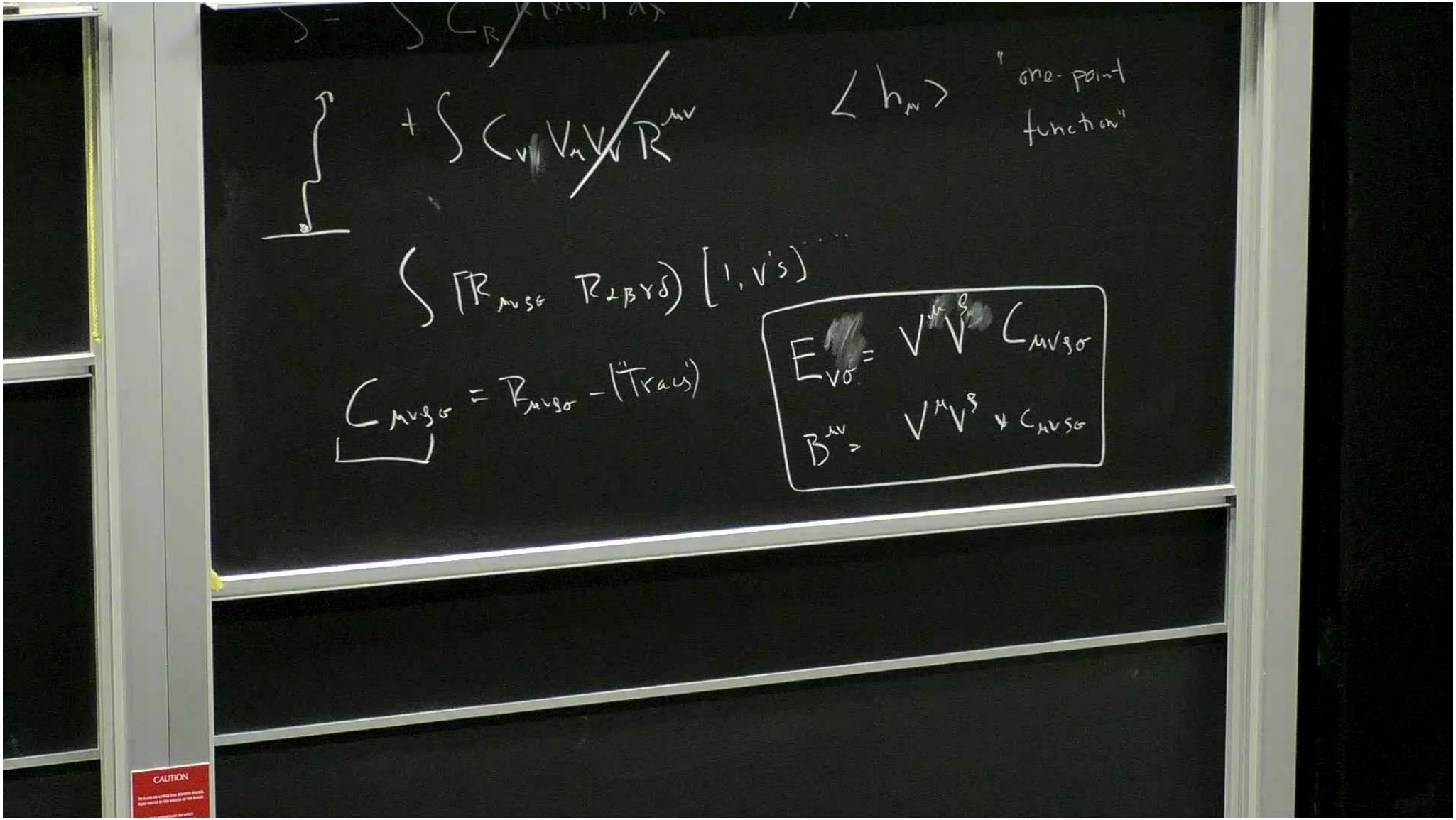
$$\left(d/dt + \omega_0 \right)$$

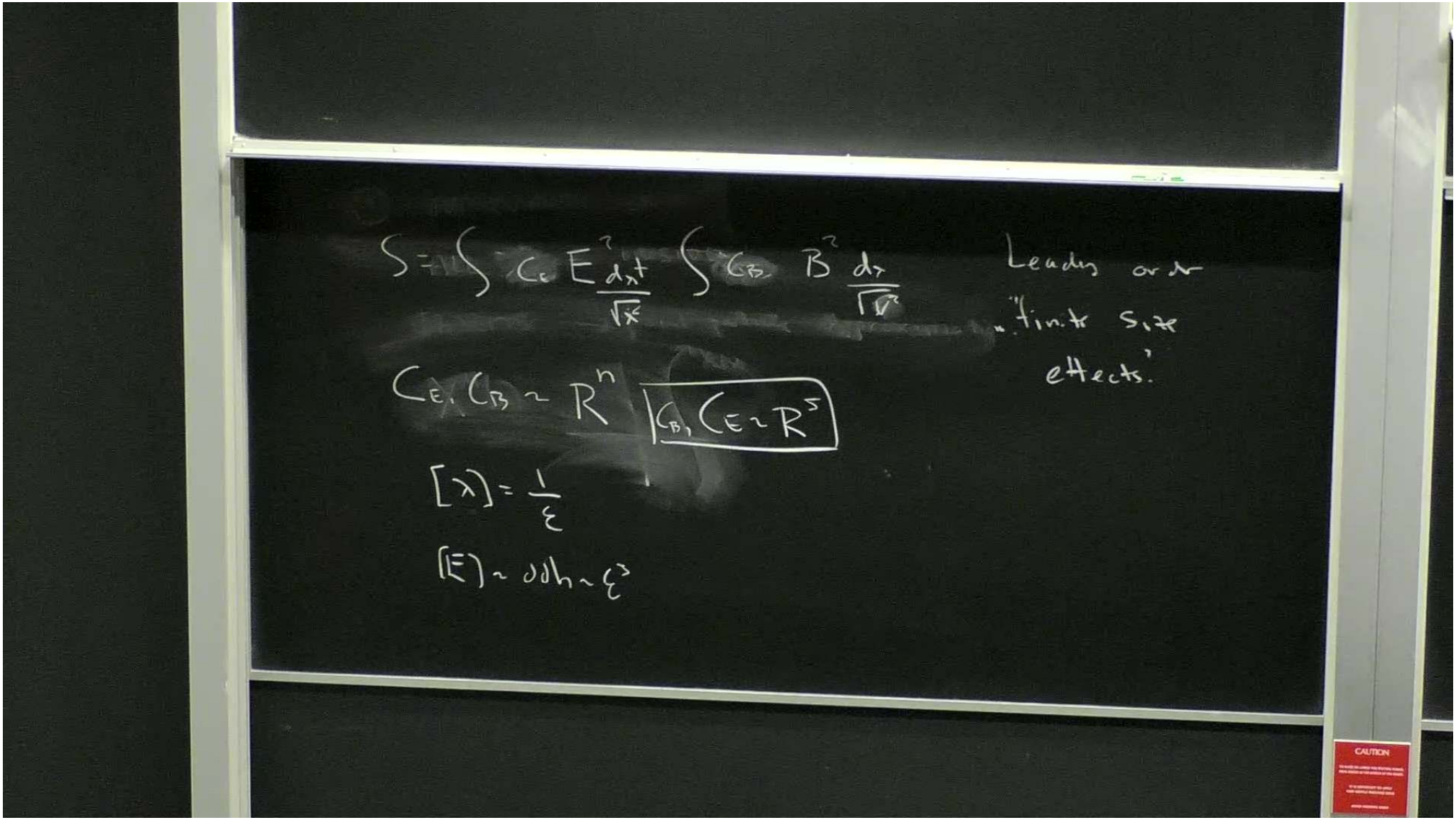
$$L = \int dt \left[x\, \sigma x + \sigma x\, F(x) \right]$$

$$x \to x - F/2$$

$$= \int dt \left[(x - F/2)\, \sigma (x - F/2) + (\sigma x)\, F(x) \right.$$

$$\frac{F \sigma F}{4} \to \text{new}$$


$+ \int C_\nu V_\mu V_\nu R^{\mu\nu}$
$\langle h_{\mu\nu} \rangle$
"one-point function"
$\int [R_{\mu\nu\rho\sigma} \; R_{\alpha\beta\gamma\delta}) [1, V's]$
$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (\text{Tracs})$
$E_{\nu\sigma} = V^\mu V^\rho C_{\mu\nu\rho\sigma}$
$B^{\mu\nu} = V^\mu V^\rho * C_{\mu\nu\rho\sigma}$
CAUTION


