Title: Higher Categories of Bordisms with Geometric Structures and the Classification of Extended Geometric Field Theories

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Collection/Series: Mathematical Physics

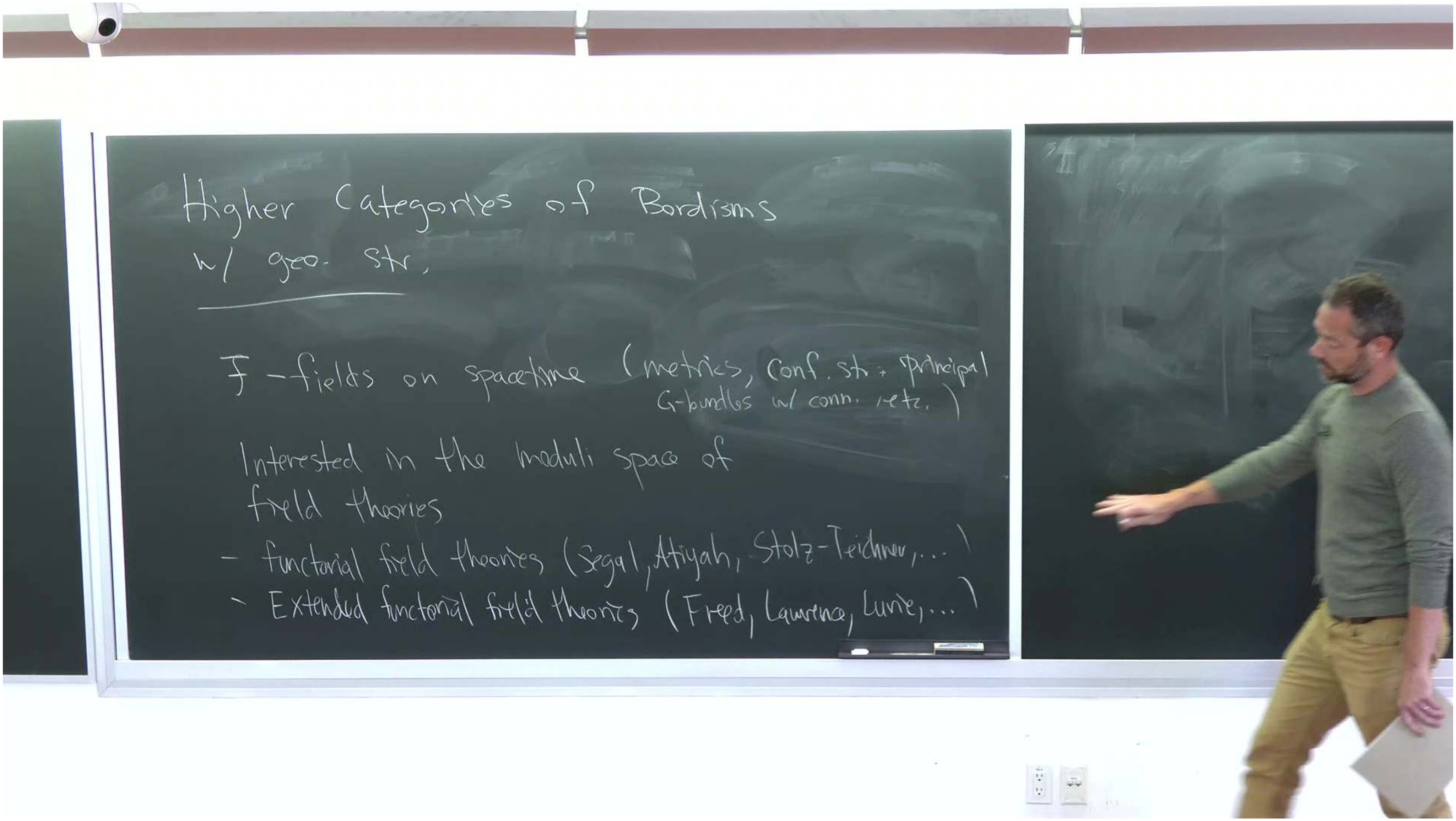
Subject: Mathematical physics

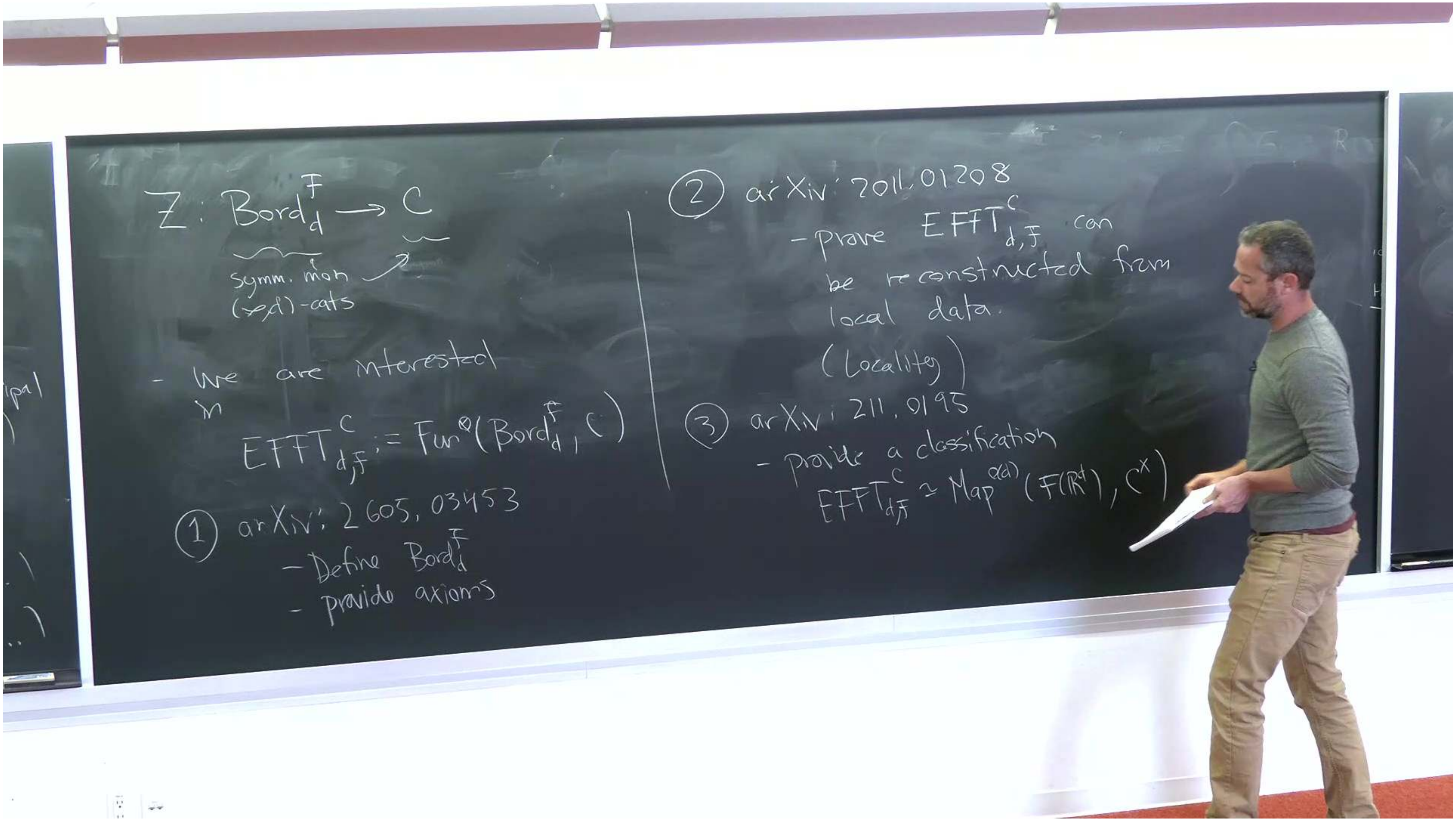
Date: May 29, 2026 - 6:30 PM

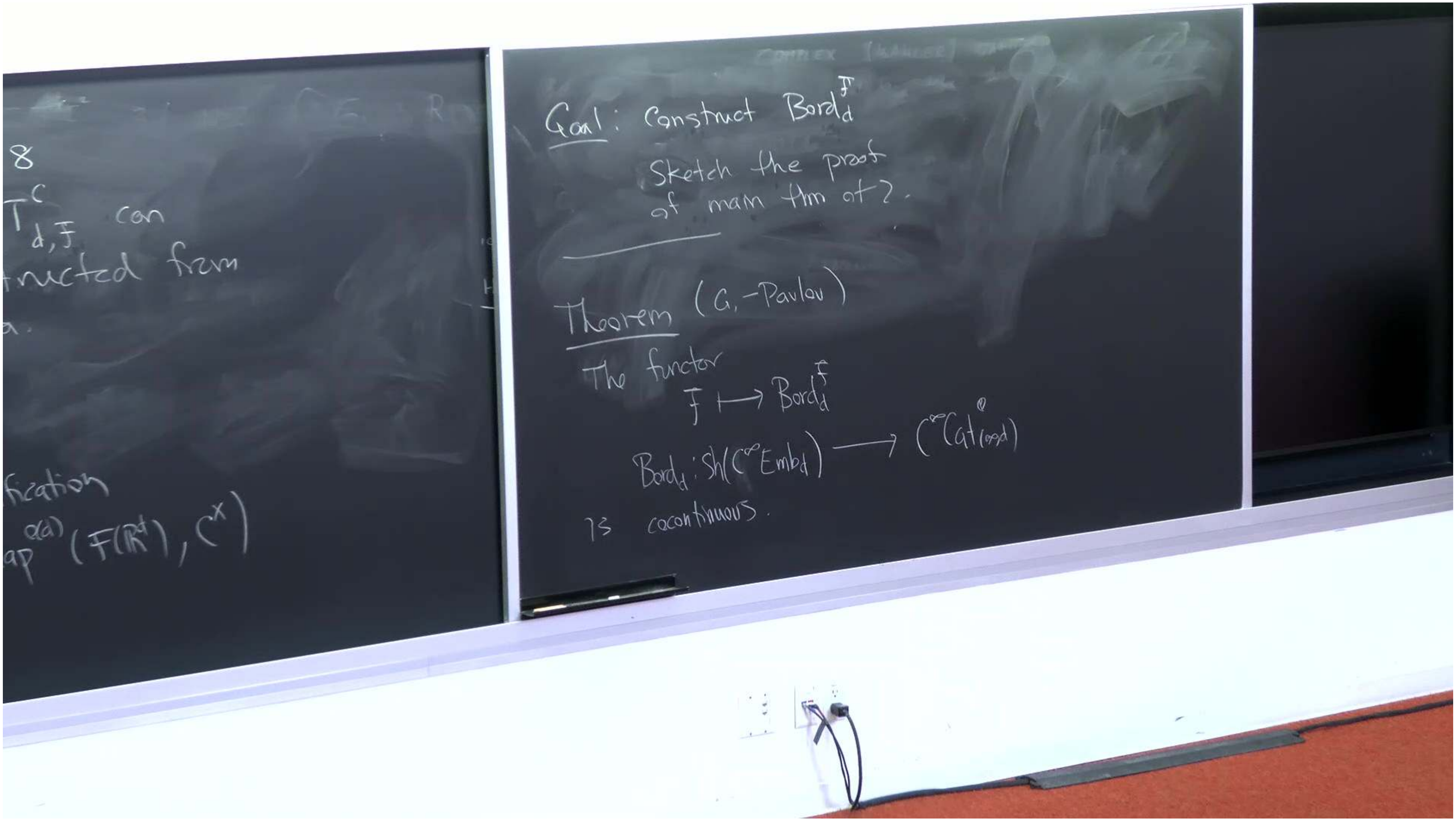
URL: https://pirsa.org/26050066

Abstract:

In this talk, I will survey joint work with Dmitri Pavlov on the construction and classification of extended geometric functorial field theories. After briefly reviewing some background and classical definitions, I will discuss the construction of the extended geometric bordism category and present the two main theorems. I will then sketch the proof of the first theorem, establishing locality for extended field theories, and, if time permits, conclude with some applications.


Higher Categories of Bordisms
w/ geo. str.
F - fields on spacetime (metrics, conf. str., principal
G-bundles w/ conn., etc.)
Interested in the moduli space of
field theories
- functorial field theories (Segal, Atiyah, Stolz-Teichner, ...)
- Extended functorial field theories (Freed, Lawrence, Lurie, ...)


