Title: Standard Model

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Subject: Particle Physics

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QCD + Chiral Symmetry Breaking

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \sum_{\text{quarks}} \left(-\bar{q}\left(\not{\partial} + m_q\right) q + \frac{i g_3}{2} G^{a}_{\mu} \bar{q} \gamma^{\mu} \lambda_a q \right)$$

Gell-Mann matrices $a = 1, 2, \ldots, 8$

$$\alpha_s = \frac{g_3^2}{4\pi}$$

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(M_Z)} + \frac{1}{12\pi}\left(2n_q - 33\right) \ln\left(\frac{M_Z^2}{\mu^2}\right)$$

$\mu = m_b \simeq 5\ \text{GeV}$, $n_q = 5$

$$b_5 = \frac{1}{12\pi}\left(2n_q - 33\right) = -0.61$$

very important!

$\alpha_s(M_Z) = 0.12$
$\alpha_s(m_b) = 0.20$
$\alpha_s(M_S) = 1.7$?

Λ_{QCD} : $\frac{1}{\alpha_s} \to 0$

$$\frac{1}{\alpha_s(\Lambda_{QCD})} = \frac{1}{\alpha_s(M_z)} + b_5 \ln\left(\frac{M_z^2}{\Lambda_{QCD}^2}\right) = 0$$

$$\Lambda_{QCD} = M_z \exp\left(\frac{1}{2 b_5 \alpha_s(M_z)}\right) : \text{QCD scale}$$