$S = \int C_E E^2 \frac{d_x t}{\sqrt{\dots}} \int C_B B^2 \frac{d_x}{\sqrt{\dots}}$
Leading order
"finite size effects"
$C_E, C_B \sim R^n$
$C_B, C_E \sim R^5$
$[\lambda] = \frac{1}{\ell}$
$[E] \sim \dots \sim \ell^3$

(EQUIVALENCE THEOREM)

$h = h_B + \delta h$

↓

Background

$E = E_B + \delta E$

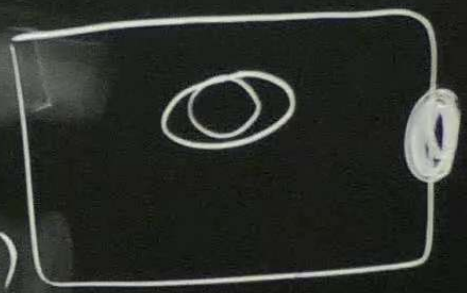
$\vec{E} \to \boxed{E_B}\,\delta E + \vec{E}_B + O(\delta E)$

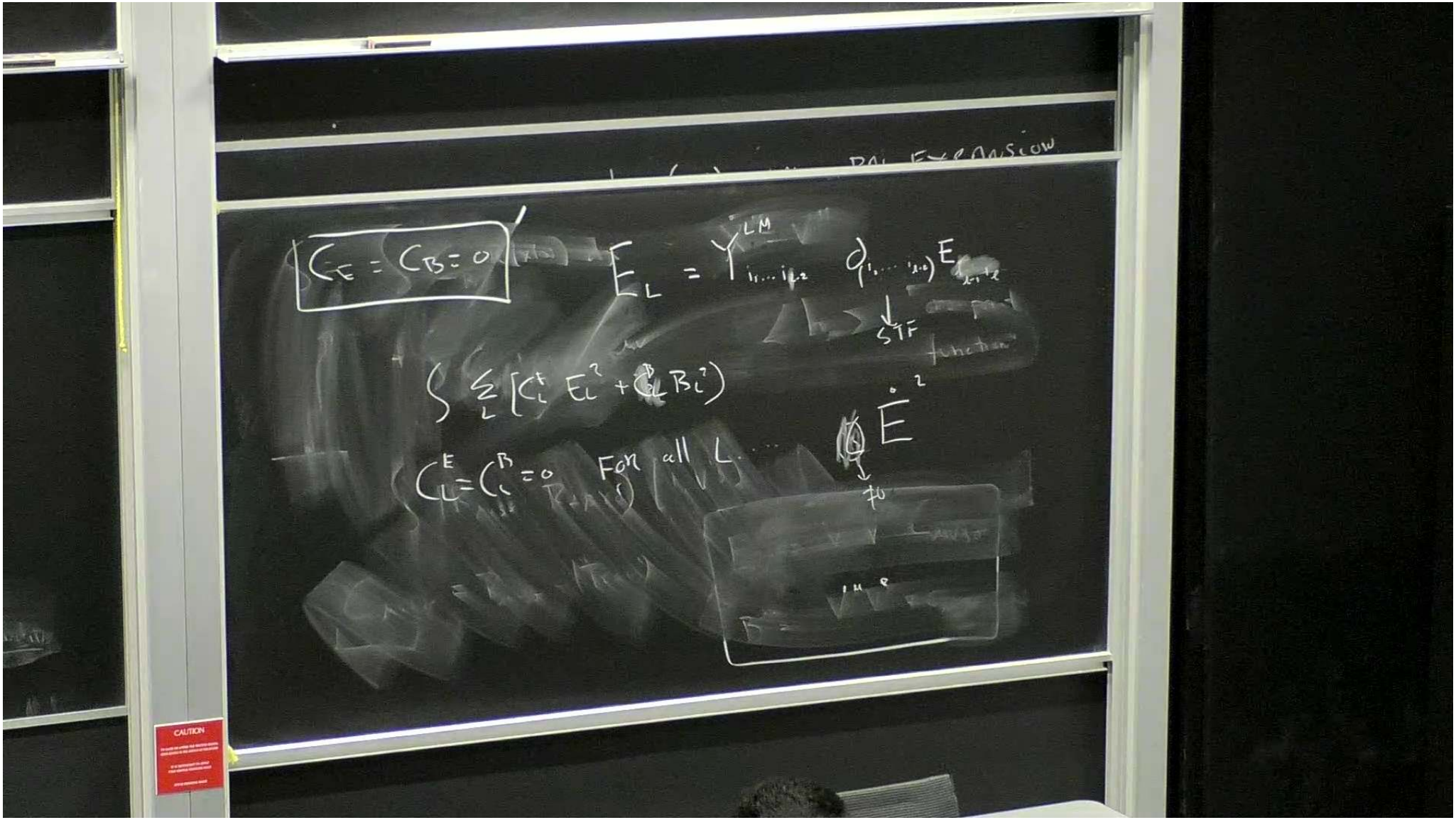
$S = \int C_E\, E_B(x)\, \delta E$

$C_E \equiv$ "Love #"

TIDAL Polarizability

$E_B \rightsquigarrow h(x)$

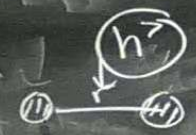



$C_E = C_B = 0$
$E_L = Y^{LM}_{i_1 \ldots i_{L-2}} q_{i_1 \ldots i_{L-2}} E$
STF
$\sum_L \left[C^E_L E_L^2 + C^B_L B_L^2 \right]$
$C^E_L = C^B_L = 0$ For all L
$\dot{E}^2$
$\neq 0$
CAUTION

Remove all terms

(EQUIVALENCE Theorem)

Step (2) (Eliminate r)