$Z : \mathrm{Bord}_d^{\mathcal{F}} \longrightarrow \mathcal{C}$
symm. mon
(∞,d)-cats
- We are interested in
$\mathrm{EFFT}^{\mathcal{C}}_{d,\mathcal{F}} := \mathrm{Fun}^{\otimes}(\mathrm{Bord}_d^{\mathcal{F}}, \mathcal{C})$
(1) arXiv: 2605.03453
- Define $\mathrm{Bord}_d^{\mathcal{F}}$
- provide axioms
(2) arXiv: 2011.01208
- prove $\mathrm{EFFT}^{\mathcal{C}}_{d,\mathcal{F}}$ can be reconstructed from local data.
(Locality)
(3) arXiv: 211.0195
- provide a classification
$\mathrm{EFFT}^{\mathcal{C}}_{d,\mathcal{F}} \simeq \mathrm{Map}^{O(d)}(\mathcal{F}(\mathbb{R}^d), \mathcal{C}^{\times})$


Goal: Construct $\mathrm{Bord}_d^{\mathcal{F}}$
Sketch the proof of main thm of ?.
Theorem (G.-Pavlov)
The functor
$\mathcal{F} \longmapsto \mathrm{Bord}_d^{\mathcal{F}}$
$\mathrm{Bord}_d : \mathrm{Sh}(C^\infty\mathrm{Emb}_d) \longrightarrow C^\infty\mathrm{Cat}^{\otimes}_{(\infty,d)}$
is cocontinuous.
8
$T^{c}_{d,\mathcal{F}}$ can
structed from
fication
$\mathrm{ap}^{o(d)}(\mathcal{F}(\mathbb{R}^d), \mathbb{C}^\times)$

Corollary: $\mathcal{F} = \operatorname{colim}_i \mathcal{F}_i$

$\mathbb{B}^n_\nabla U(1)$

$$\mathrm{EFFT}^c_{d,\mathcal{F}} \simeq \lim_i \mathrm{EFFT}^c_{d,\mathcal{F}_i}$$

Ex: $\mathcal{F}$ = Smooth maps to X

$\{U_i\}$ is an open cover,

$$X = \operatorname*{colim}_{[n]\in\Delta^{op}} \coprod_{\alpha:[n]\to A} U_{\alpha_0} \cap U_{\alpha_1} \cap \cdots \cap U_{\alpha_n}$$

$$\Rightarrow \mathrm{EFFT}^c_{d,X} \simeq \lim_{[n]\in\Delta^{op}} \prod_{\alpha:[n]\to A} \mathrm{EFFT}^c_{d,U_\alpha}$$

Smooth symm. mon
(∞,d)-cats

$\Delta^{\times d}$ obj $\mathbf{m} = ([m_0],[m_1],\dots,[m_d])$

Mor: $\mathbf{m} \to \mathbf{n}$ tuples of order preserving maps

Cart: Obj: $U \subset \mathbb{R}^n$, $U \cong \mathbb{R}^n$

Mor: $f: U \to V$

Γ obj: $\langle \ell \rangle = \{*, 1, \dots, \ell\}$

Mor: $\langle \ell \rangle \to \langle k \rangle$, $\{*, 1, \dots, k\} \to \{*, 1, \dots, \ell\}$

$$C^\infty \mathrm{Cat}^{\otimes}_{(\infty,d)} = \mathrm{PSh}(\Delta^{\times d} \times \mathrm{Cart} \times \Gamma)_{\mathrm{loc}}$$

loc: Segal gluing
+ globular
+ sheaf condition
+ univalence

$\{*, 1, \ldots, k\} \to \{*, 1, \ldots, \ell\}$

loc

egal gluing
globular
sheaf condition
univalence

Def: We define $C^\infty\mathrm{Emb}_d$

ob: submersions $p: M \to U$
w/ d-dim fibers

Mor:
$$\begin{tikzcd} M \arrow[r, "g"] \arrow[d, "p"'] & N \arrow[d, "q"] \\ U \arrow[r, "f"] & V \end{tikzcd}$$
s.t. g restricts to an open emb. fiberwise

