Title: A Non-Semisimple Categorical Symmetry on the Lattice

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Abstract:

Semisimple categorical symmetries, and their physical implications have been studied in great detail in the continuum and on the lattice. In particular, the nice properties of semisimple categories allows one to construct hamiltonians which enjoy this symmetry, and study its symmetric gapped phases. Using the anyonic chain framework in (1+1)d, I will present a model that has a finite but non-semisimple categorical symmetry. Sacrificing Hermiticity, I will present several symmetric, frustration-free, gapped Hamiltonians with real spectra and analyze their ground state subspaces. I then discuss two intriguing phenomena. First, I will identify a smooth path of gapped symmetric Hamiltonians whose ground states transform inequivalently under the symmetry. Second, I will discuss a model where a product state and the so-called W state spontaneously break the symmetry, and propose an explanation for the indistinguishability of these two states in the infinite-volume limit in terms of the symmetry category.

Taft alg T_4

$T_4 = \mathbb{C}\langle x, g \mid x^2 = 0,\ g^4 = 1,\ xg = -gx \rangle$

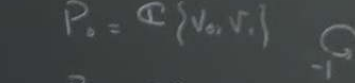
$\Delta(g) = g \otimes g$ $\quad \Delta(x) = x \otimes 1 + g \otimes x$

$\varepsilon(g) = 1$ $\quad \varepsilon(x) = 0$

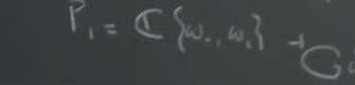
$S(g) = g$ $\quad S(x) = -gx$

$\mathrm{Mod}(T_4)$: $S_0 = \mathbb{C}\{w_0\}$ w/ $xw_0 = 0$, $gw_0 = w_0$,

$S_1 = \mathbb{C}\{v_0\}$ w/ $xv_0 = 0$, $gv_0 = -v_0$,

$P_0 = \mathbb{C}\{v_0, v_1\}$ 

,

$P_1 = \mathbb{C}\{w_0, w_1\}$ 

$0 \to S_0 \to P_1 \to S_1 \to 0$

$0 \to S_1 \to P_0 \to S_0 \to 0$

Monoidal Structure $(\alpha_1 \neq \alpha_2)$

$S_{\alpha_1} \otimes S_{\alpha_2} = S_0$, $S_{\alpha_1} \otimes S_{\alpha_1} = S_1$, $S_{\alpha_1} \otimes P_{\alpha_1} = P_0$

$S_{\alpha_1} \otimes P_{\alpha_1} = P_1$, $P_{\alpha_1} \otimes P_{\alpha_1} = P_0 \otimes P_1$, $P_{\alpha_1} \otimes P_{\alpha_2} = P_0 \otimes P_1$

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A non-Semi simple Categorical Symmetry on the Lattice

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Categorify finite Sym ⟶ fusion categories

↓

are rigid Semi-simple $\mathbb{C}$-linear monoidal cats w/ finitely many Simple objects.

↘ Direct sums of Simple objects.

↓

X is simple if every subobject $A \hookrightarrow X$ is either trivial or an equiv.

* No local topological operators.

What have we done with fusion cats $\mathcal{C}$.

Symmetric gapped phases of Systems with $\mathcal{C}$ sym $\longleftrightarrow$ (finite s.s.) $\mathcal{C}$-module cats. $\mathcal{M}$.

