Title: The Discrete Geometries of Schwinger-Keldysh and Correlation Functions

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Collection/Series: Mathematical Physics

Subject: Mathematical physics

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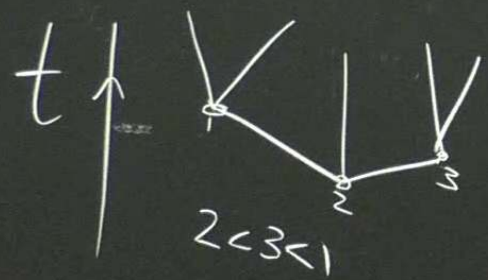
Abstract:

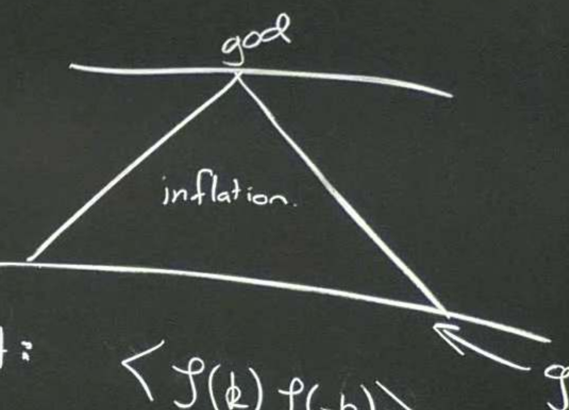
Cosmological correlation functions probe the origins of structure in the universe. We view them as a prototype for time-dependent correlators and finite-temperature correlators in QFT. I will share recent work on the simple discrete geometries and spacetime pictures that control these correlators at tree-level, give a single origin of the range of polytopes previously studied in connection with cosmological correlators, and recast the Born rule as one expansion among many. Based on 2602.21194 and ongoing work. N.B. A math version of this talk will be given on the same day in the Waterloo combinatorics seminar, and people may wish to "pick their flavor" of talk.

Discrete, SK, Cosmology

Lotter 2602.21194
Glew



- Context



Cosmologist: $\langle \zeta(k)\,\zeta(-k) \rangle$

$$g \sim -N^2 dt^2 + e^{2\zeta} a^2(t) \times (dx + \ldots)^2$$

$$\langle \zeta(k_1)\,\zeta(k_2)\,\zeta(k_3) \rangle$$

$$f^{NL} = -0.9 \pm 5.4$$

SIMPLE QFT:

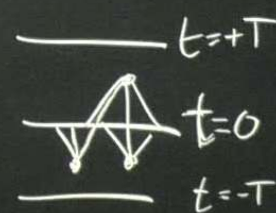
$$S = \int (\partial\varphi)^2/2 + m^2\varphi^2 + \frac{1}{3!}\varphi^3$$

$$\Psi[\varphi_{t=0}] = \langle\varphi|\, \mathcal{U}_{-\infty,0}\, |\Omega\rangle \qquad e^{i\int H_I}$$

$$\langle\varphi(k_1)\cdots\varphi(k_n)\rangle = \underbrace{\langle\Omega|\,\mathcal{U}^\dagger_{-\infty,0}}\,\varphi\cdots\varphi\,\underbrace{\mathcal{U}_{-\infty,0}|\Omega\rangle}$$

$$= \int D\varphi\, |\Psi|^2\, \varphi(k_1)\cdots\varphi(k_n).$$

SK: $\langle\Omega_{in}|\,\underbrace{\mathcal{U}^\dagger_{-T,0}}\,\varphi\cdots\varphi\,\underbrace{\mathcal{U}_{-T,0}}\,|\Omega_{in}\rangle \qquad T\to\infty \quad \& \ \Omega_{in}=\Omega$



$$\mathrm{Tr}\, e^{-\beta H} = \frac{1}{Z_\beta}\mathrm{Tr}(e^{-\beta H}\varphi_1\cdots\varphi_n)$$

