Title: Lecture - Quantum Gravity, PHYS 644

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Subject: Quantum Gravity

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3d diffeo constr. $H_{\parallel}^{j} = \bar{\nabla}_i \pi^{ij} = 0$

Hamiltonian c. $H_{\perp} = \frac{1}{2\sqrt{h}} \underbrace{G_{ijke}}_{\text{DeWitt metric on } \mathrm{Riem}(\Sigma)} \pi^{ij}\pi^{ke} + \underbrace{V(h)}_{\text{potential}} = 0$ $\quad \frac{\sqrt{h}}{2}\left(\bar{R}(h) - 2\Lambda\right)$

$G_{ijke} = h_{ik}h_{je} + h_{jk}h_{ie} - h_{ij}h_{ke}$

$H_{\parallel}(X) = \int_{\Sigma} H_{\parallel}^{j} X_j$ $\quad \sim$ Noether charges for $\xi = X + Nn$

$H_{\perp}(N) = \int_{\Sigma} H_{\perp} N$

$\{H_{\parallel}(X),$

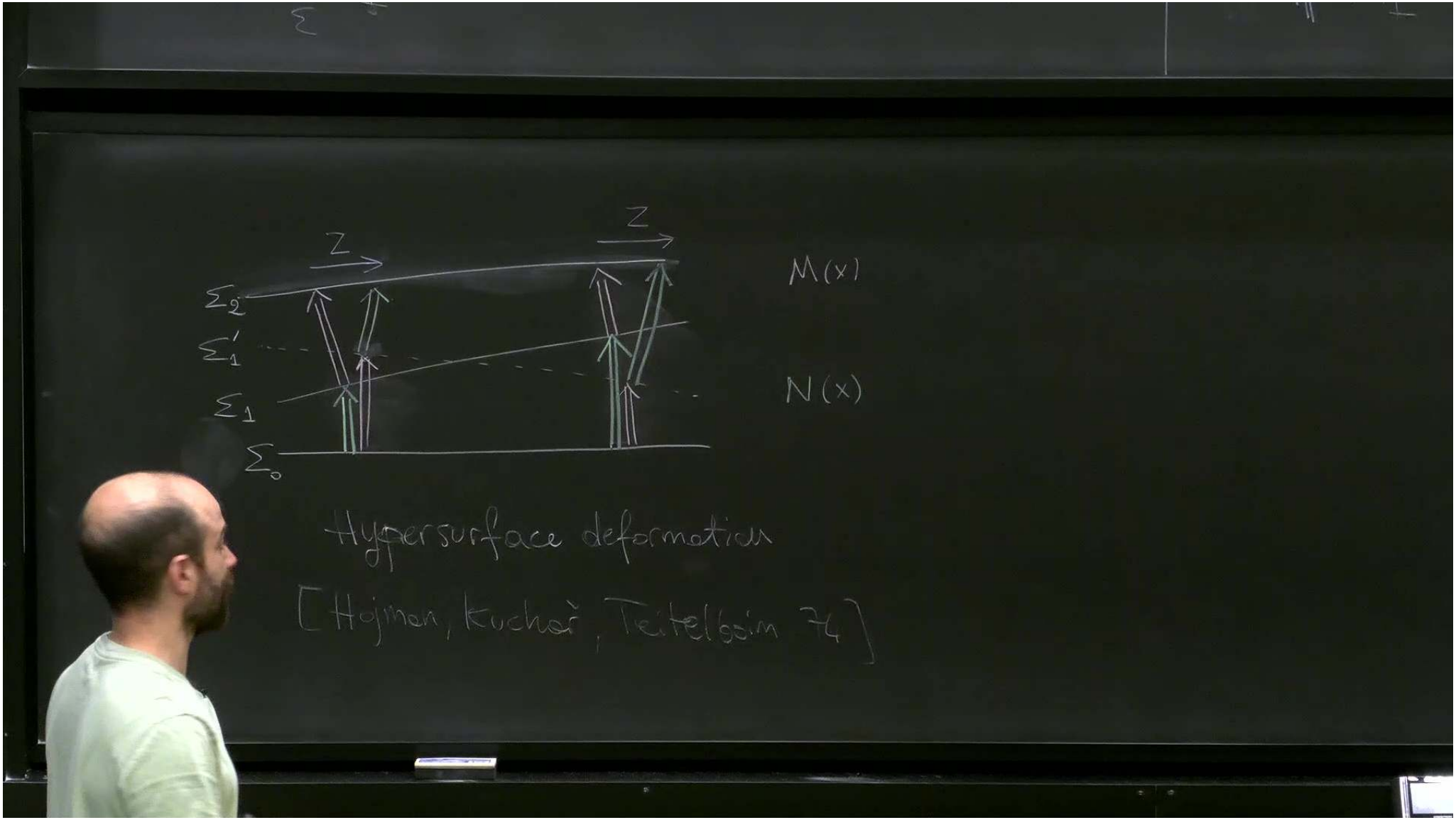
① $\{H_{\parallel}(X), H_{\parallel}(Y)\} = H_{\parallel}([X,Y]_{T\Sigma})$