$n_q = 5$: $\Lambda_{QCD} \simeq 80\,MeV$

$n_q = 3$ $\Lambda_{QCD} \simeq 200\,MeV$

short distance

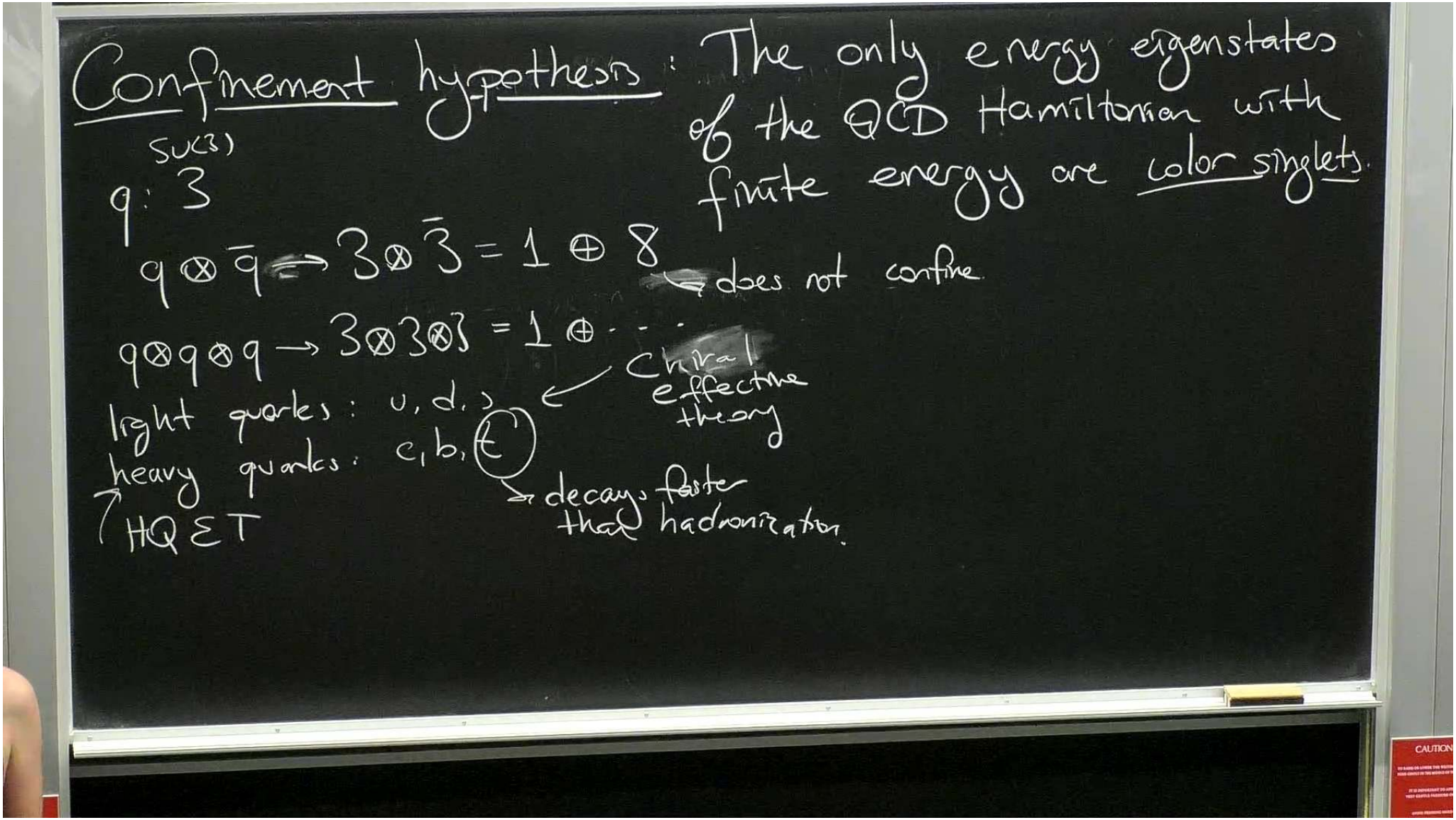
$V_s \sim \frac{\alpha_s}{r}$

longer distances

$V \sim \sigma r$

(hypothetical)

$\Lambda_c = \Lambda_{QCD}$, $r_c : \frac{1}{\Lambda_{QCD}}$


Confinement hypothesis: The only energy eigenstates of the QCD Hamiltonian with finite energy are color singlets.
SU(3)
q: 3
q ⊗ q̄ → 3 ⊗ 3̄ = 1 ⊕ 8
does not confine
q ⊗ q ⊗ q → 3 ⊗ 3 ⊗ 3 = 1 ⊕ ...
Chiral effective theory
light quarks: u, d, s
heavy quarks: c, b, t
decays faster than hadronization
HQET

$$\mathcal{L}_{QCD} = \mathcal{L}_{chiral} + \mathcal{L}_{mass}$$

$$\mathcal{L}_{chiral} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - (\bar{u}\ \bar{d}\ \bar{s}) \begin{pmatrix} \not{\partial} & 0 & 0 \\ 0 & \not{\partial} & 0 \\ 0 & 0 & \not{\partial} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\mathcal{L}_{mass} = (\bar{u}\ \bar{d}\ \bar{s}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$m_u, m_d, m_s \ll \Lambda_{QCD}$

Symmetries of $\mathcal{L}_{chiral}$:

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \xrightarrow{U_L(3) \times U_R(3)} e^{-i\theta_L^a \frac{\lambda_a}{2} P_L + -i\theta_R^a \frac{\lambda_a}{2} P_R} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$a=0 \quad : \quad \lambda_0 = \sqrt{\frac{2}{3}}\, \mathbb{1}_{3\times 3} \qquad \mathrm{tr}(\lambda_a \lambda_b) = \delta_{ab}.$

$a = 1, 2, \ldots, 8 \quad \lambda_a =$ Gell-Mann matrices

$U(3) = SU(3) \times \underbrace{U(1)}_{\text{anomalous symmetry.}}$

The anomaly-free (good) symmetries of $\mathcal{L}_{ch}$:

$$G = SU_L(3) \times SU_R(3) \times U_V(1)$$

$a=0$: $\lambda_0 = \sqrt{\frac{2}{3}}\, \mathbb{1}_{3\times 3}$ $\quad\quad$ $\mathrm{tr}(\lambda_a \lambda_b) = \delta_{ab}$.

