Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 30 - May 1 2026

Subject: Condensed Matter

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Fractional Quantum Hall Effect (FQHE).

(2+1)d: Dual formulation in dynamical U(1) gauge field.

Superfluid.

$$Z[A] = \int Da_\mu \, e^{i\int d^3x \left(f_{\mu\nu}^2 + \frac{1}{2\pi} A \wedge da \right)}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu.$$

Trivial Insulator $A^2 \sim (A-\partial\theta)^2$

Add CS term.

$$S_{cs}[a] = -\frac{k}{4\pi}\int a \wedge da\,, \quad k \in \mathbb{Z}$$

① Gapped. $\frac{\delta S}{\delta a} = 0 \Rightarrow$ mass gap $m \sim g^2 |k|$.

Maxwell irrelevant at low energy.

Energy $\ll m \sim g^2 |k|$, ~~Maxwell~~.

$$\Rightarrow S[a, A] = \int \left[-\frac{k}{4\pi} a da + \frac{1}{2\pi} a dA \right] \qquad \text{Topological QFT}$$

$$\int \mathcal{D}a_0 \, e^{i a_0 (\partial_x a_y - \partial_y a_x)}$$

$\Rightarrow \int \mathcal{D}a$ imposes constraint. $k \, da = dA$.

Turn off $A = 0$. $da = 0$.

Locally: $a = 0$ No local d.o.f

$U(1)_k$ CS theory. Laughlin state.

Turn on A. Locally, $a = \frac{A}{k}$

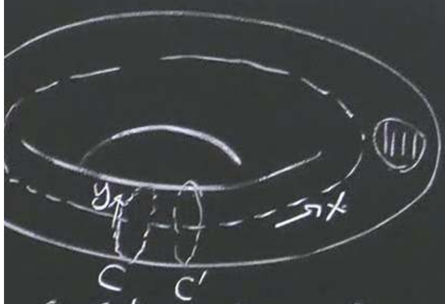
$$S[A] = \int \frac{1/k}{4\pi} A dA. \Rightarrow \text{Hall conductivity } \sigma_{xy} = \frac{1}{2\pi} \frac{1}{k}$$

"Trivial insulator": $\sigma_{xy} \in \frac{1}{2\pi} \mathbb{Z}$. $k=1$ O.K

$\uparrow$

No interesting d.o.f at low energy

$|k|>1$: "FQHE" $\rightarrow$ Still interesting stuff at low energy.



$(S^1 \times S^1) \times \mathbb{R}$

$\boxed{\text{Space}}$: Torus $\quad q \in \mathbb{Z}$.

$da = 0, \quad e^{iq\oint_{C_{x,y}} \vec{a} \cdot d\vec{l}}$: Wilson loop.

$\int Da_{=}$

Gauge transform: $\vec{a} \to \vec{a} + \nabla\varphi$

φ can wind by $2\pi n$ around $C_{x,y}$.

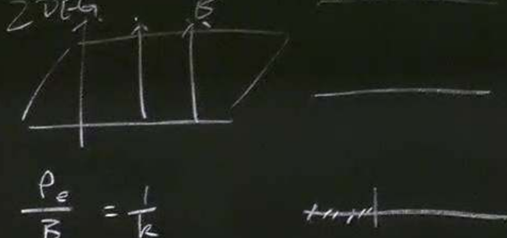
$\int_C \vec{a} \cdot dl \to \int_C \vec{\nabla}\varphi \cdot dl = 2\pi n$ "Large gauge transform",

$U(1)_k$ CS theory. Laughlin state.

Locally, $a = \frac{A}{k}$

$A] = \int \frac{1/k}{4\pi} A dA . \Rightarrow$ Hall conductivity $\sigma_{xy} = \frac{1}{2\pi} \frac{1}{k}$

2DEG $\vec{B}$



$\frac{\rho_e}{B} = \frac{1}{k}$

...vial insulator" : $\sigma_{xy} \in \frac{1}{2\pi} \mathbb{Z}$. $k=1$ O.K

$\uparrow$

...esting d.o.f

low energy

$|k|>1$: "FQHE" $\to$ Still interesting stuff at low energy.