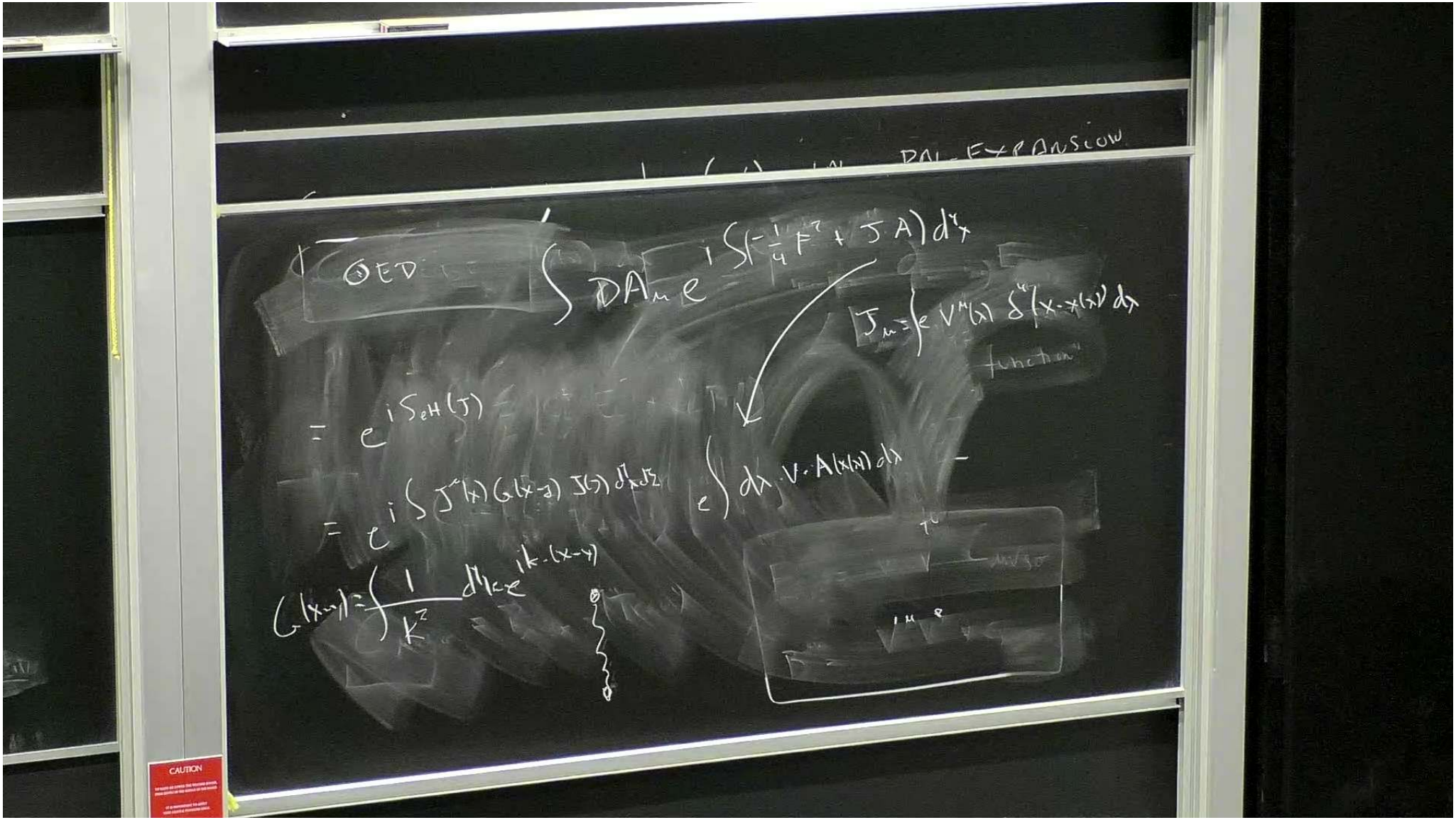


$h(k) = h^{<}(k) + h^{>}(k)$

$h^{>}(k)$	$k > \frac{1}{r}$	('Potential')
$h^{<}(k)$	$k < \frac{1}{r}$	('Radiative')