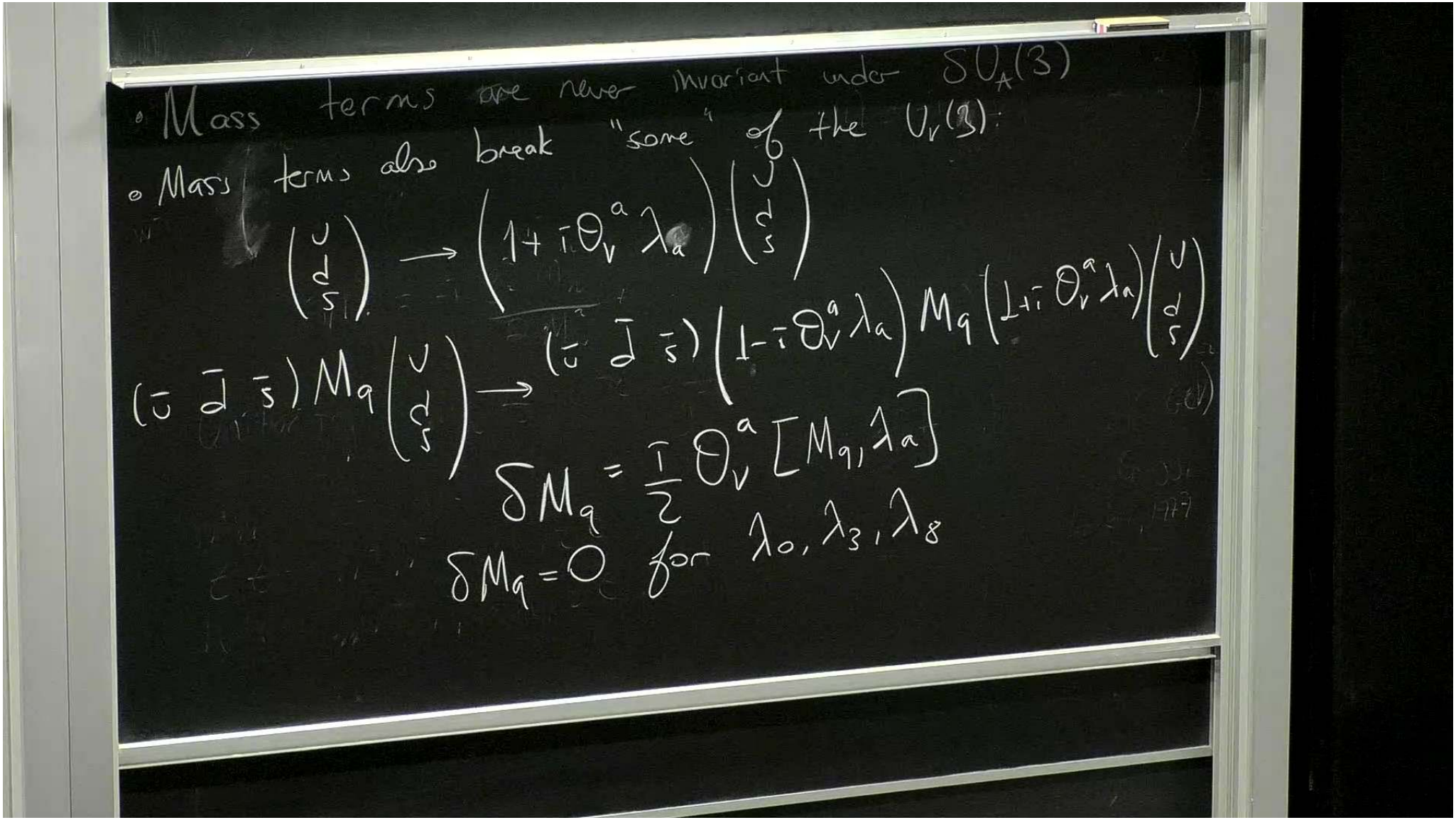
$a = 1, 2, \ldots, 8$ $\quad$ λ_a = Gell-Mann matrices

$U(3) = SU(3) \times \underbrace{U(1)}_{\text{anomalous symmetry.}}$

The anomaly-free (good) symmetries of Lch:

$$G = \underbrace{SU_L(3) \times SU_R(3)}_{SU_V(3) \times SU_A(3)} \times \underbrace{U_V(1)}_{U_B(1)}$$

↳ baryon number.