W_x, W_y commute only up to $e^{i\frac{2\pi}{k}}$.

$q = k$, $W_x^k = e^{ik\int_C a\, dl}$ Commutes with everything.

$= \text{Identity}$.

Space: genus g

$GSD = |k|^g$.

W_x: Z_k

$k=2$: qubit, two-dim. degenerate g.S.

k-fold. G.S.D.

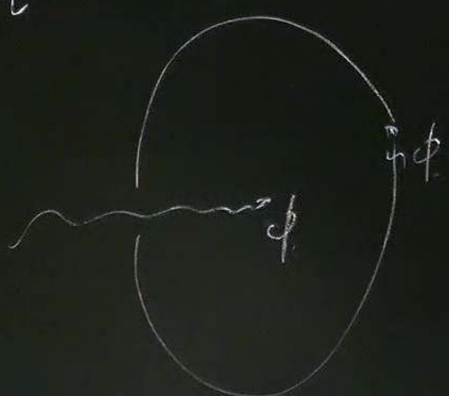
$k=1$: trivial algebra. Unique. g.S. $\Rightarrow$ IQHE

Introduce scalar field with gauge charge, gapped

$$S[\phi, a] = \int d^3x \left[-\frac{1}{2} |(\partial_\mu - i a_\mu)\phi|^2 + m^2 |\phi|^2 \right] \qquad m^2 >> 0$$

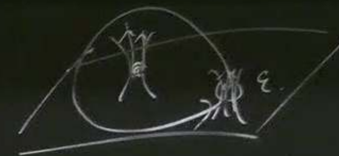
Open Wilson line: $\phi_2^\dagger \left[e^{i\int_{C_{12}} \vec{a}\cdot d\vec{l}} \right] \phi_1$: particle-moving operator.

$\Rightarrow$ Braiding phase $e^{i\frac{2\pi}{k}}$



Exchange phase $e^{i\frac{\pi}{k}}$, $|k|>1$. "Anyon".

$S[a_\mu, j_\mu] = \int d^3x \left(-\frac{k}{4\pi} a\,da + a\cdot j + \frac{1}{2\pi} a\,dA\right)$ c, $c_\alpha c^\dagger_\beta c_\gamma$



$A=0$: $\int \mathcal{D}a_0$, $\int d^2x \left[\nabla\times\vec{a} = \frac{2\pi}{k} j^0\right] \Rightarrow$ Gauge charge q. traps gauge flux $\frac{2\pi q}{k}$.

$\frac{2\pi q^2}{k}$

δA_0: $Q = \int d^2x \, \frac{\nabla\times\vec{a}}{2\pi} = \frac{q}{k}$ Fractional charge $q<|k|$.

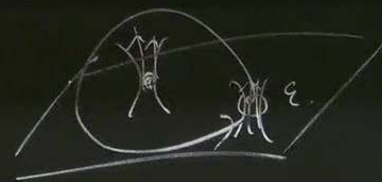
If $q=k$, $\int \nabla\times\vec{a} = 2\pi$. local operator. no string attached: monopole

$Q=1$ k: odd electron.

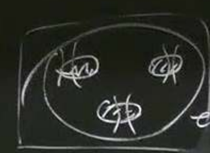
$\theta_{exchange} = k\pi$ k = even. local $Q=1$ boson

$c_\alpha c^{+}_\beta c_\gamma$

traps gauge flux $\frac{2\pi q}{k}$.