$$\int Dh_{>} Dh_{<} \, e^{iS(h_{>}, h_{<})}$$

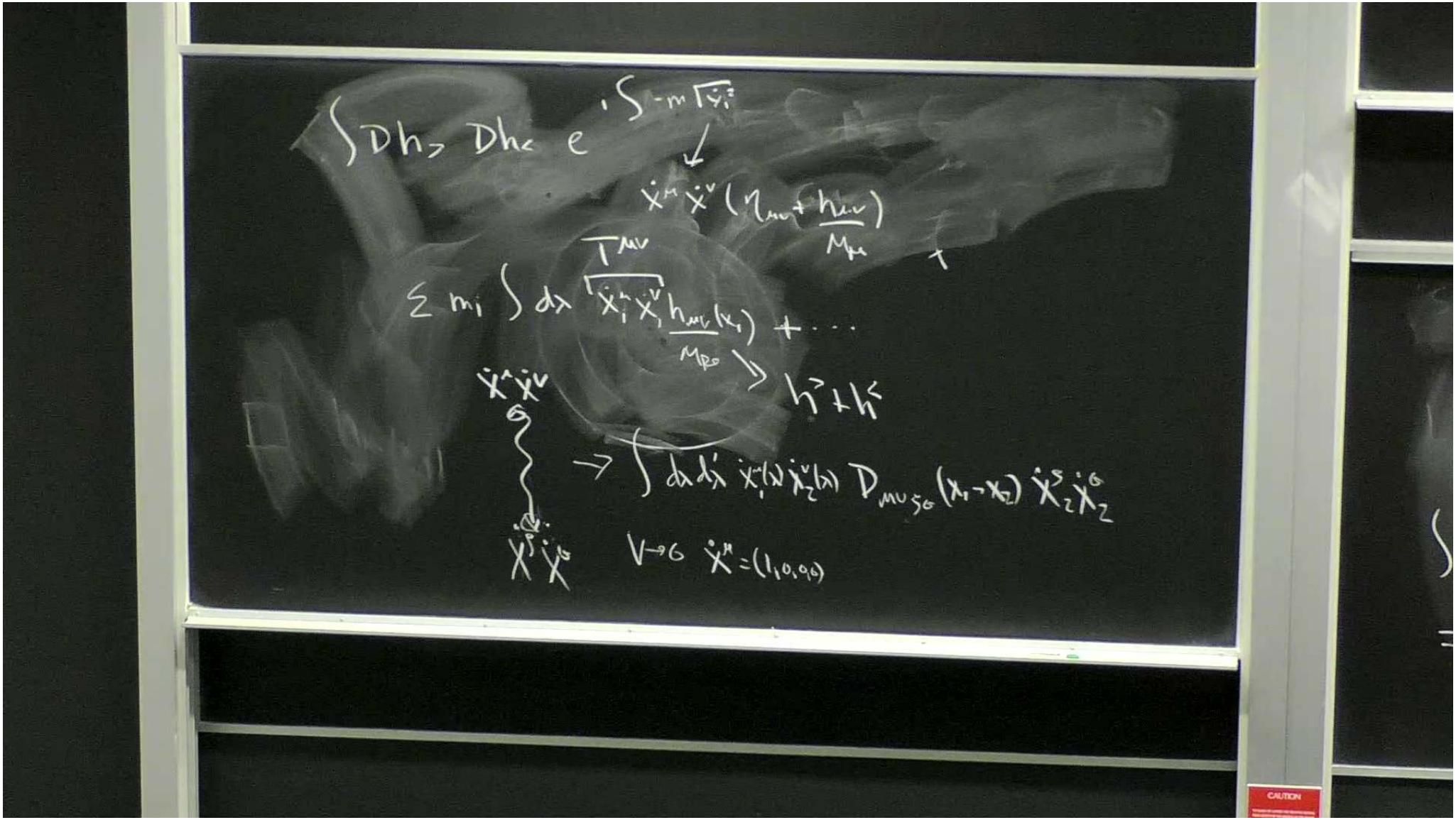
$$= \int Dh_{<} \, e^{iS_{eff}(h_{<})}$$