Ground States $\longleftrightarrow$ Simple objects in $\mathcal{M}$

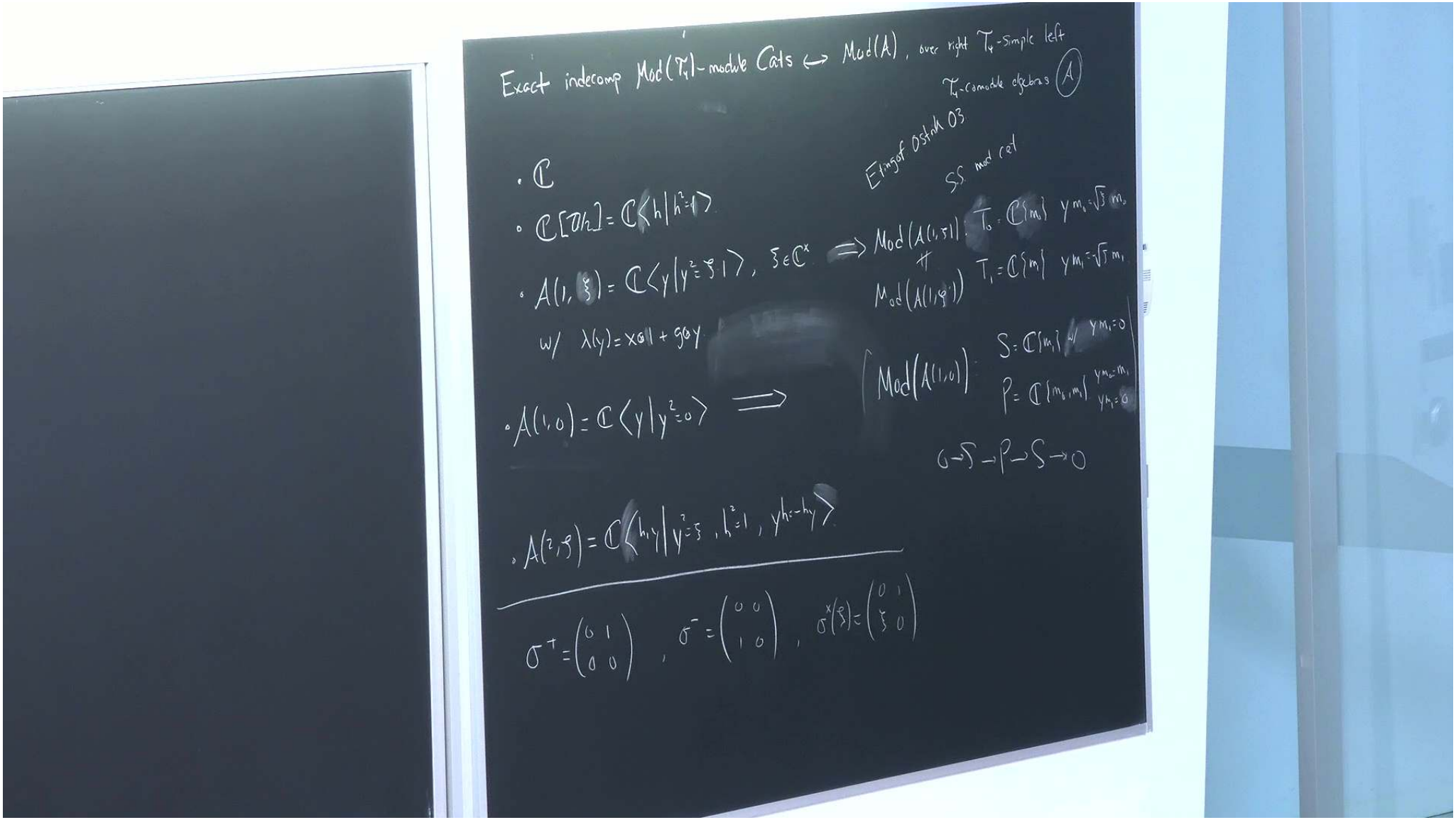
Goal: Construct a (1+1)d lattice model with a action by a non-s.s. tensor cat.

Q| Do sym gapped phases correspond to exact indecomp mod cats?

Q| What role does the sym enforce about unitarity of the Hamiltonian?

Q| Is there already something out there that can secretly be described by a non-s.s. cat sym?