SK Calculation

$$\varphi(t,\underline{x}) = \int d\tilde{k}\; e^{-k\cdot x}\,\varphi(t,k).$$

Prop. $(-\partial_t^2 + E^2)\, G(t;E) = \delta(t)$ $\qquad E = \|k\|$

$G(t,E) \to 0 \quad |t| \to \infty.$

$$G(t,t';E) = \frac{1}{2E}\, e^{-E|t-t'|}.$$

Dirichlet. $G_D(t,t';E) \to 0 \quad t \to 0$

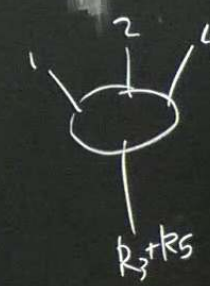
$G_D(t,t';E) = G(t,t';E) - G(\ldots$

Correlator: $C_G = \int_{-\infty}^{\infty} dt\; e^{-\sum E_i |t_i|} \prod_{e=ij} G(t_i,t_j;E) \qquad E_I = \| \sum_{i\in I} k_i \|$

Wavefunction: $\Psi_G = \int_{-\infty}^{0} dt\; e^{-E|t_i|} \prod_{e=ij} G_D(t_i,t_j,E).$

1 2 3 4 5

(124) 35



Born rule: $\langle \varphi_1 \cdots \varphi_n \rangle = \sum_{\substack{\pi \text{ part} \\ \{1\cdots n\}}} \; \sum_{\substack{\text{Trees} \\ \text{on part}}} \prod_e \frac{1}{2E_e} \prod_{\pi_i} \left(2\Psi_{\pi_i, T}\right)$

If color ordered: $\langle \varphi_1 \cdots \varphi_n \rangle_{col} = \sum_{\substack{\pi \text{ noncrossing} \\ \text{partition} \\ \text{of } 1\cdots n}} \boxed{\text{Same.}}$

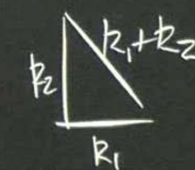
Wavefunction

$\sum k_i = 0$ $E_I = \left\| \sum_{i \in I} k_i \right\|$

e.g. $\psi_{123} = \frac{1}{E_1 + E_2 + E_3}$; $\psi_{1234} = \frac{1}{E_1} \underbrace{\frac{1}{E_{12}+E_1+E_2}\frac{1}{E_{12}+E_3+E_4}}$.

$E_{12} - E_1 - E_2$

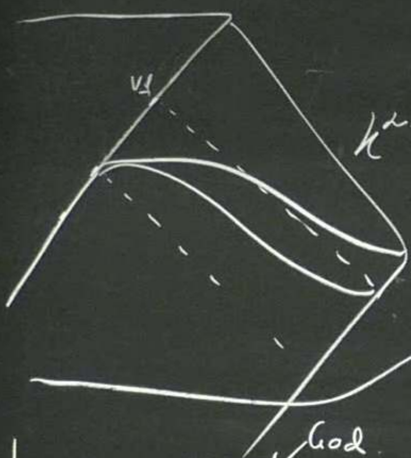
$$\psi_{12345} = \frac{E_\uparrow + E_{123}^+ + E_{12}^+}{E_\uparrow\, E_{12}^+\, E_{123}^+ (\quad)(\quad)(\quad)}.$$



$\boxed{\psi_n = \sum_G \psi_G}$ $|V| + |E|$ $E_{12}^+ = E_{12}+E_1+E_2$ $E_{123}^+ = E_{123}+E_1+E_2+E_3$

$$\psi_G = \mathrm{Vol}\,\mathrm{Cone}(W_t \mid t \text{ of } G) \qquad W_t \cdot E = E_t.$$

$$= \int_\nabla dt^{|V|+|E|}\, e^{-t \cdot E} = \frac{\det[W_{t_1} \cdots W_{t_{|V|+|E|}}]}{\prod W_{t_i} \cdot E}. \qquad E = (E_1, E_2, \dots)$$



$$k^\mu \left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box + g_{\mu\nu}\right)\phi = -8\pi G_N T_{\mu\nu}$$