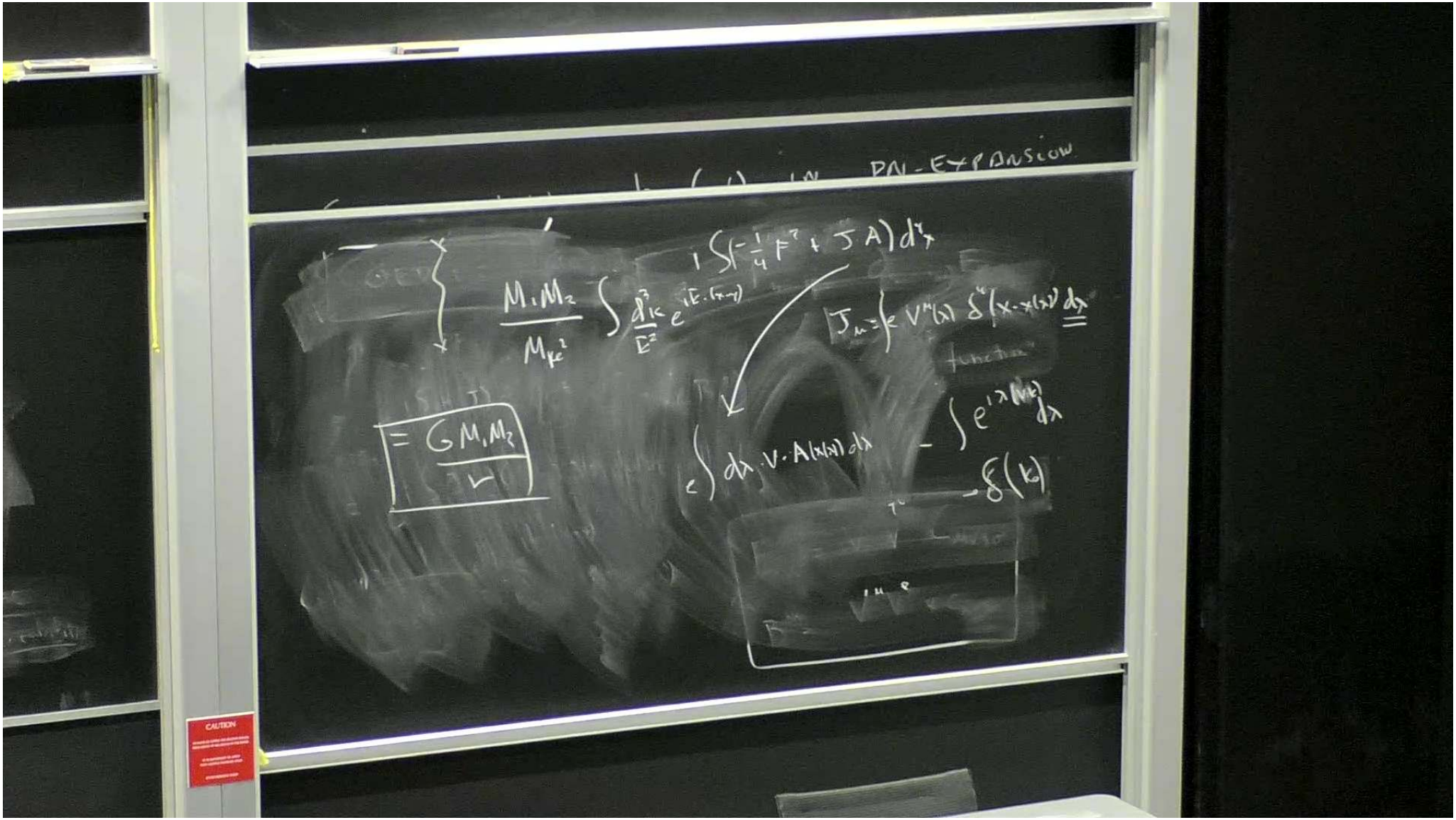

QED
$\int DA_\mu e^{iS(-\frac{1}{4}F^2 + JA)d^4x}$
$J_\mu = \int e V^\mu(\lambda) \delta^4(x - x(\lambda)) d\lambda$
function
$= e^{iS_{eff}(J)}$
$= e^{iS J^\mu(x) G(x-y) J(y) d^4x d^4y}$
$e^{\int d\lambda \, V \cdot A(x(\lambda)) d\lambda}$
$G(x-y) = \int \frac{1}{k^2} d^4k \, e^{ik\cdot(x-y)}$
CAUTION