Running Example: $\mathcal{C} = \mathrm{Mod}(T_4)$


Exact indecomp Mod(T_4)-module Cats ↔ Mod(A), over right T_4-Simple left T_4-comodule algebras (A)
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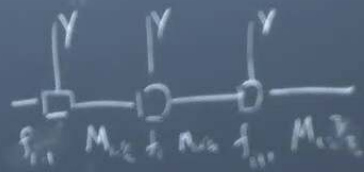
SS mod cat
$\mathbb{C}$
$\mathbb{C}[\mathbb{Z}/2] = \mathbb{C}\langle h \mid h^2 = 1\rangle$
$A(1,\xi) = \mathbb{C}\langle y \mid y^2 = \xi 1\rangle$, $\xi \in \mathbb{C}^\times$ ⟹ Mod(A(1,ξ)) ≠ Mod(A(1,ξ'))
w/ $\lambda(y) = x \otimes 1 + g \otimes y$
$T_0 = \mathbb{C}[m_0]$ $ym_0 = \sqrt{\xi}\, m_0$
$T_1 = \mathbb{C}[m_1]$ $ym_1 = \sqrt{\xi}\, m_1$
$A(1,0) = \mathbb{C}\langle y \mid y^2 = 0\rangle$ ⟹
Mod(A(1,0)): $S = \mathbb{C}[m_1]$ w/ $ym_1 = 0$
$P = \mathbb{C}[m_0, m_1]$ $ym_0 = m_1$, $ym_1 = 0$
$0 \to S \to P \to S \to 0$
$A(2,\xi) = \mathbb{C}\langle h, y \mid y^2 = \xi,\ h^2 = 1,\ yh = -hy\rangle$
$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $\sigma^x(\xi) = \begin{pmatrix} 0 & 1 \\ \xi & 0 \end{pmatrix}$

Study Hamiltonians & their ground states for $\mathcal{C}$ R $\mathcal{A}(1,0)$ and $\mathcal{A}(1,S)$.

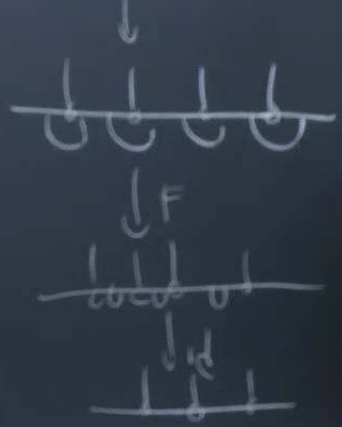
Anyon Chain: Pick $\Lambda \subset \mathbb{Z}$, $\mathcal{C}$, $M \in \mathrm{Mod}(\mathcal{C})$ and $Y \in \mathcal{C}_M^* = \mathrm{Fun}_{\mathcal{C}}(M,M)$. (M: vect; $= \mathrm{Cond}(T_4)$)

$H_\Lambda = \mathrm{Span}_{\mathbb{C}}\{$  $\}$

$\cong \bigoplus_{\{M\},\, i \in \Lambda} \bigotimes \mathrm{Hom}_{\mathcal{M}}(M_{i-1} \triangleleft Y, M_{i+1})$

S.