is

$\{*, 1, \ldots, k\} \to \{*, 1, \ldots, \ell\}$

$\times \Gamma)$

$\underbrace{\text{loc}}$

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covering families
$$\left\{ \begin{tikzcd} M_i \arrow[r, hook, "g_i"] \arrow[d, "p"'] & M \arrow[d, "p"] \\ U_i \arrow[r, hook, "f_i"] & U \end{tikzcd} \right\}$$
s.t. $\{ M_i \overset{g_i}{\hookrightarrow} M \}$

is

Def: A **field stack** is an object

$$\mathcal{F} \in Sh(C^\infty Emb_d)$$

Def: let $p: M \to U \in C^\infty Emb_d$. A **cut** is a triple $C = (C_<, C_=, C_>)$

$$C_<, C_=, C_> \subset M$$

s.t. $\exists\, h: M \to \mathbb{R}$ s.t. 0 is reg val

$$h^{-1}(0) = C_=, \quad h^{-1}(-\infty, 0) = C_<, \quad h^{-1}(0, \infty) = C_>$$

$^{-1}(0,\infty) = C_{>}$

The set of cuts is a poset

$C \leq C' \iff C_{\leq} \subset C'_{\leq}$

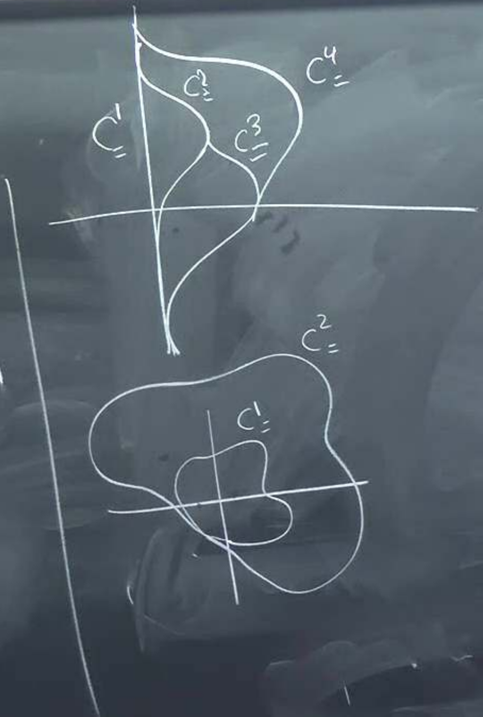
A cut [m] $\in \Delta$ -tuple is a functor

$$[m] \longrightarrow \mathrm{Cut}(P)$$

$C^1_{=} \quad C^2_{=} \quad C^3_{=}$ $\qquad d=2$

ex:

$p : \mathbb{R}^2 \to P^+$



<u>Def</u>: We define a functor called the <u>cut</u> <u>grid</u> <u>functor</u>

$\mathrm{Cut}_\phi : (\Delta^{\times d})^{op} \times C^\infty\mathrm{Emb}_d \to \mathrm{Set}$

$\mathrm{Cut}_\phi([m], p: M \to U)$

- $\forall\, 1 \le i \le d$, a cut $[m_i]$-tuple

ϕ: $\forall\, j: \{1,2,\dots,d\} \to \mathbb{Z}$

define

submers

w/

is

a

the

mb$_d \to$ Set

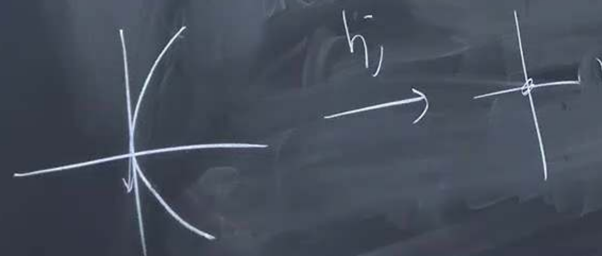
t. $[m_i]$-tuple

$\to \mathbb{Z}$

s.t. $0 \le j_i \le m_i$

$\exists\, h_j : M \to \mathbb{R}^d$ s.t. $\pi_i \circ h_j : M \to \mathbb{R}$
is a height funct for C_{j_i}

and fiberwize reg values form
an open neighborhood of $0 \in \mathbb{R}^d$



is

We define a functor

$$\mathrm{Bord}_d^{\mathcal{F}} : \mathrm{Cart}^{op} \times (\Delta^{\times d})^{op} \times P^{op} \longrightarrow \mathrm{sSet}$$