② $\{H_{\parallel}(X), H_{\perp}(N)\} = H_{\perp}(L_X N)$

③ $\{H_{\perp}(N), H_{\perp}(M)\} = H_{\parallel}(Z)$ $\quad Z^i = N h^{ij}\partial_j M - M h^{ij}\partial_j N$

1 & 2 are easy because $\{H_{\parallel}(X), \begin{vmatrix} h_{ij} \\ \pi^{ij} \end{vmatrix}\} = L_X \begin{vmatrix} h_{ij} \\ \pi^{ij} \end{vmatrix}$ generates 3d diffeos as expected

and $H_{\parallel}$ & $H_{\perp}$ are tensorial expressions


Z
Z
Σ_2
Σ_1'
Σ_1
Σ_0
M(x)
N(x)
Hypersurface deformations
[Hojman, Kuchař, Teitelboim 74]

and $H_{\parallel}$ & $H_{\perp}$ are tensorial expressions

Thm (Lovelock)

In 4d, the Einstein-Hilbert action (w/ CC) is unique in $\mathcal{F} = \mathrm{pRiem}(M)$

① diffeo inv. functional

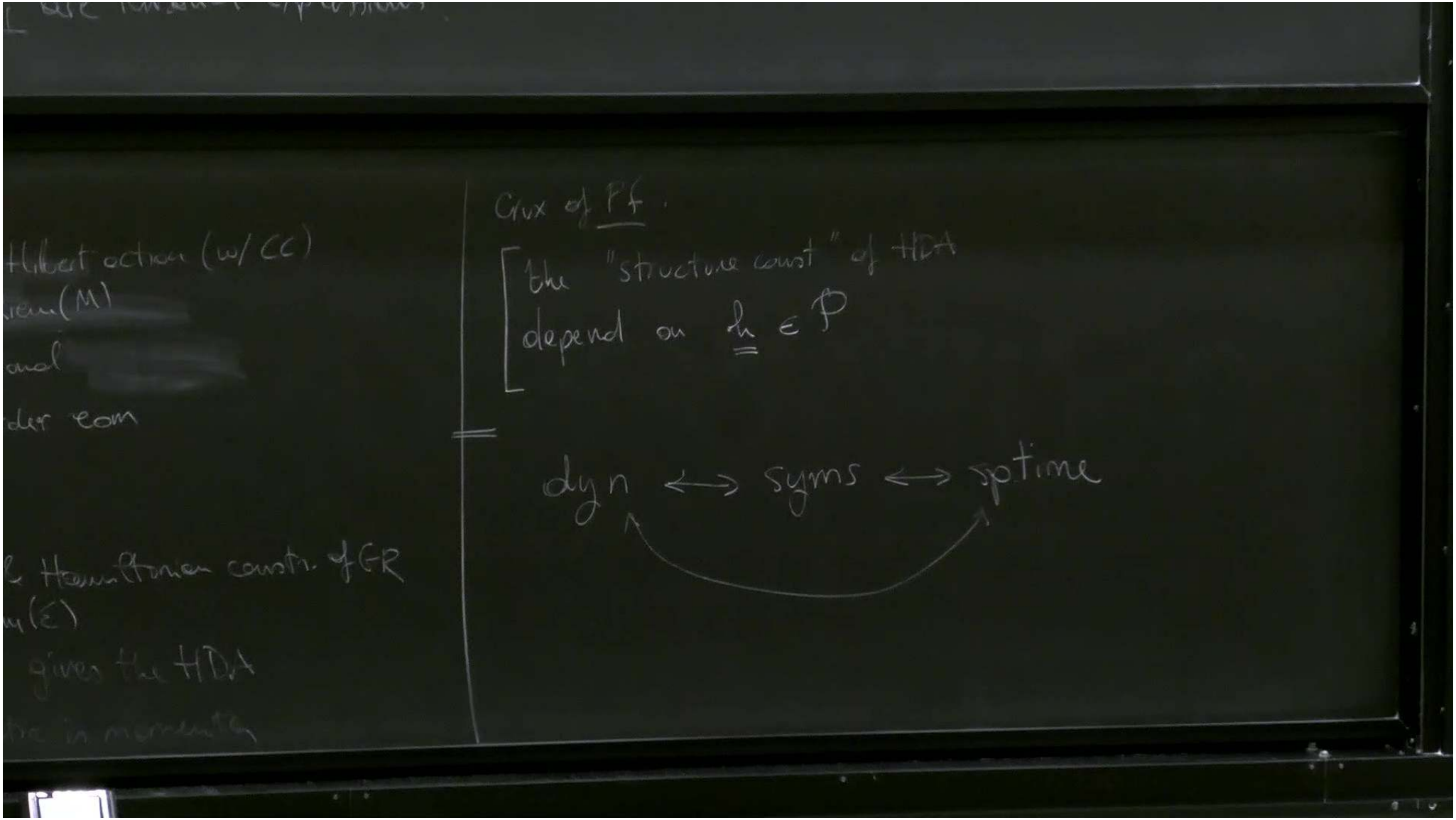
② leads to 2nd order eom

Thm (HKT)

In 4d, the diffeo & Hamiltonian constr. of GR are unique in $\mathcal{P} = T^*\mathrm{Riem}(\Sigma)$

① their Poisson alg gives the HDA

② $H_{\perp}$ is quadratic in momenta


Hilbert action (w/ cc)
eom
der eom
& Hamiltonian constr. of GR
Crux of Ff:
[the "structure const" of HDA
depend on h ∈ P
dyn ⟷ syms ⟷ spacetime

② H_L is quadratic in momenta

(Euclidean)

"Lorentz" $\downarrow$ IJ — $su(2)$ str. const.