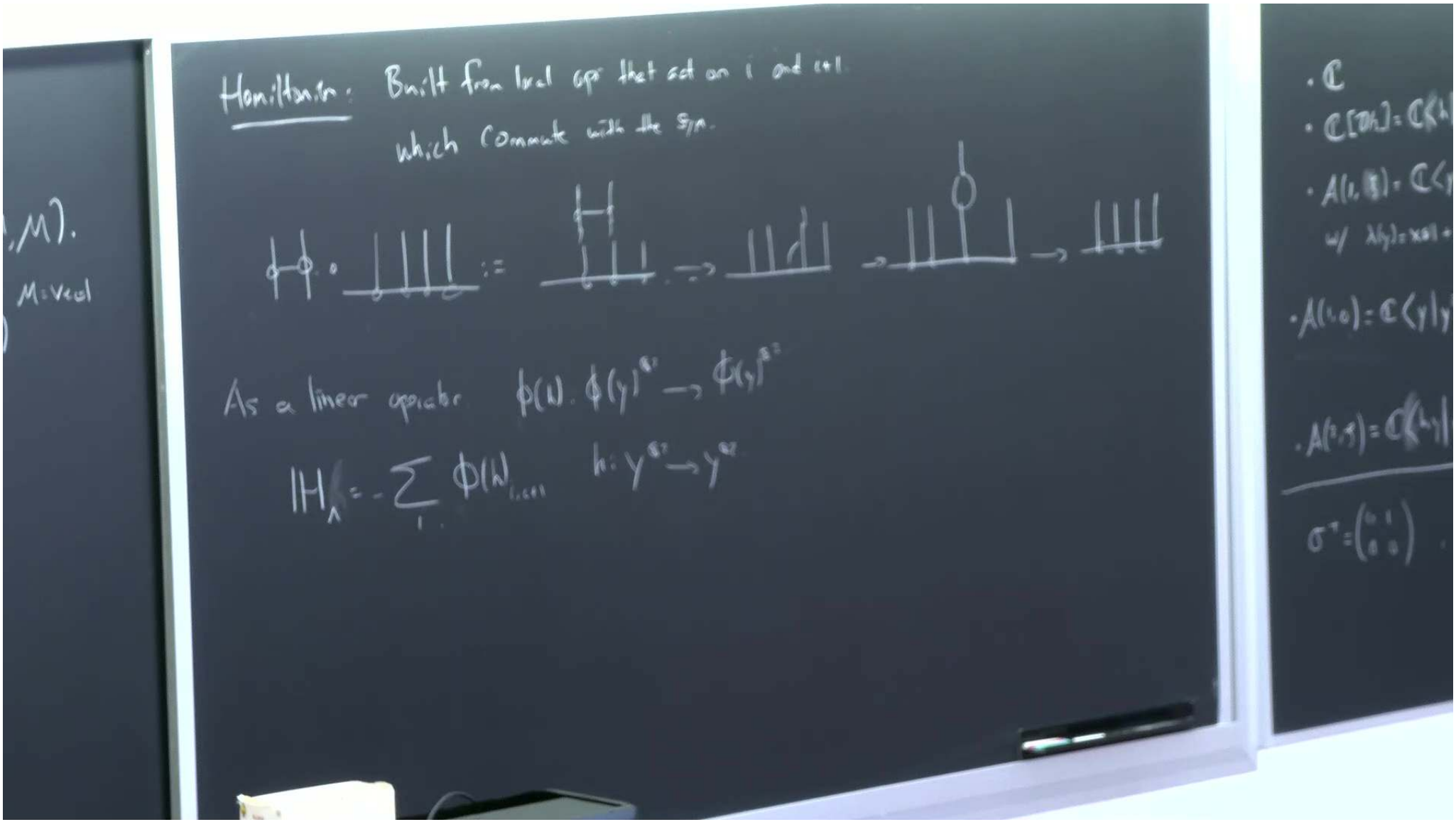
$\phi : \mathrm{Mod}(T_4) \to \mathrm{Vect}$



s.s. tensor cat.

amiltonian?

ed by a non-s.s. cat sym?


Hamiltonian: Built from local ops that act on i and i+1
which commute with the sym.
As a linear operator
$H_\Lambda = -\sum_i \phi(h)_{i,i+1}$
$h: y^{q^2} \to y^{q^2}$

Find Hamiltonians for ground states labelled by $Mod(A(1,q))$.

Let $Y = A(1,q)$

turn the morphism $A(1,q)^{\otimes 2} \longrightarrow A(1,q)^{\otimes 2}$

$\Delta \circ m : A(1,q)^{\otimes 2} \xrightarrow{m} A(1,q) \xrightarrow{\Delta} A(1,q)^{\otimes 2}$

$H(q) = -\sum \phi(\Delta \circ m)_{i,i+1}$

$\mathbb{1}_i \otimes (\sigma^+\sigma^-)_{i+1} + \sigma^x(q)_i \otimes \sigma^-_{i+1}$