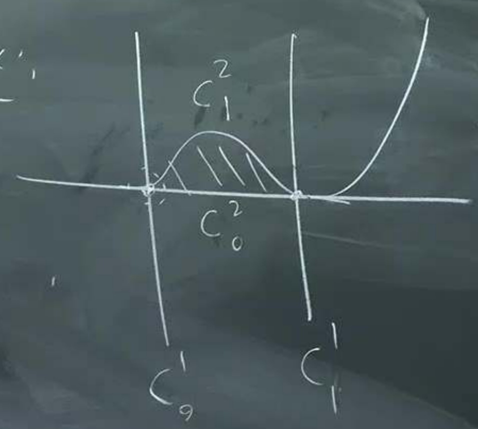
$B \in \mathrm{Bord}_d^{\mathcal{F}}(U, m, \langle \ell \rangle)_0$ is

(1) $p : M \to U$

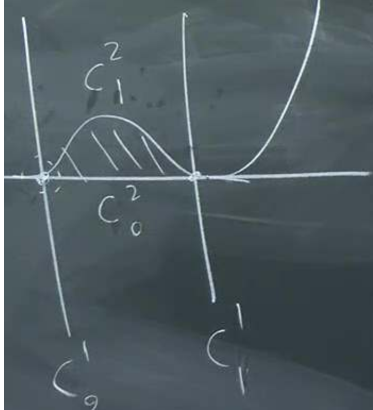
(2) $C \in \mathrm{Cut}_\phi(p)$

(3) $\sigma \in \mathcal{F}(p)$.

ex:



Set



Axioms:

(A1) Bord_d defines a functor

$$\mathcal{F} \longmapsto \mathrm{Bord}_d^{\mathcal{F}}$$

(A2) $\mathcal{F} \longmapsto \mathrm{Bord}_d^{\mathcal{F}}$ cocontinuous at the level of presheaves