• Mass terms are never invariant under $SU_A(3)$
• Mass terms also break "some" of the $U_V(3)$:
$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \left(1 + i\theta_V^a \lambda_a\right)\begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$(\bar{u}\ \bar{d}\ \bar{s})\, M_q \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow (\bar{u}\ \bar{d}\ \bar{s})\left(1 - i\theta_V^a \lambda_a\right) M_q \left(1 + i\theta_V^a \lambda_a\right)\begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$\delta M_q = \frac{i}{2}\theta_V^a [M_q, \lambda_a]$
$\delta M_q = 0$ for $\lambda_0, \lambda_3, \lambda_8$

$$\mathcal{L}_{QCD} = \mathcal{L}_{chiral} + \mathcal{L}_{mass}$$

$$\mathcal{L}_{chiral} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - (\bar{u}\ \bar{d}\ \bar{s}) \begin{pmatrix} \not{\partial} & 0 & 0 \\ 0 & \not{\partial} & 0 \\ 0 & 0 & \not{\partial} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\mathcal{L}_{mass} = (\bar{u}\ \bar{d}\ \bar{s}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = m_0 \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_8}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + m_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_0 = \frac{1}{\sqrt{6}} (m_u + m_d + m_s)$$

$$m_8 = \frac{1}{2\sqrt{3}} (m_u + m_d - 2m_s) \qquad m_3 = \frac{1}{2} (m_u - m_d)$$

- $m_0 \lambda_0$ does not break any symmetries
- $m_8 \lambda_8$ commutes w/ a $SU_I(2) \times U_S(1)$ subgroup of $U(3)$

($SU_I(2)$: isospin; $U_S(1)$: "strangeness")

$I(u) = \frac{1}{2}$, $I(d) = -\frac{1}{2}$, $S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $S(s) = -1$

- $m_3 \lambda_3$ breaks everything down to $U_u(1) \times U_d(1) \times U_s(1)$

Mesons $q\bar{q}$

$$M = \begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u}\ \bar{d}\ \bar{s}) = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$: meson matrix

$$M = \sum_{a=0}^{8} \mu_a \lambda_a = \begin{pmatrix} \mu_3 + \frac{1}{\sqrt{3}}\mu_8 + \sqrt{\frac{2}{3}}\mu_0 & \mu_1 - i\mu_2 & \mu_4 - i\mu_5 \\ \mu_1 + i\mu_2 & -\mu_3 + \frac{1}{\sqrt{3}}\mu_8 + \sqrt{\frac{2}{3}}\mu_0 & \mu_6 - i\mu_7 \\ \mu_4 + i\mu_5 & \mu_6 + i\mu_7 & -\frac{2}{\sqrt{3}}\mu_8 + \sqrt{\frac{2}{3}}\mu_0 \end{pmatrix}$$

π^0 η_8 π^+ K^+ π^- η_0 K^- K^0

$$\pi^0 \sim \frac{1}{2}(u\bar{u} - d\bar{d})$$

$$\pi^+ \sim u\bar{d}, \quad \pi^- \sim \bar{u}d$$

$$K^+ \sim u\bar{s}, \quad K^- \sim \bar{u}s$$

$$K^0 \sim d\bar{s}, \quad \bar{K}^0 \sim s\bar{d}$$

$$\eta_8 \sim \frac{1}{2\sqrt{3}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\eta_0 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = m\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_8}{\sqrt{3}}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + m_3\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L}_{QCD} = \mathcal{L}_{chiral} + \mathcal{L}_{mass}$$

$$\mathcal{L}_{chiral} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - (\bar{u}\ \bar{d}\ \bar{s}) \begin{pmatrix} \not{\partial} & 0 & 0 \\ 0 & \not{\partial} & 0 \\ 0 & 0 & \not{\partial} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