$k=3$



$\frac{2\pi q^2}{k}$

Electron : "fractionalized"

$q < |k|$

ttached : monopole

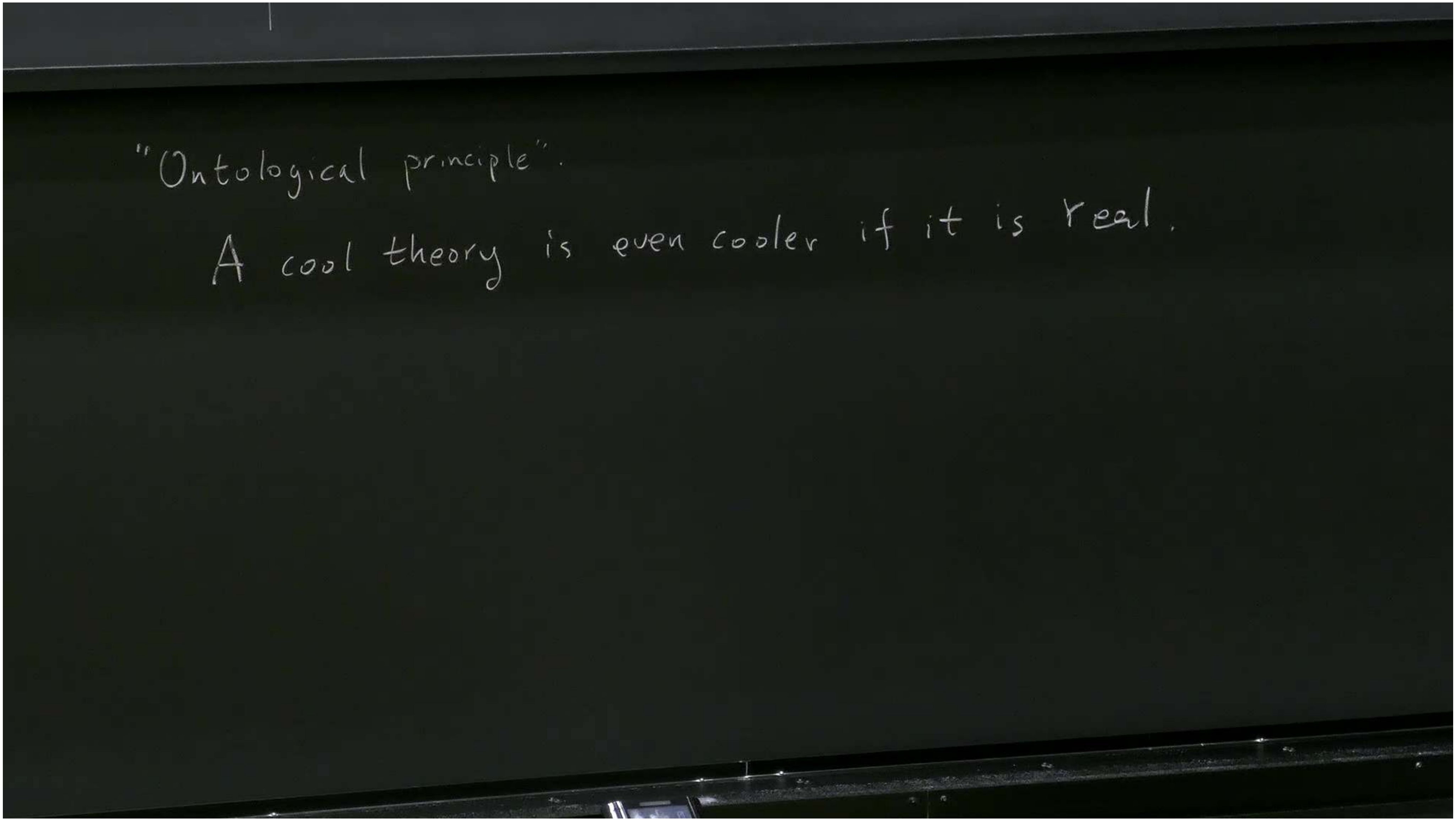
el boson

FHQE ($G_{xy} = \frac{e^2}{h} \frac{1}{3}, \frac{2}{5} \dots \frac{1}{2}$) : 1980's

Theory : 1980's - 1990's.

Fractional charge exp.: $\sim$2000.

Anyon statistics exp.: $\sim$2020


"Ontological principle".
A cool theory is even cooler if it is real.