Title: Standard Model

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Subject: Particle Physics

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$$S = \int d^4x \, \mathcal{L}$$

$\mathcal{L}$: Lagrangian (density)

$$\mathcal{L}\left(\phi(x^\mu), \partial_\mu \phi(x^\mu)\right)$$

- real
- invariant under Poincare sym
- an analytical function of the fields
- local

Model = Particle content + Lagrangian

- should be invariant under some symmetry.
- Include every term allowed by the symmetries.
- Renormalizable.

$$\frac{\tilde{\lambda}\phi^6}{\Lambda^2}$$

Standard Model

$SU_c(3) \times SU_L(2) \times U_Y(1)$ → hypercharge.

	Field	Charges
gauge bosons	G_μ^a, $a=1,\dots,8$	$(8,1,0)$
	W_μ^a, $a=1,2,3$	$(1,3,0)$
	B_μ	$(1,1,0)$
leptons $m=e,\mu,\tau$	$l_L^m = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_m$	$(1,2,-1/2)$
	e_R^m	$(1,1,-1)$
quarks $m=1,2,3$	$q_L^m = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_m$	$(3,2,1/6)$
	u_R^m	$(3,1,2/3)$
	d_R^m	$(3,1,-1/3)$

	Field	Charge
Higgs field	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$(1,2,+1/2)$

+ antiparticles

No ν_R^m : $(1,1,0)$

A bit of group theory

$\phi_a \longrightarrow U \phi_a U^* \equiv M_{ab} \phi_b$

ϕ_a: transforms under some sym. group.

M_{ab}: $n \times n$ matrix that depends on how ϕ_a transforms.

Lagrangian is invariant under these transf.

Lie groups: A group which is also a manifold.

any element of the group: $g = \mathbb{1} + i\theta^\alpha t_\alpha$ (t_α: Lie algebra; θ^α: infinitesimal parameter)

$[t_\alpha, t_\beta] = i f^\gamma_{\alpha\beta} t_\gamma$

$f^\gamma_{\alpha\beta}$: structure constants

$$M(\mathbb{1} + i\theta^\alpha t_\alpha) \equiv \mathbb{1} + i\omega^\alpha T_\alpha$$

representation particular to M.

$$[T_\alpha, T_\beta] = i f_{\alpha\beta}^{\ \ \gamma} T_\gamma$$

- Every group has a representation where $T_\alpha = 0$: singlet.
- Fundamental rep: $T_\alpha = t_\alpha$ $\quad \dim = N$
- Adjoint rep: $(T_\alpha)_{bc} = -i f_{bc}$ *

$M(\mathbb{1} + i\theta^\alpha t_\alpha) \equiv \mathbb{1} + i\omega^\alpha T_\alpha$ — T_α: representation particular to M.