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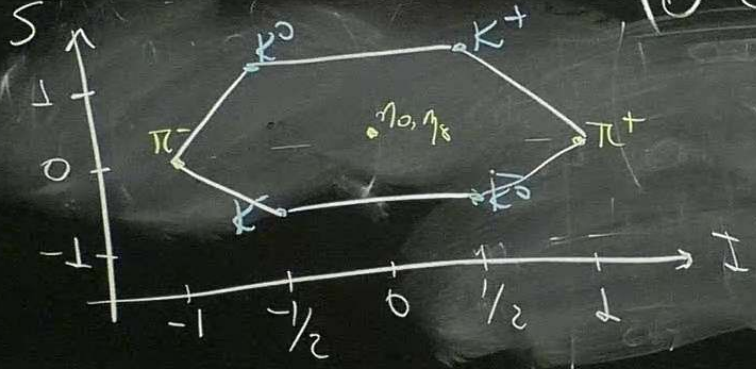


"eightfold way"

$SU_f(3)$

Spontaneous Chiral Symm. Breaking

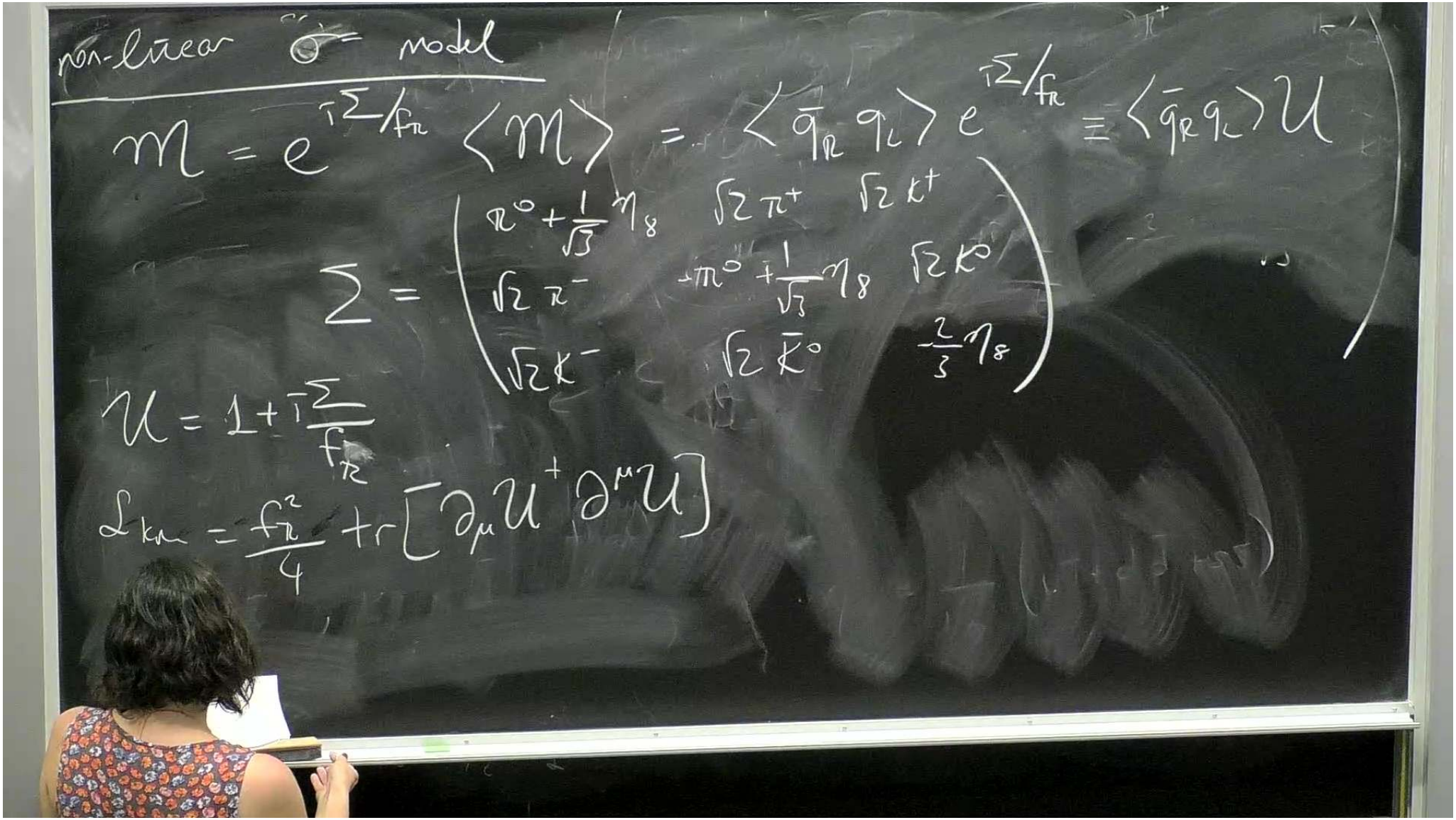
$SU_L(3) \times SU_R(3) \times U_B(1) \xrightarrow{\langle \bar{q}_R q_L \rangle} SU_f(3) \times U_B(1)$

