Title: Lecture - Quantum Fields & Strings, PHYS 77

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Collection/Series: Quantum Fields & Strings (Elective), March 30 - May 1, 2026

Subject: Quantum Fields and Strings

Date: April 14, 2026 - 3:30 PM

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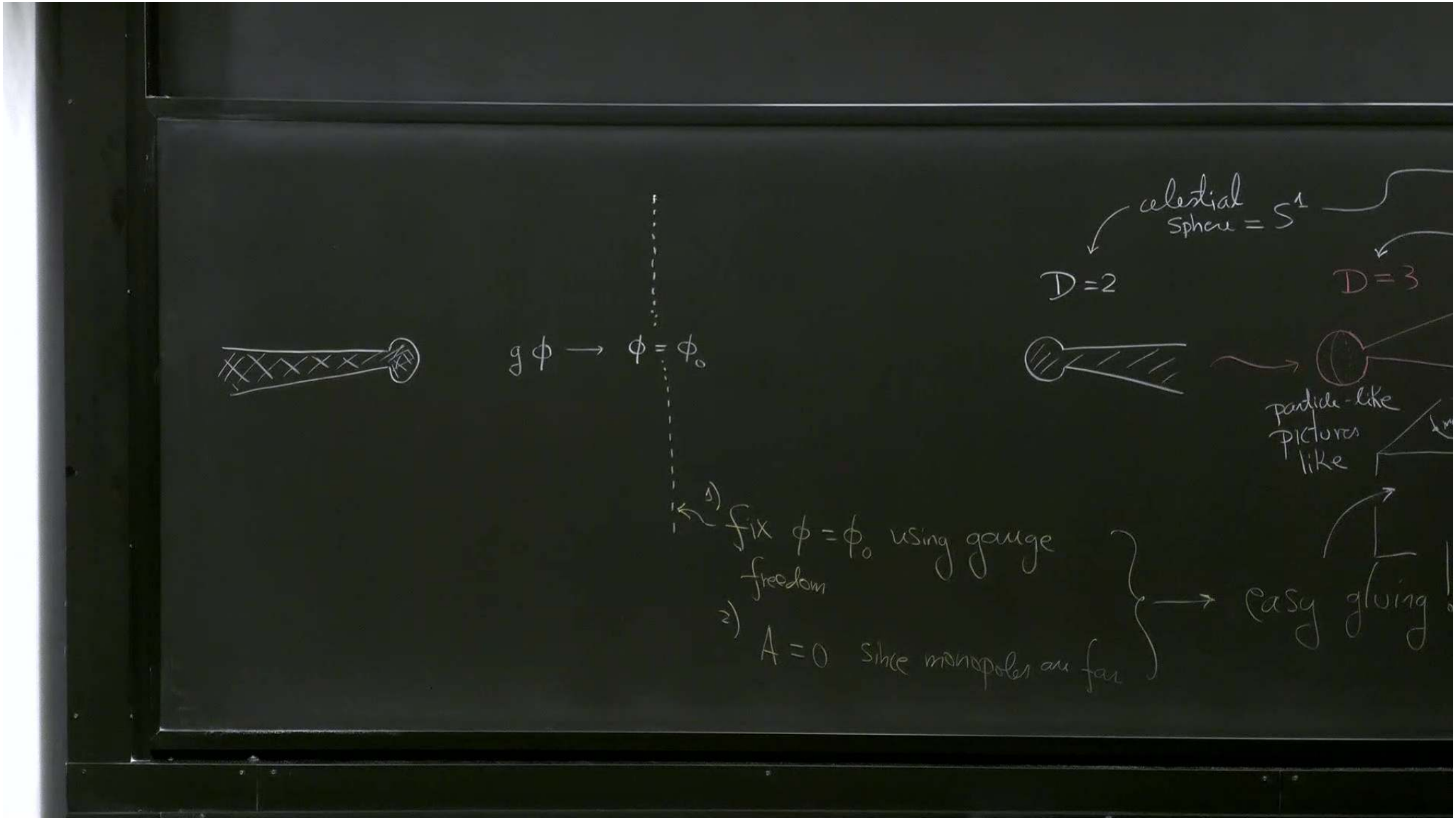
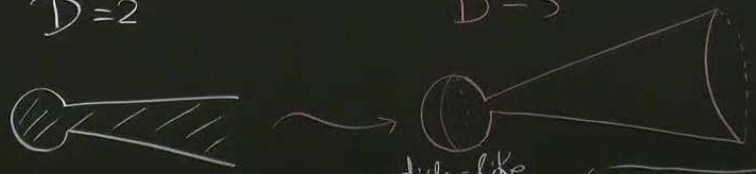