$$\partial_v^2 \phi = -8\pi G_N^{(2)} T_{vv}$$

$t_I^\pm \quad E_I^\pm$

$J =$ 1234

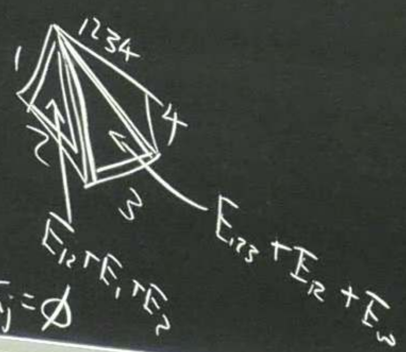
123 234

12 23 34

K_1

$K_i \subseteq J \quad K_i \cap K_j = \emptyset$

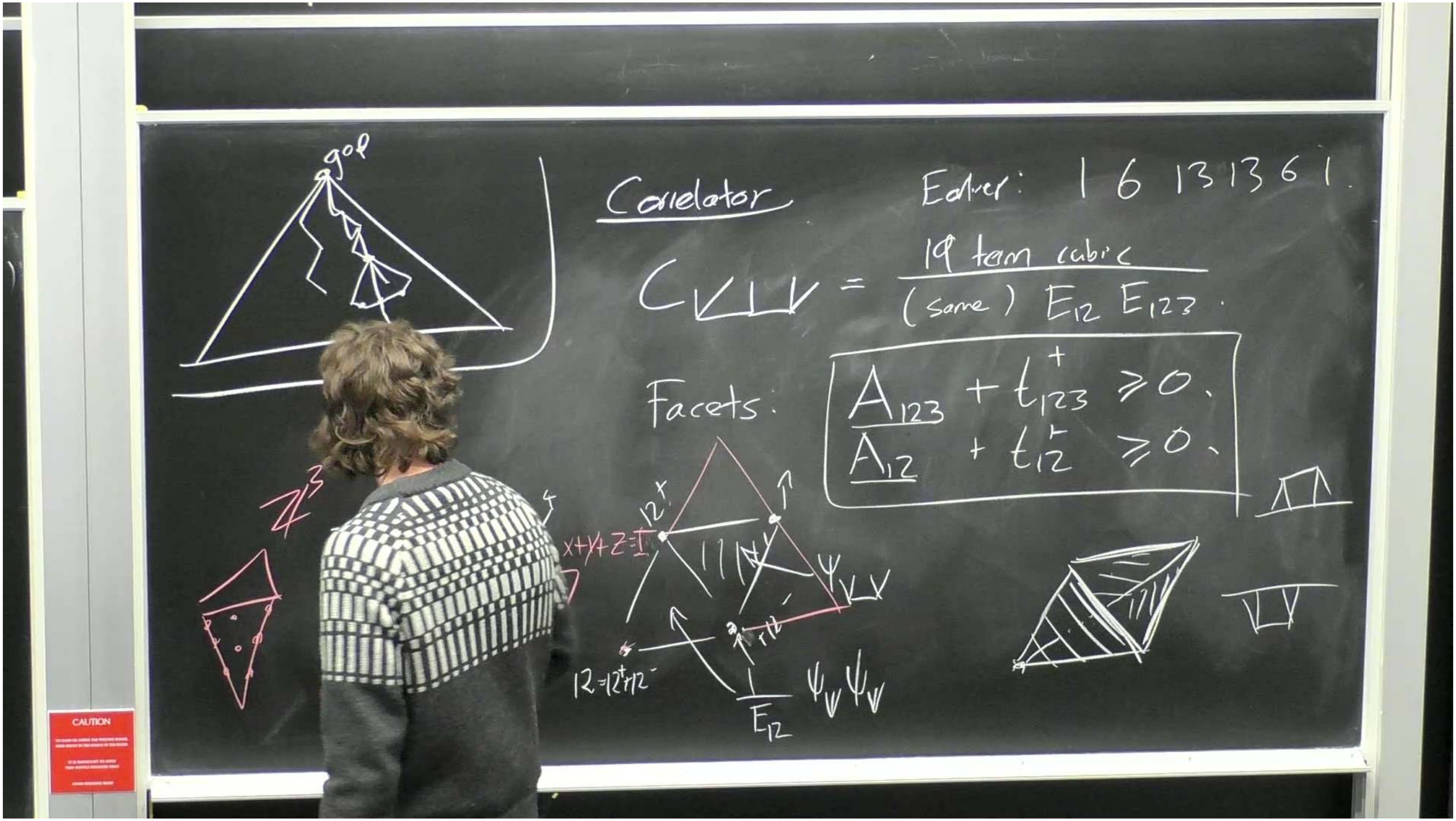
Facets of Cone:

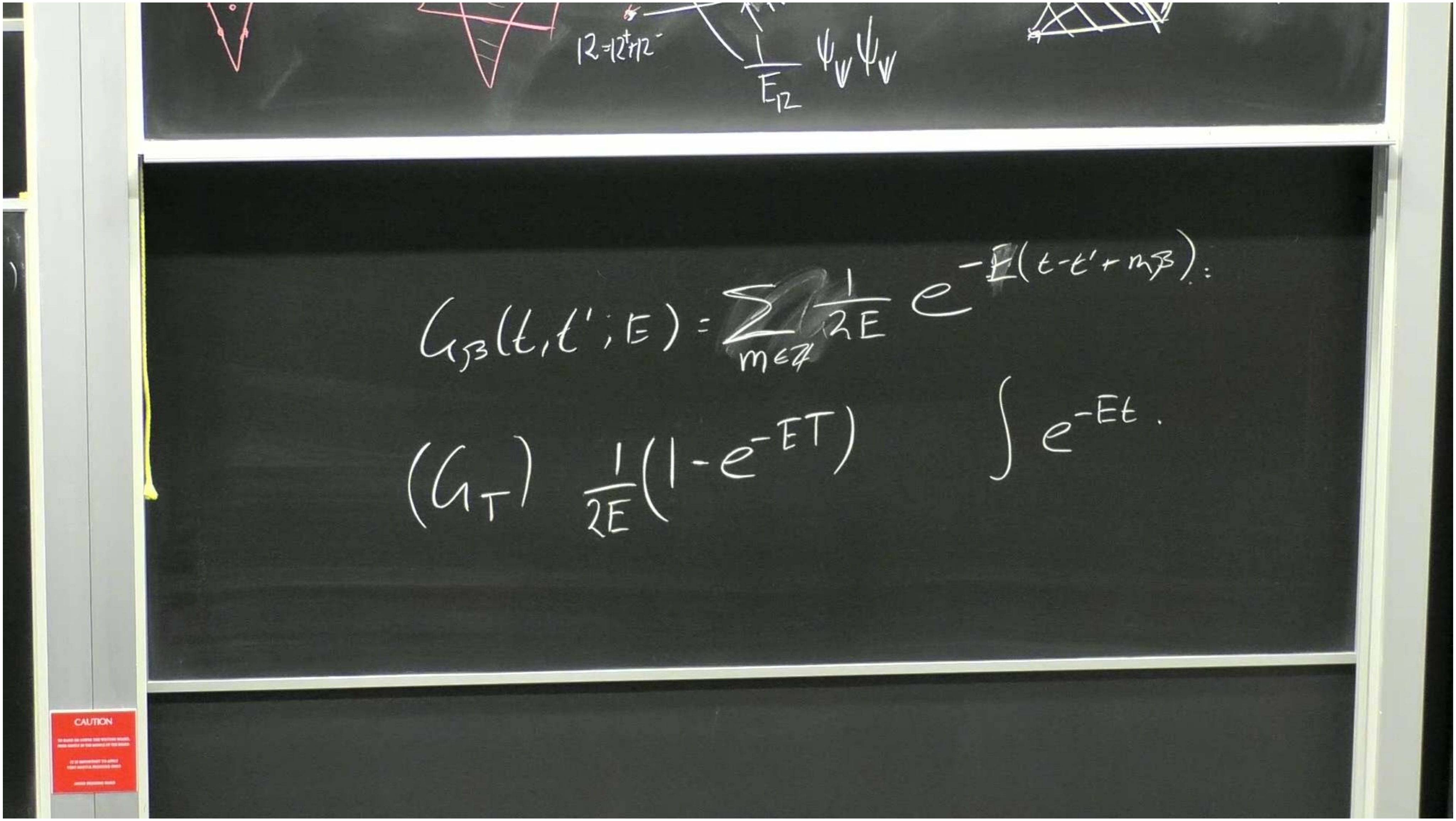


$$A_{123} = t_1 - t_{123}^- \geq 0.$$

$$A_{12} = t_1 - t_{123}^- + t_{123}^+ - t_{12}^- \geq 0$$

$$t_I^\pm \geq 0.$$


Correlator
Euler: 1 6 13 13 6 1
19 term cubic
(same) E12 E123
Facets:
A123 + t123 ≥ 0
A12 + t12 ≥ 0
x+y+z = I
12 = 12+ + 12-
E12


$12 = 12^+ + 12^-$
$\frac{1}{E_{12}} \psi_V \psi_V$
$G_\beta(t, t'; E) = \sum_{m \in \mathbb{Z}} \frac{1}{2E} e^{-E(t-t'+m\beta)}$:
(G_T) $\frac{1}{2E}(1 - e^{-ET})$
$\int e^{-Et}$.