In 3d $\quad e_\mu^I, \omega_\mu^{IJ} \sim \omega_\mu^K \epsilon_K^{IJ}$

$$S = \int_{M_3} \mathrm{tr}\left(e \wedge F(\omega)\right) \qquad \text{BF action}$$

"Poincaré" $= iso(3) = (\mathbb{R}^3, +) \oplus su(2)$

$$\sim \int_{M_3} \mathrm{tr}\left(A\,dA + \frac{2}{3} A \wedge A \wedge A\right) \qquad A = e^I P_I + \omega^I J_I$$

In 3d: $\quad Riem = Ric + \underbrace{Weyl}_{\equiv 0} \approx 0$

T*Riem(Σ)
n dlg gives the HDA
adratic in momenta

You can trade
diffeos for "Poincaré" gauge sym!

$$L_\xi A = d\, i_\xi A + i_\xi\left(dA + \tfrac{1}{2}[A \wedge A]\right) - [i_\xi A \wedge A]$$

$$= d_A\underbrace{(i_\xi A)}_{\text{(field dep) gauge trsnf.}} + i_\xi \underbrace{F(A)}_{=0}$$

You can trade the Hamiltonian constr. w/
the gauge constr. $F(A) = 0$

T*Riem(Σ)

n dif gives the HDA

adhetic in momenta

You can trade diffeos for "Poincaré" gauge sym!

$$L_\xi A = d\, i_\xi A + i_\xi\left(dA + \tfrac{1}{2}[A \wedge A]\right) - [i_\xi A \wedge A]$$

$$= d_A\underbrace{(i_\xi A)}_{\text{(field dep) gauge trsf.}} + i_\xi \underbrace{F(A)}_{=0}$$

You can trade the Hamiltonian constr. w/ the gauge constr. $F(A) = 0$

On "lattice":

you can trade diffeo inv. for re-triangulation inv. (topological)

i.e. things don't depend on choice of lattice.