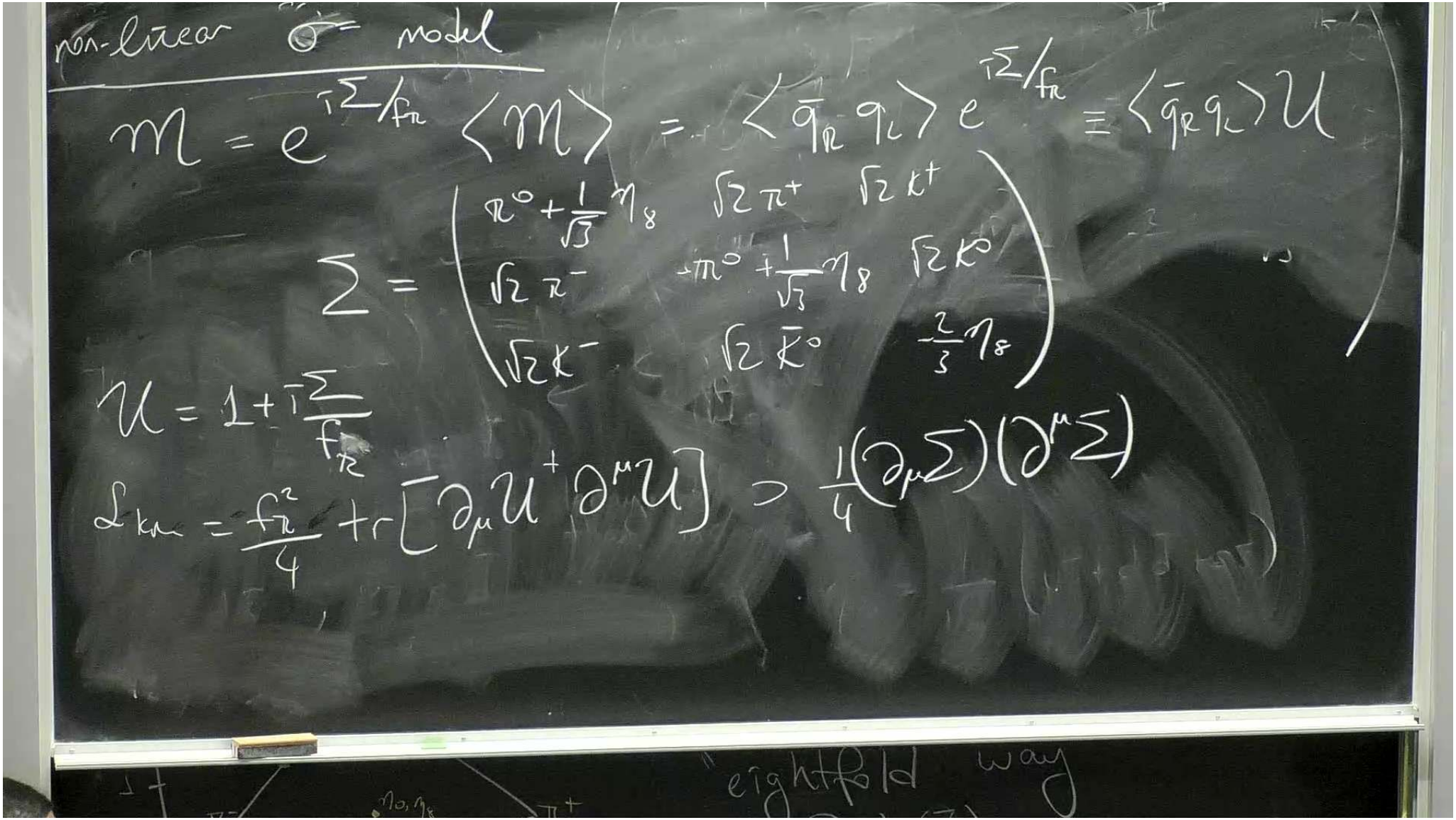
$\langle \bar{q}_R q_L \rangle \sim \Lambda_{QCD}^3$ (naive dimensional analysis) (Ann Nelson)