(A3) For reps $q: N \to V$

defines a

$\mathrm{Bord}_d^{\mathcal{F}}$

Bord_d

$\mathrm{Bord}_d^{\mathcal{F}}$

at the

presheaves

ps $N \to V \in C^\infty\mathrm{Emb}_d$

Bord_d^{yq} is w.e

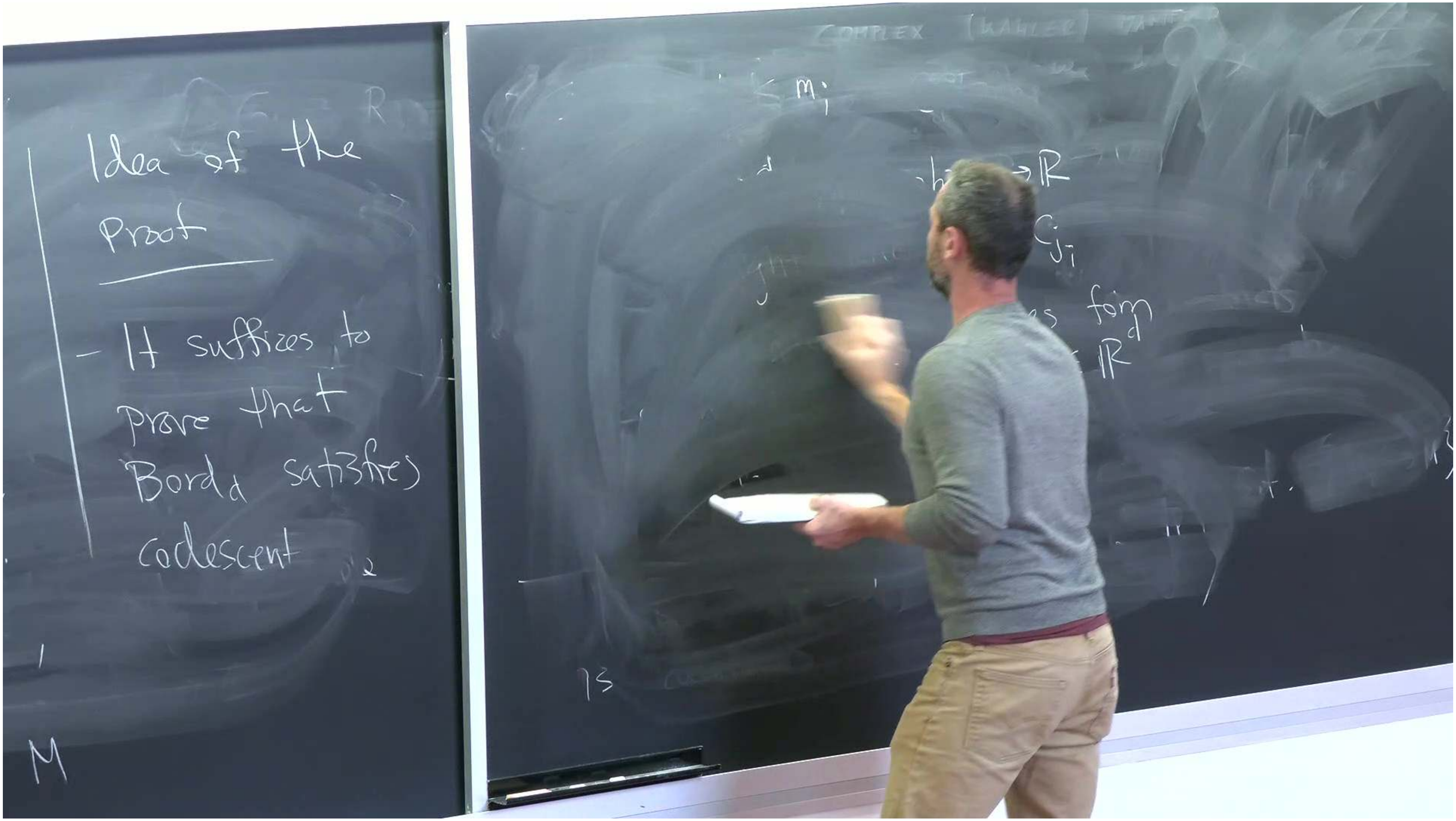
to EBord_d

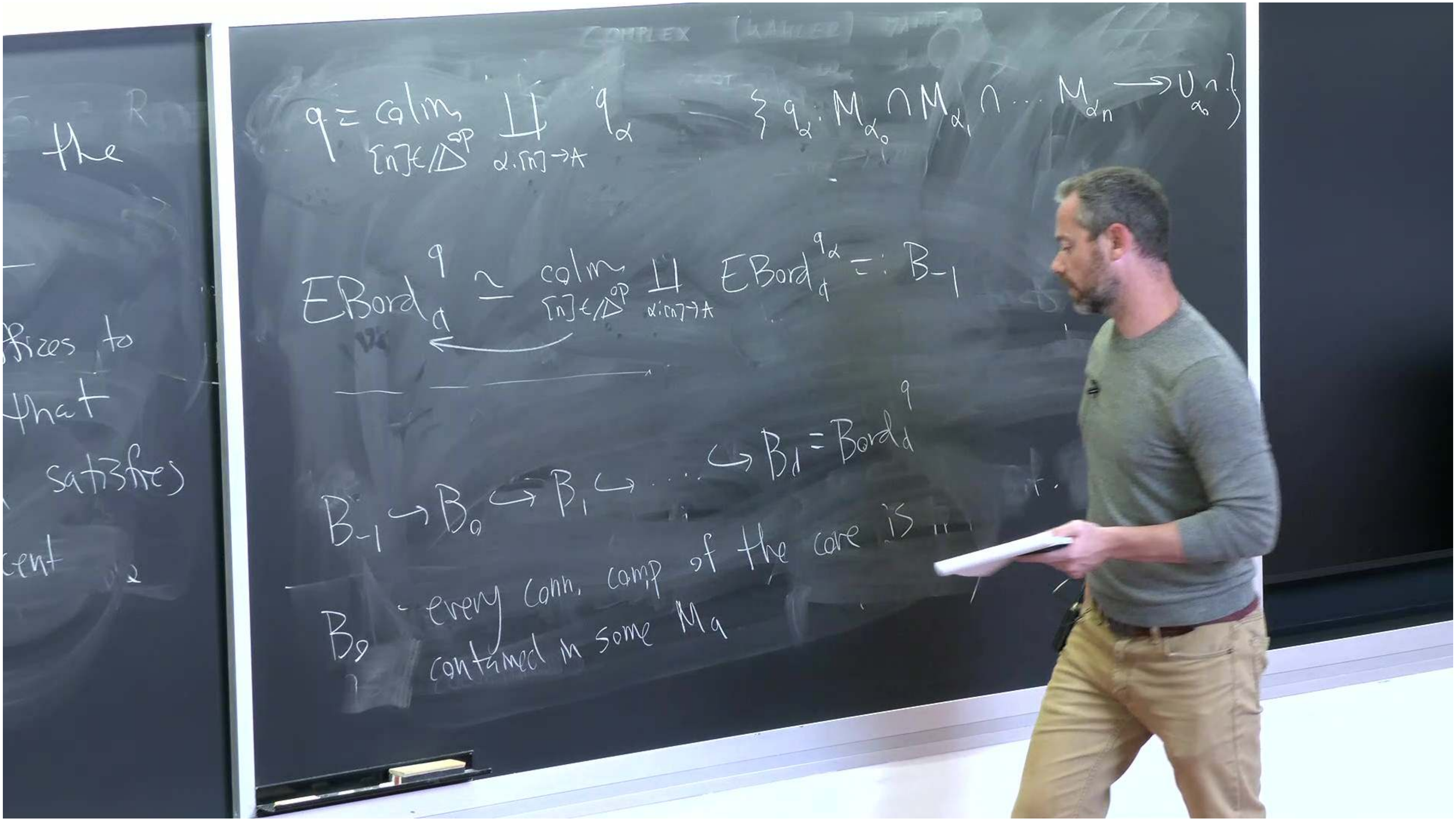
<u>Def</u>: $\mathrm{EBord}_d^q(U, \mathfrak{m})$