Properties: Gapped, frustration-free, non-Hermitian, real spectrum

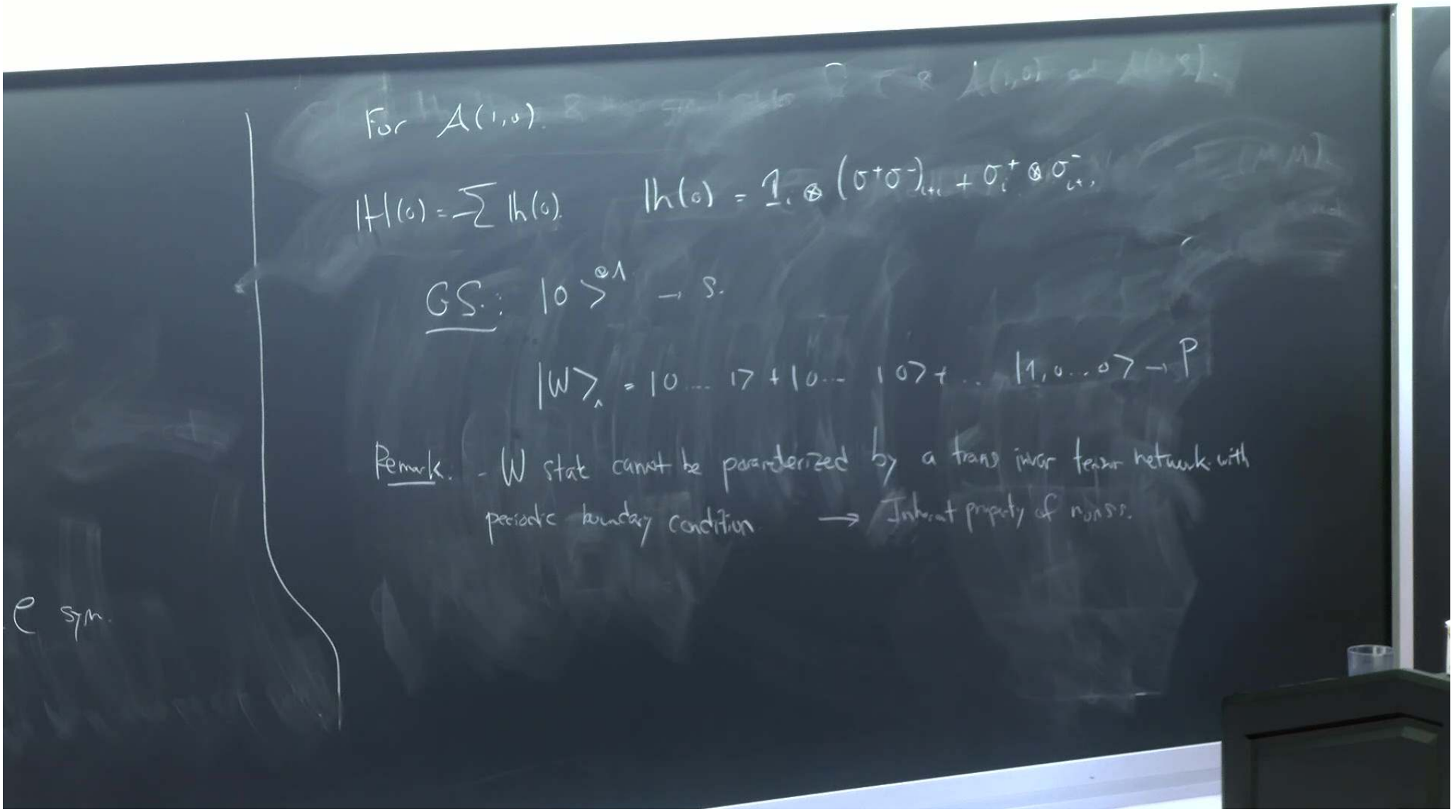
Two degenerate ground states

$|+\xi\rangle^{\otimes\Lambda}$ with $|+\xi\rangle = |0\rangle + \sqrt{\xi}|1\rangle \to T_0$

$|-\xi\rangle^{\otimes\Lambda}$ with $|-\xi\rangle = |0\rangle - \sqrt{\xi}|1\rangle \to T_1$

Result: For $\xi \neq \xi'$ the ground states transform differently under the $\mathcal{C}$ sym.