Goldstone theorem: When a global symmetry (w/ N_G generators) spontaneously breaks down to a smaller symm (w/ N_S generators), there will be $N_G - N_S$ numbers of massless bosons. ← Goldstone

If the symm is not exact,

$m_{goldstone} \sim \Lambda_{sym.\ breaking}$


non-linear σ-model
$M = e^{i\Sigma/f_\pi} \langle M \rangle = \langle \bar{q}_R q_L \rangle e^{i\Sigma/f_\pi} \equiv \langle \bar{q}_R q_L \rangle U$
$\Sigma = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}$
$U = 1 + i\frac{\Sigma}{f_\pi}$
$\mathcal{L}_{kin} = \frac{f_\pi^2}{4} + r\left[\partial_\mu U^+ \partial^\mu U\right]$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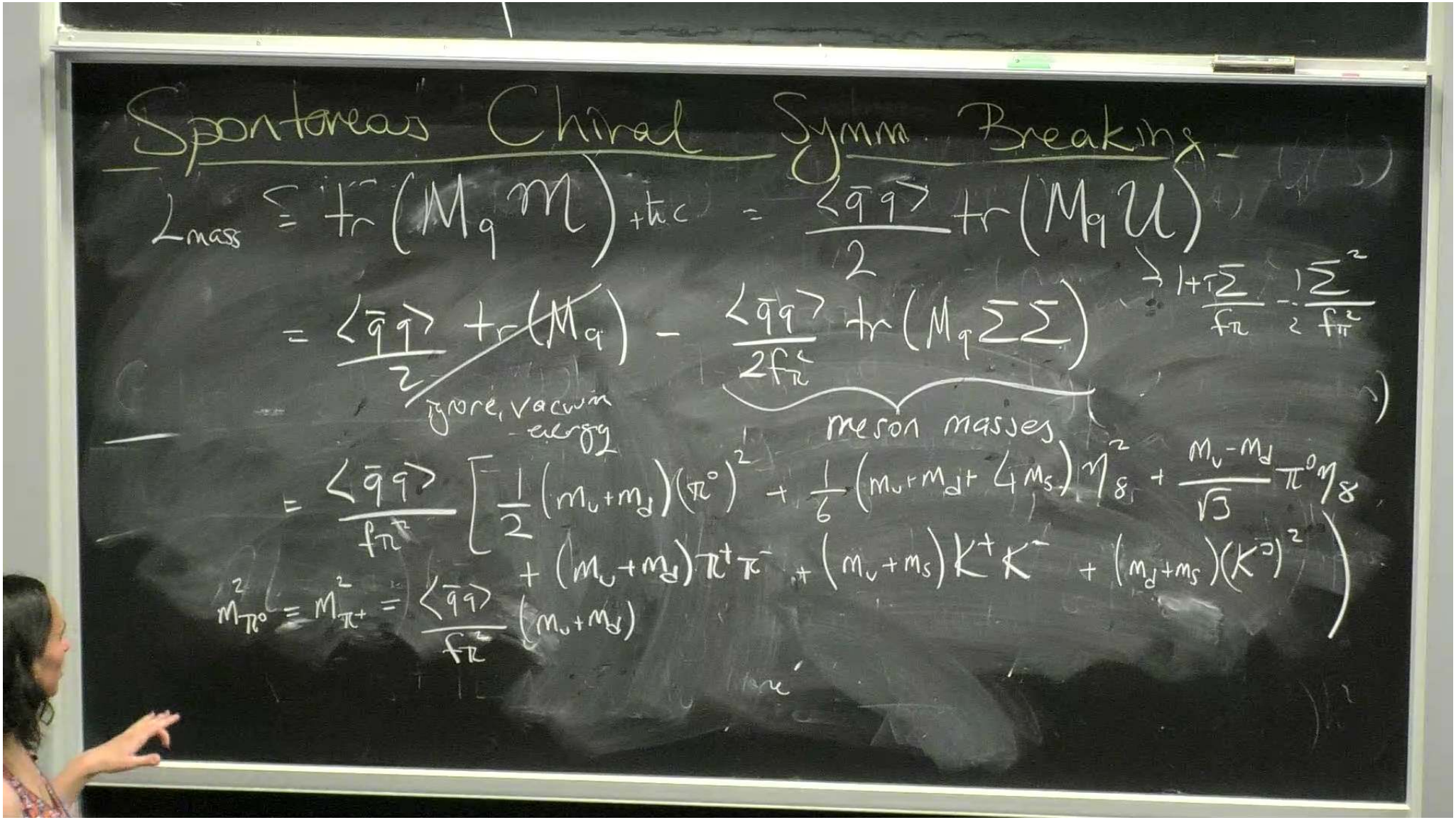
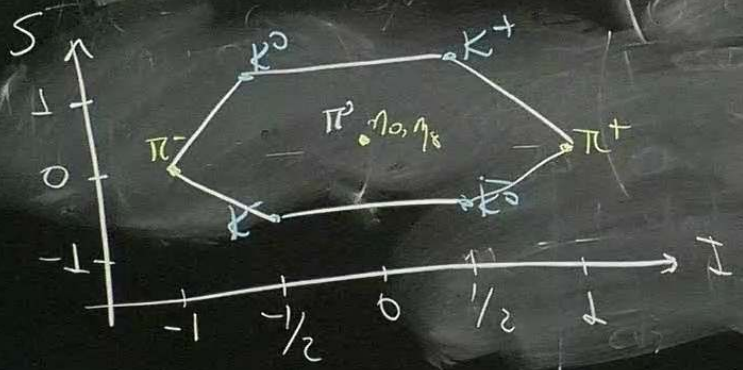
$\Sigma = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}$
$U = 1 + i\frac{\Sigma}{f_\pi}$
$\mathcal{L}_{kin} = \frac{f_\pi^2}{4} + r[\partial_\mu U^+ \partial^\mu U] = \frac{1}{4}(\partial_\mu \Sigma)(\partial^\mu \Sigma)$
eightfold way