(1) sm map $f: U \to V$

(2) an open subset $M \subset f^*N$

(3) a cut grid on M


Idea of the Proof
- It suffizes to prove that Bord$_d$ satisfies codescent



the
fizes to
that
satisfies
cent
$q = \mathrm{colim}_{[n]\in\Delta^{op}} \coprod_{\alpha:[n]\to A} q_\alpha$
$\{q_\alpha : M_{\alpha_0}\cap M_{\alpha_1}\cap\ldots M_{\alpha_n} \to U_{\alpha_0}^n\}$
$EBord_d^q \simeq \mathrm{colim}_{[n]\in\Delta^{op}} \coprod_{\alpha:[n]\to A} EBord_d^{q_\alpha} =: B_{-1}$
$B_{-1} \to B_0 \hookrightarrow B_1 \hookrightarrow \ldots \hookrightarrow B_d = Bord_d^q$
B_0 : every conn. comp of the core is contained in some M_a

the

fizes to

that

satisfies

cent

$$q = \operatorname{colim}_{[n]\in\Delta^{op}} \coprod_{\alpha:[n]\to A} q_\alpha \qquad \{ q_\alpha : M_{\alpha_0} \cap M_{\alpha_1} \cap \dots M_{\alpha_n} \longrightarrow U_{\alpha_0}^{n} \}$$

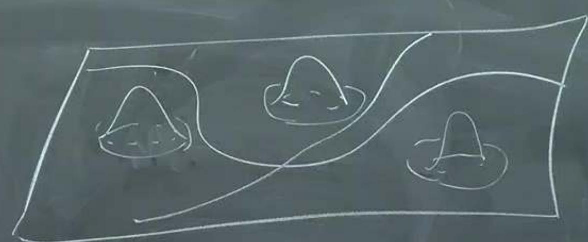
$$\mathrm{EBord}_d^{q} \simeq \operatorname{colim}_{[n]\in\Delta^{op}} \coprod_{\alpha:[n]\to A} \mathrm{EBord}_d^{q_\alpha} =: B_{-1}$$

$$B_{-1} \to B_0 \hookrightarrow B_1 \hookrightarrow \dots \hookrightarrow B_d = \mathrm{Bord}_d^{q}$$

B_0 - every conn. comp of the core is contained in some M_a

B_i - every conn comp of the i-core is in Ma.

$d=3,\ i=2$



$B_{i-1} \hookrightarrow B_i$