celestial sphere = S^1
D = 2
D = 3
$g\phi \to \phi = \phi_0$
particle-like pictures like
1) fix $\phi = \phi_0$ using gauge freedom
2) $A = 0$ since monopoles are far
→ easy gluing

celestial sphere = S^1 → $\Pi_1(G/H)$

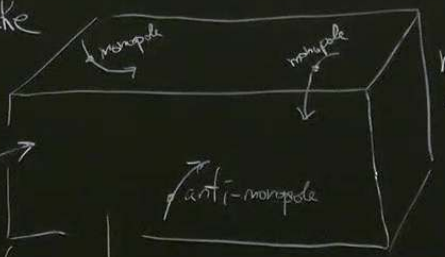
celestial sphere = S^1 → $\Pi_2(G/H)$

D = 2

D = 3



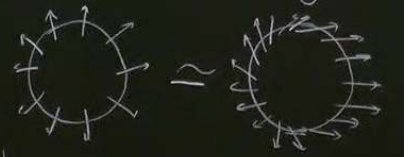
particle-like pictures like



make sense!

different sectors

= disconected components of the space of no

= homotopy classes 

[= elements of the homotopy group Π

gauge

→ easy gluing!

poles are far

$\Pi_1(G/H)$ $\Pi_2(G/H)$

stial sphere = S^1

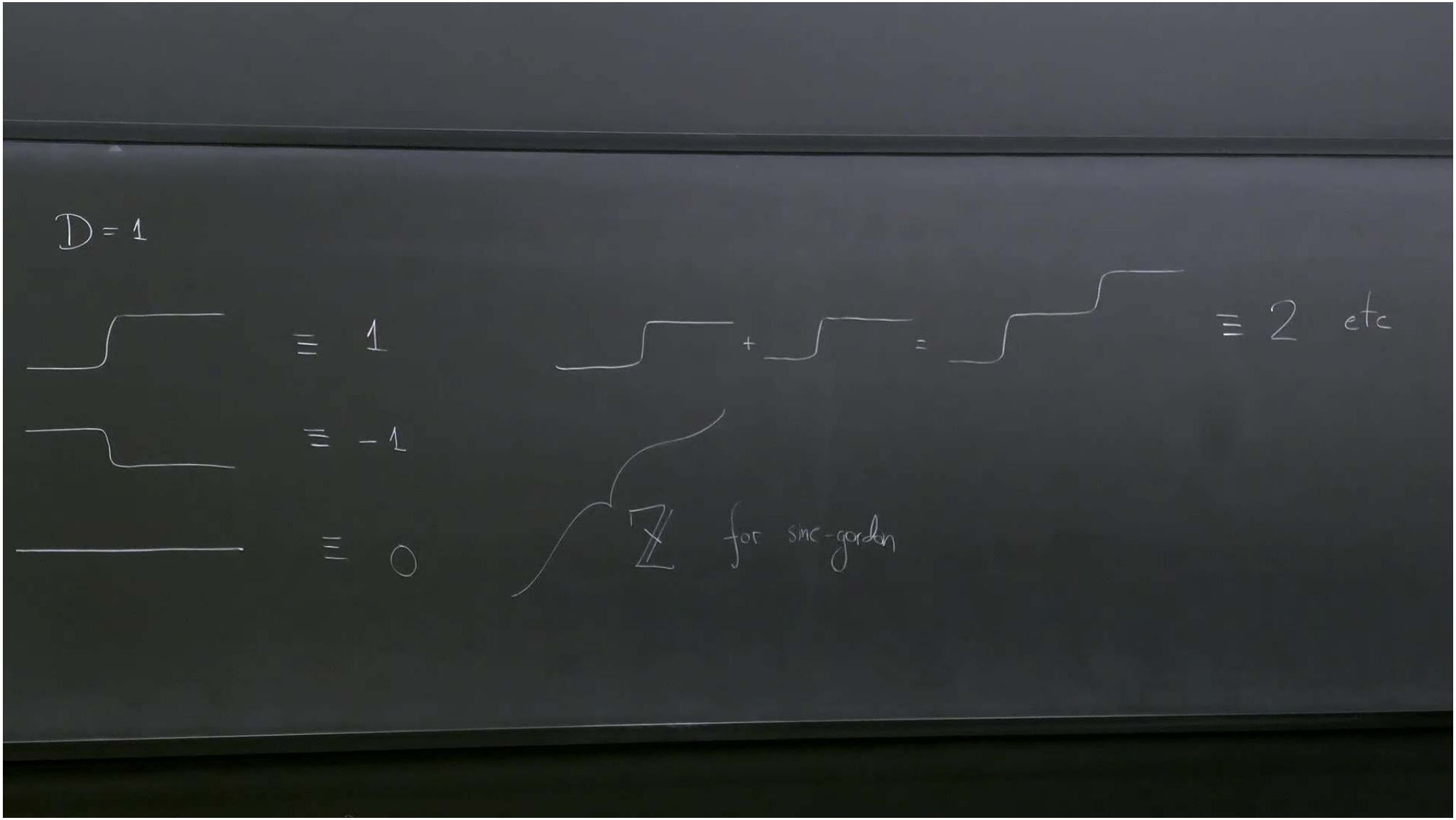
make sense.