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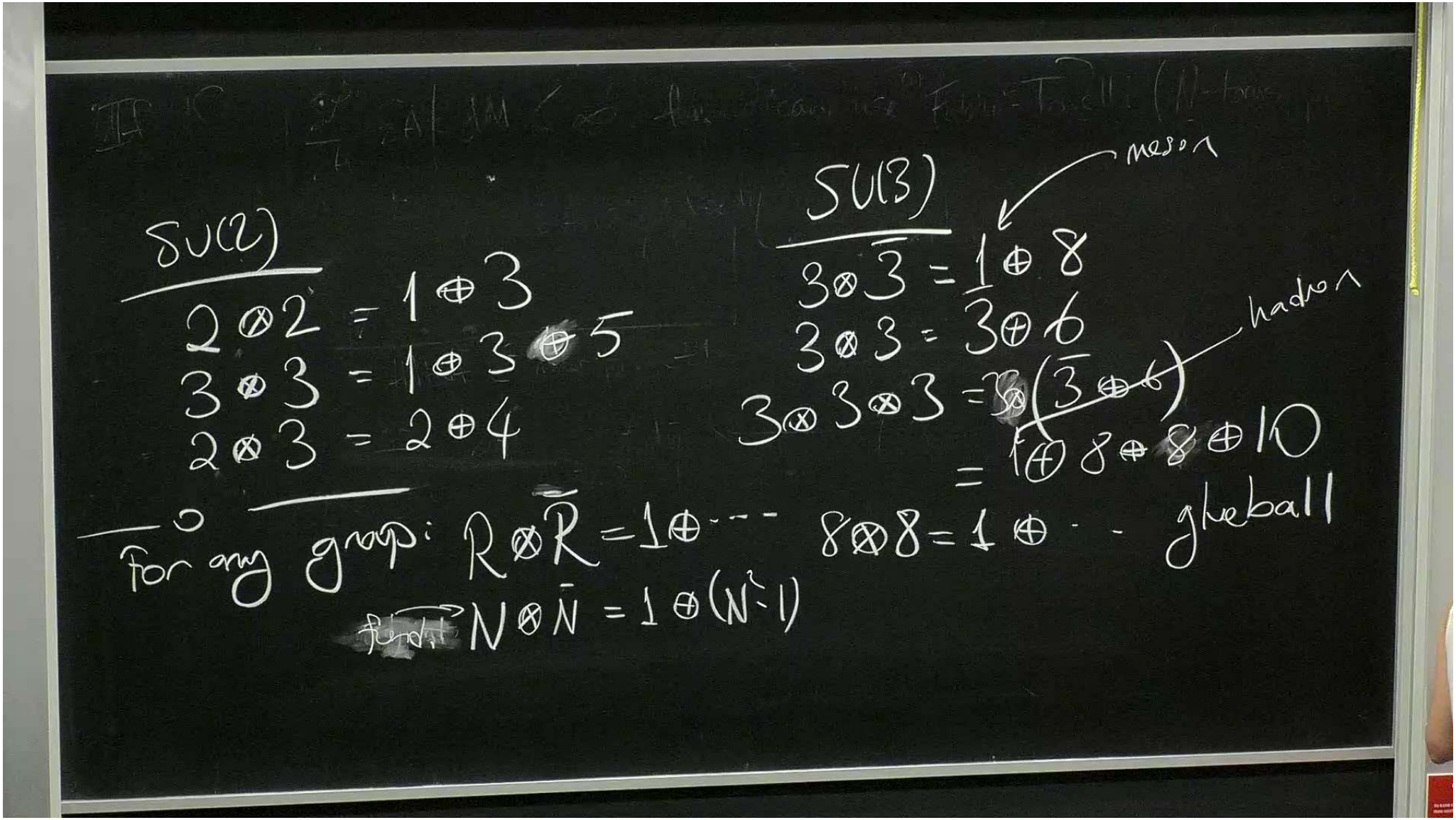
$\phi_a \to M_{ab}\phi_b$

$\chi_a \to N_{ab}\phi_b$

$U(g')\phi_a\chi_b U(g) = \underbrace{M_{ac}N_{bd}}_{\text{tensor product}}\phi_c\chi_d$


SU(2)
2 ⊗ 2 = 1 ⊕ 3
3 ⊗ 3 = 1 ⊕ 3 ⊕ 5
2 ⊗ 3 = 2 ⊕ 4
SU(3)
meson
3 ⊗ 3̄ = 1 ⊕ 8
3 ⊗ 3 = 3 ⊕ 6
hadron
3 ⊗ 3 ⊗ 3 = 3 ⊗ (3̄ ⊕ 6)
= 1 ⊕ 8 ⊕ 8 ⊕ 10
For any group: R ⊗ R̄ = 1 ⊕ ···
fundamental N ⊗ N̄ = 1 ⊕ (N² − 1)
8 ⊗ 8 = 1 ⊕ · · glueball

The SM Lagrangian

$$\mathcal{L}_{SM} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{1}{2} i \bar{L}_m \not{D} L_m - \frac{1}{2} i \bar{\Sigma}_m \not{D} \Sigma_m - \frac{1}{2} i \bar{Q}_m \not{D} Q_m$$

$$- \frac{1}{2} i \bar{U}_m \not{D} U_m - \frac{1}{2} i \bar{D}_m \not{D} D_m$$

$$- \left(Y^e_{mn} \bar{L}_m P_R \Sigma_n \Phi + Y^d_{mn} \bar{Q}_m P_R D_n \Phi + Y^u_{mn} \bar{Q}_m P_R U_n \tilde{\Phi} + h.c. \right)$$

$$- (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$$l_L \equiv P_L L = \begin{pmatrix} P_L \nu \\ P_L \Sigma \end{pmatrix} \qquad \Sigma = \begin{pmatrix} e_L \\ \epsilon e_L^* \end{pmatrix} \quad \Xi = \begin{pmatrix} -\epsilon e_R^* \\ e_R \end{pmatrix} \Big\} \text{ Majorana fields} \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$e_R = P_R \Xi$$