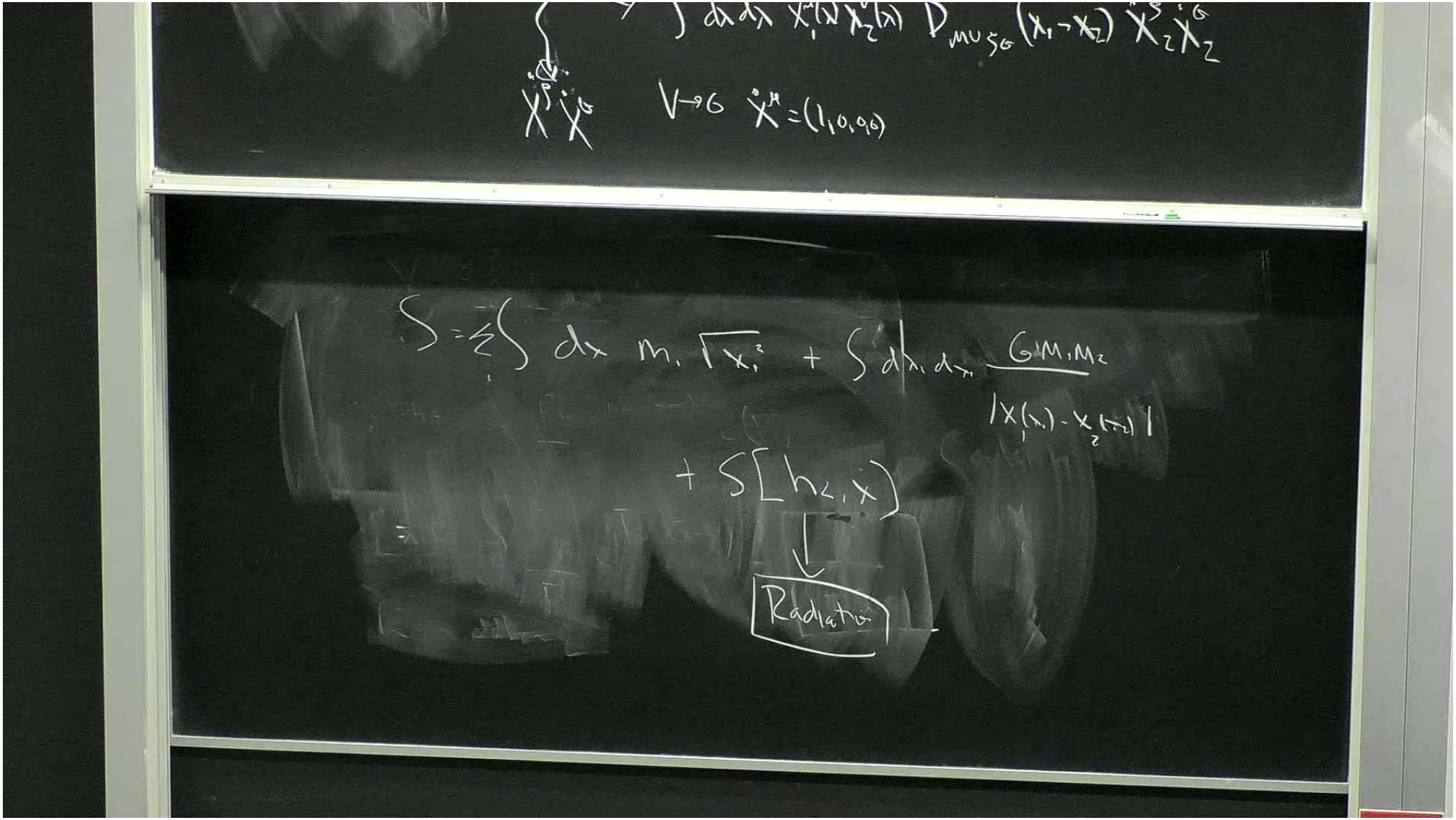
$V^M = (1, v, 0, 0)$

$$S_{eff} = \int \frac{d^4k}{k^2} e^{ik(x-y)} \delta(k_0)$$

$$= e^2 \int \frac{d^3k}{\vec{k}^2} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} = \frac{e^2}{4\pi|\vec{x}-\vec{y}|}$$


$\int Dh_> Dh_< e^{iS}$
$-m\sqrt{\dot{x}_i^2}$
$\dot{x}^\mu \dot{x}^\nu (\eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{Pl}})$
$T^{\mu\nu}$
$\sum m_i \int d\lambda \dot{x}_i^\mu \dot{x}_i^\nu \frac{h_{\mu\nu}(x_i)}{M_{Pl}} + \dots$
$h_> + h_<$
$\dot{x}^\mu \dot{x}^\nu$
$\dot{x}^\rho \dot{x}^\sigma$
$\int d\lambda d\lambda' \dot{x}_1^\mu(\lambda) \dot{x}_2^\nu(\lambda') D_{\mu\nu\rho\sigma}(x_1 - x_2) \dot{x}_2^\rho \dot{x}_2^\sigma$
$V \to G \quad \dot{x}^\mu = (1,0,0,0)$