Spontaneous Chiral Symm. Breaking:
$\mathcal{L}_{mass} = \mathrm{tr}(M_q m) + h.c. = \frac{\langle \bar{q}q \rangle}{2} \mathrm{tr}(M_q U)$
$\to 1 + i\frac{\Sigma}{f_\pi} - \frac{1}{2}\frac{\Sigma^2}{f_\pi^2}$
$= \frac{\langle \bar{q}q \rangle}{2} \mathrm{tr}(M_q) - \frac{\langle \bar{q}q \rangle}{2 f_\pi^2} \mathrm{tr}(M_q \Sigma\Sigma)$
ignore, vacuum energy
meson masses
$= \frac{\langle \bar{q}q \rangle}{f_\pi^2} \left[\frac{1}{2}(m_u + m_d)(\pi^0)^2 + \frac{1}{6}(m_u + m_d + 4m_s)\eta_8^2 + \frac{m_u - m_d}{\sqrt{3}}\pi^0\eta_8 \right.$
$\left. + (m_u + m_d)\pi^+\pi^- + (m_u + m_s)K^+K^- + (m_d + m_s)(K^0)^2 \right)$
$m_{\pi^0}^2 = m_{\pi^+}^2 = \frac{\langle \bar{q}q \rangle}{f_\pi^2}(m_u + m_d)$

$$m_\eta^2 \simeq \frac{1}{3}\left(2\left(m_{K^+}^2 + m_{K^0}^2\right) - m_\pi^2\right)$$

Gell-Mann Okuba formula



"eightfold way"

$SU_f(3)$

$\eta_0 = u\bar{u} + d\bar{d} + s\bar{s}$