Dirac spinor: $e = P_L \Sigma + P_R \Xi$

$$\tilde{\Phi} = \epsilon \Phi^*$$

Pauli matrices

$$D_\mu L_m = \partial_\mu L_m - \frac{i}{2} g_2 \tau_a W_\mu^a (P_L L_m) + \frac{i}{2} g_2 \tau_a W_\mu^a (P_R L_m) + \frac{i}{2} g_1 B_\mu (P_L L_m) - \frac{i}{2} g_1 B_\mu (P_R L_m)$$

$$D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2} g_2 \tau_a W_\mu^a \Phi - \frac{i}{2} B_\mu \Phi$$

$M(\mathbb{1} + i\theta^\alpha t_\alpha) \equiv \mathbb{1} + i\omega^\alpha T_\alpha$ — T_α: representation particular to M.

$[T_\alpha, T_\beta] = i f_{\alpha\beta}^{\ \ \gamma} T_\gamma$

$m_e \bar{e}_L e_R \Phi$

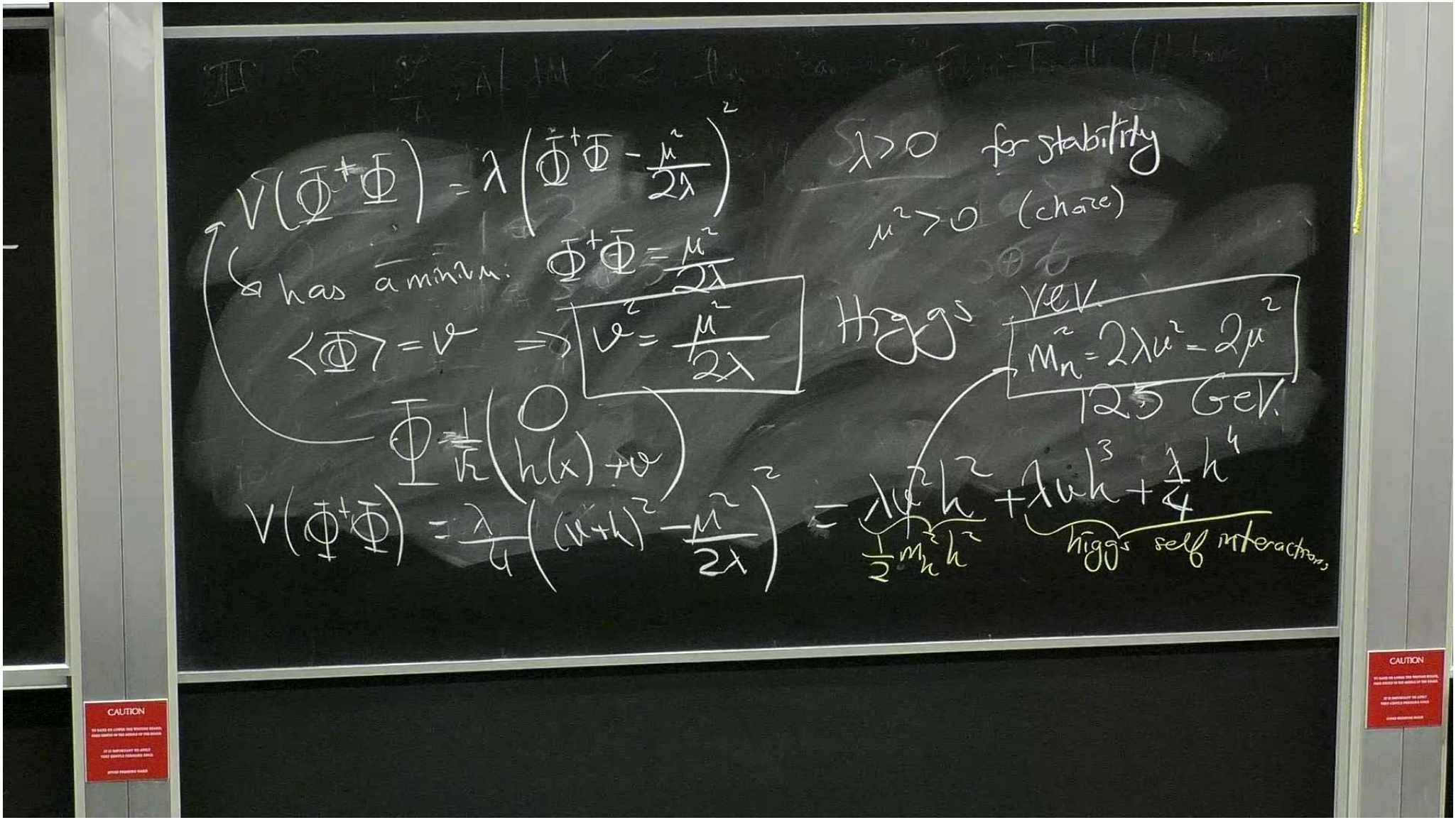
$2 \otimes 1 \otimes 2 = 1 \oplus 3$

- Every group has a representation where $T_\alpha = 0$: singlet.
- Fundamental rep: $T_\alpha = t_\alpha$, $\dim = N$
- Adjoint rep: $(T_\alpha)_{bc} = -i f_{bc}$

$\phi_a \to M_{ab} \phi_b$

$\chi_a \to N_{ab} \phi_b$

$U(g) \phi_a \chi_b U^\dagger(g) = \underbrace{M_{ac} N_{bd}}_{\text{tensor product}} \phi_c \chi_d$


$V(\Phi^\dagger\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{\mu^2}{2\lambda}\right)^2$
$\lambda > 0$ for stability
$\mu^2 > 0$ (chaze)
has a minimum: $\Phi^\dagger\Phi = \frac{\mu^2}{2\lambda}$
$\langle\Phi\rangle = v \Rightarrow v^2 = \frac{\mu^2}{2\lambda}$
Higgs vev
$m_h^2 = 2\lambda v^2 = 2\mu^2$
125 GeV
$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h(x) + v \end{pmatrix}$
$V(\Phi^\dagger\Phi) = \frac{\lambda}{4}\left((v+h)^2 - \frac{\mu^2}{2\lambda}\right)^2 = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4}h^4$
$\frac{1}{2}m_h h^2$
Higgs self interactions
CAUTION
CAUTION