PAL-EXPANSION
$\frac{M_1 M_2}{M_{pl}^2} \int \frac{d^3k}{E^2} e^{iE\cdot(x-y)}$
$= \frac{GM_1M_2}{r}$
$\int (-\frac{1}{4}F^2 + JA)\,d^4x$
$J_\mu = \int V_\mu(\lambda)\,\delta^4(x - x(\lambda))\,d\lambda$
$e \int d\lambda\, V \cdot A(x(\lambda))\,d\lambda$
$\int e^{ik\cdot x(\lambda)}\,d\lambda$
$\delta(k)$
CAUTION



$$\int d\lambda\, d\lambda\; x_1^\mu(\lambda)\, x_2^\nu(\lambda)\; D_{\mu\nu\rho\sigma}(x_1 - x_2)\; \dot{x}_2^\rho \dot{x}_2^\sigma$$

$$\dot{x}^\rho \dot{x}^\sigma \qquad V \to 0 \quad \dot{x}^\mu = (1,0,0,0)$$

$$S = \sum_i \int dx\; m_i \sqrt{x_i^2} + \int d\lambda_1\, d\lambda_2\; \frac{G m_1 m_2}{|x_1(\lambda_1) - x_2(\lambda_2)|}$$

$$+ S[h_{<}, x]$$

$$\downarrow$$

$$\boxed{\text{Radiation}}$$