$\mathrm{Mod}(\mathcal{A}(1,\xi))$

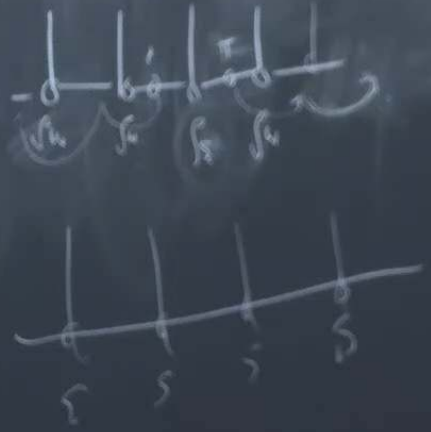

For A(1,0)
GS: $|0\rangle^{\otimes 1} \to S$.
$|W\rangle_N = |0\ldots 1\rangle + |0\ldots 10\rangle + \ldots |1,0\ldots 0\rangle \to P$
Remark. - W state cannot be parameterized by a trans invar tensor network with periodic boundary condition → Intrinsic property of nonlocality.

2). Thurgren-Yang, W-state cannot be the sole ground state of a local Hamiltonian

its accompanied by $|0\rangle$ ↪ Statement of the module cat $\mathcal{P}_{m|1}\mathcal{S}$.

3) In the inf Vol limit $|\psi\rangle$ is indistinguishable from $|0\rangle$, ∃ local defect hosting them to each other.

$S \xrightarrow{i} P \xrightarrow{\pi} S$



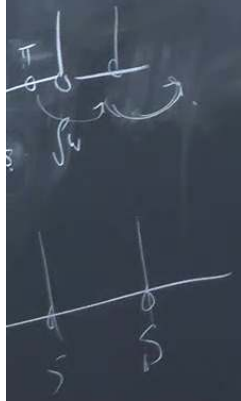
- State cannot be the single ground state of a local Hamiltonian.

ccompanied by $|0\rangle^{\wedge}$.

↳ Statement of the module cat $P \to S$.

t $|w\rangle$ is indistinguishable from $|0\rangle$, ∃ local defect taking them to each other.

$S \xrightarrow{\iota} P \xrightarrow{\pi} S$.



Exact indecomp $\mathrm{Mod}(T_4)$-module Cats $\hookleftarrow$ $\mathrm{Mod}(A)$, over right T_4-Simple left T_4-comodule algebras A

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SS mod cat

- $\mathbb{C}$
- $\mathbb{C}[\mathbb{Z}/2] = \mathbb{C}\langle h \mid h^2 = 1\rangle$
- $A(1,\xi) = \mathbb{C}\langle y \mid y^2 = \xi 1\rangle$, $\xi \in \mathbb{C}^\times$ $\Rightarrow$ $\mathrm{Mod}(A(1,\xi)) \simeq \mathrm{Mod}(A(1,\xi^{-1}))$

  w/ $\lambda(y) = x \otimes 1 + g \otimes y$

  $T_0 = \mathbb{C}[m]$ $\quad ym_0 = \sqrt{\xi} m_0$

  $T_1 = \mathbb{C}[m]$ $\quad ym_1 = -\sqrt{\xi} m_1$

- $A(1,0) = \mathbb{C}\langle y \mid y^2 = 0\rangle$ $\Rightarrow$ $\mathrm{Mod}(A(1,0))$

  $S = \mathbb{C}[m_1]$ w/ $ym_1 = 0$

  $P = \mathbb{C}[m_0, m_1]$ $\begin{matrix} ym_0 = m_1 \\ ym_1 = 0 \end{matrix}$

  $0 \to S \to P \to S \to 0$

- $A(\xi,\xi) = \mathbb{C}\langle h, y \mid y^2 = \xi,\ h^2 = 1,\ yh = -hy\rangle$

$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $\sigma^x(\xi) = \begin{pmatrix} 0 & 1 \\ \xi & 0 \end{pmatrix}$