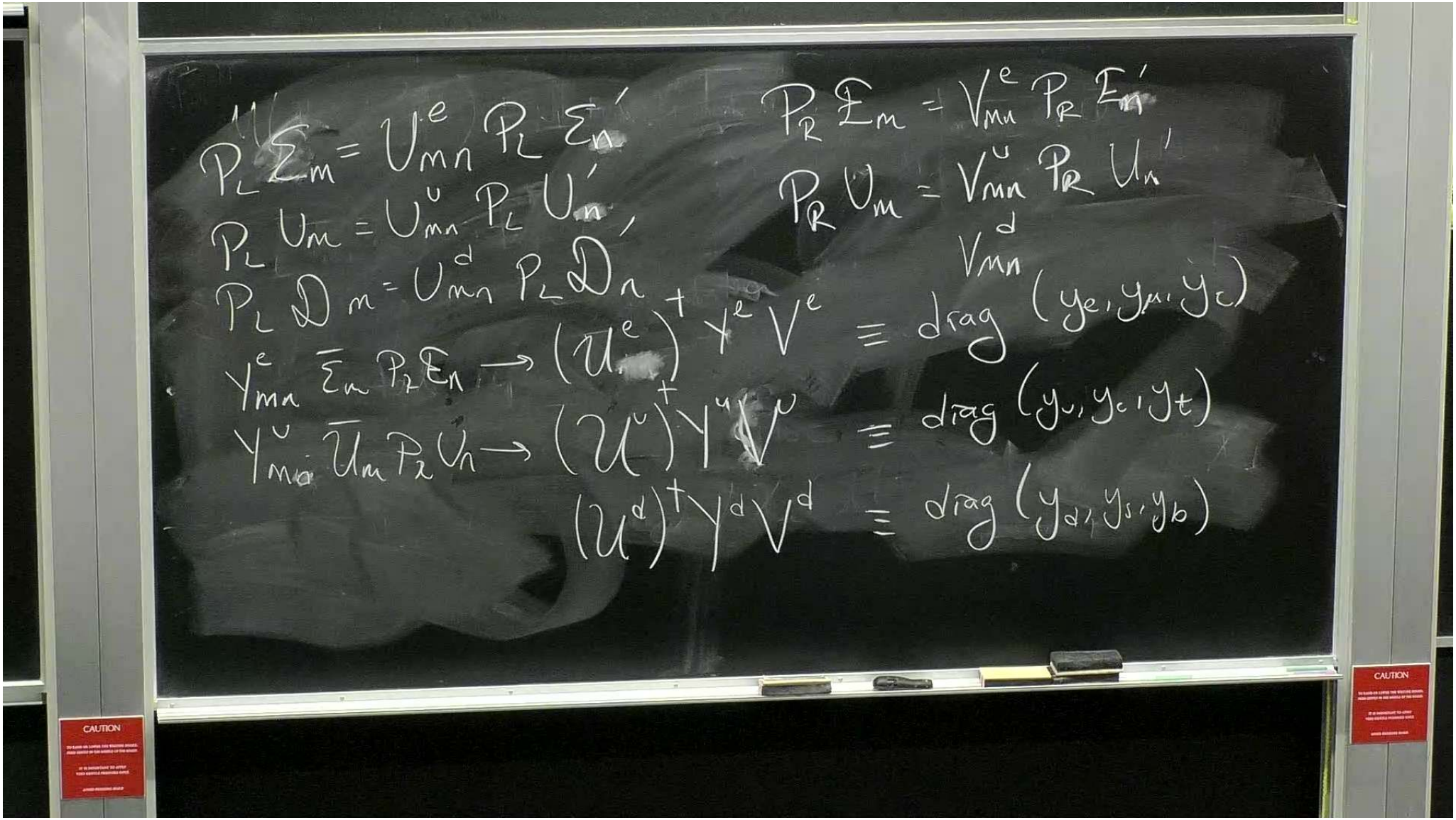
$$-(D_\mu \Phi)(D^\mu \Phi)$$

$$Y^e_{mn} \bar{L}_m P_R E_n \Phi = Y^e_{mn} \begin{pmatrix} \bar{\nu}_m \\ \bar{\varepsilon}_m \end{pmatrix}^T P_R E_n \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= Y^e_{mn} \frac{v}{\sqrt{2}} \bar{\varepsilon}_m P_R E_n + Y^e_{mn} \frac{v}{\sqrt{2}} \underbrace{h \bar{\varepsilon}_m P_R E_n}_{h \bar{f} f}$$

$$\mathcal{L} \supset -\frac{v}{\sqrt{2}} \left(Y^e_{mn} \bar{\varepsilon}_m P_R E_n + Y^u_{mn} \bar{U}_m P_R U_n + Y^d_{mn} \bar{D}_m P_R D_n + \hbar c \right)$$

fermion masses


$P_L \Sigma_m = U^e_{mn} P_L \Sigma_n'$
$P_L U_m = U^u_{mn} P_L U_n'$
$P_L D_m = U^d_{mn} P_L D_n'$
$Y^e_{mn} \bar{\Sigma}_m P_R \Sigma_n \rightarrow (U^e_L)^\dagger Y^e V^e$
$Y^u_{mn} \bar{U}_m P_R U_n \rightarrow (U^u_L)^\dagger Y^u V^u$
$(U^d_L)^\dagger Y^d V^d$
$P_R \Sigma_m = V^e_{mn} P_R \Sigma_n'$
$P_R U_m = V^u_{mn} P_R U_n'$
V^d_{mn}
$\equiv \mathrm{diag}(y_e, y_\mu, y_\tau)$
$\equiv \mathrm{diag}(y_u, y_c, y_t)$
$\equiv \mathrm{diag}(y_d, y_s, y_b)$
CAUTION
CAUTION

$$Y^e_{mn}\,\bar{L}_m P_R E_n \bar{\Phi} = Y^e_{mn}\begin{pmatrix}\bar{\nu}_m \\ \bar{\ell}_m\end{pmatrix}^T P_R E_n \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v+h\end{pmatrix}$$

$$= Y^e_{mn}\frac{v}{\sqrt{2}}\,\bar{\ell}_m P_R \ell_n + Y^e_{mn}\frac{h}{\sqrt{2}}\,\bar{\ell}_m P_R \ell_n$$

$$\mathcal{L} \supset -\frac{v}{\sqrt{2}}\left(Y^e_{mn}\,\bar{\ell}_m P_R \ell_n + Y^u_{mn}\,\bar{u}_m P_R u_n + Y^d_{mn}\,\bar{d}_m P_R d_n + \text{h.c.}\right)$$

$\mathcal{L}_{\text{fermion masses}}$

$$y_f \equiv \frac{y_f v}{\sqrt{2}}$$

$h f \bar{f}$