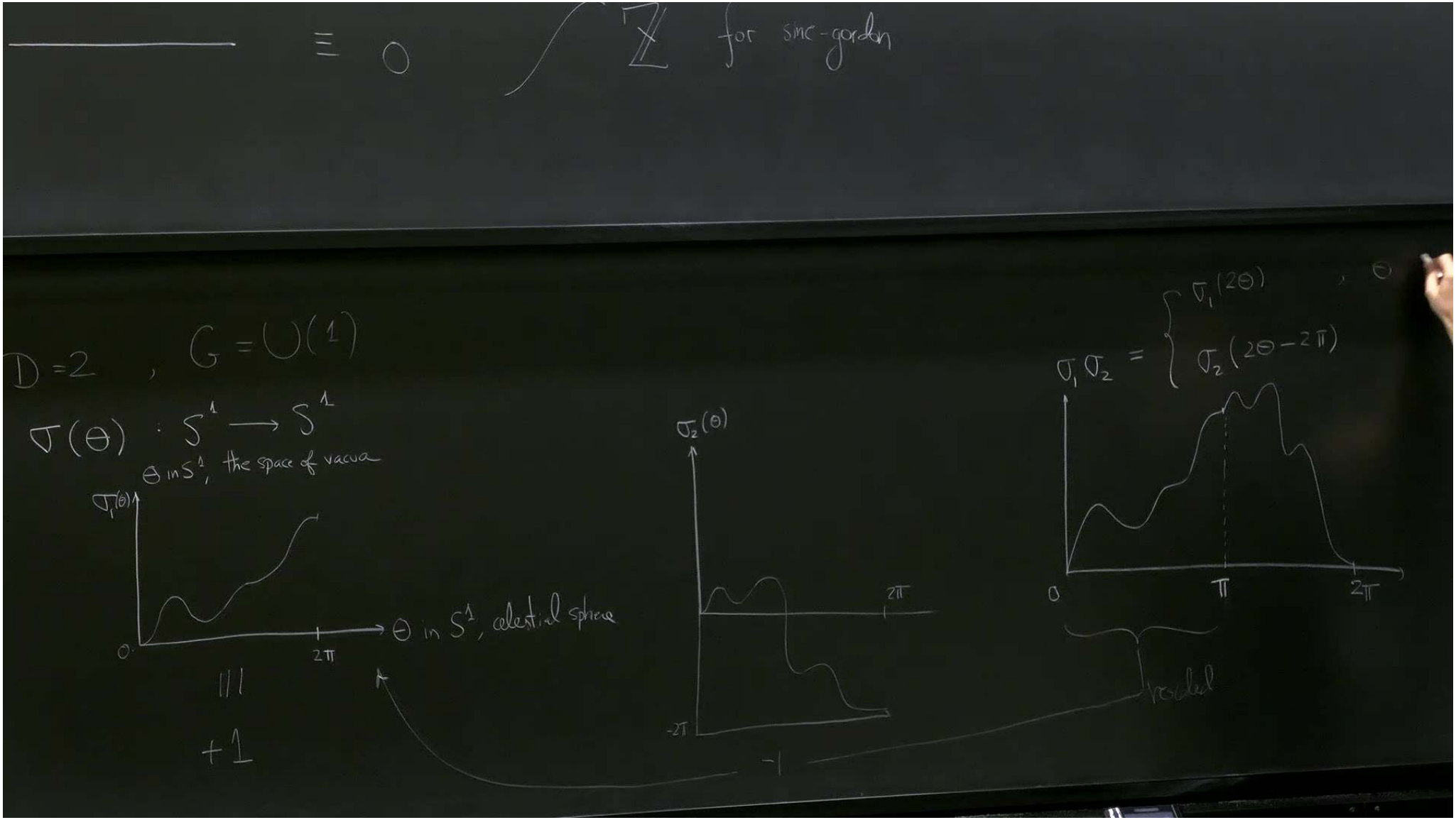
different sectors

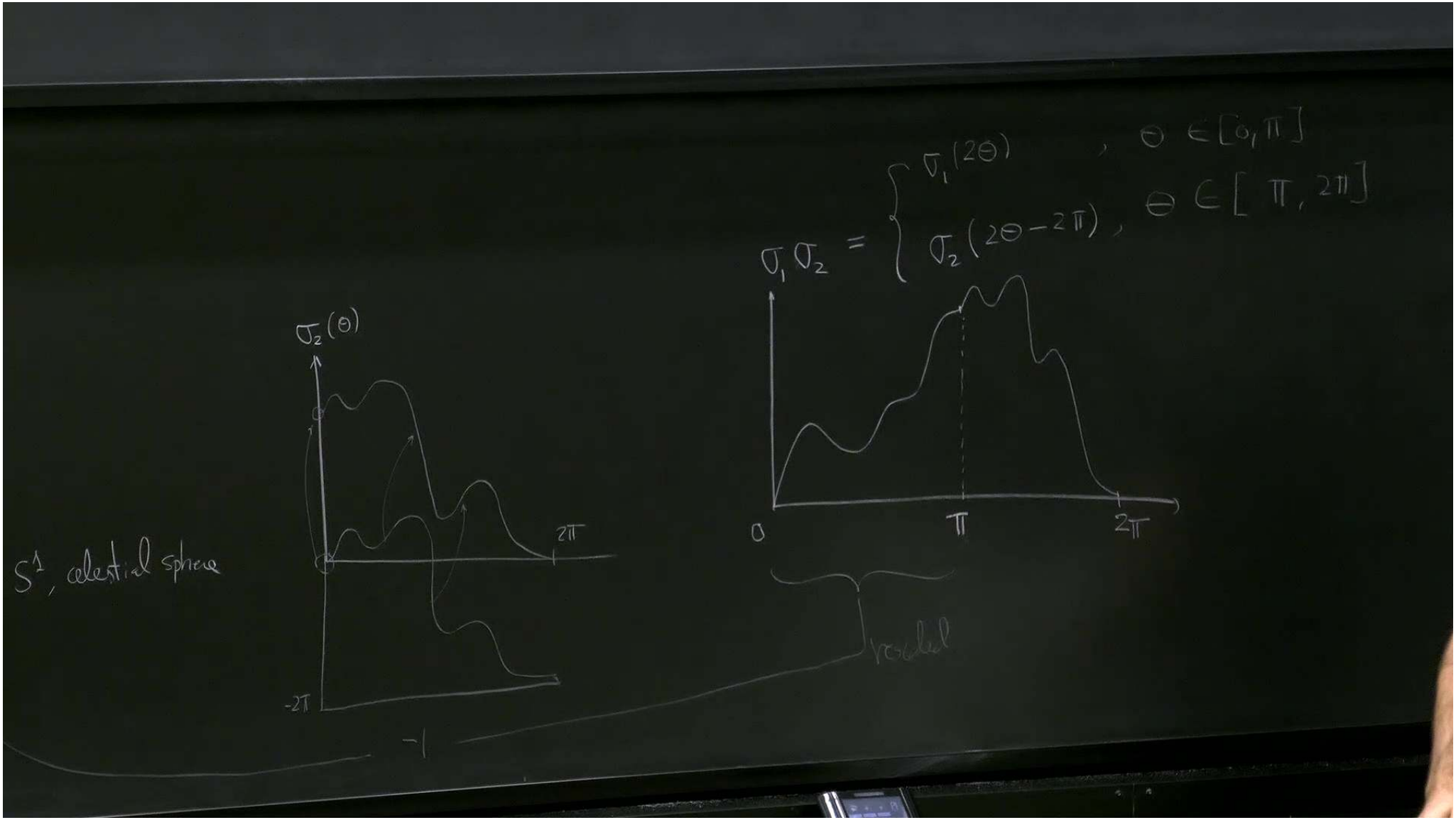
= disconnected components of the space of non-singular finite E configs

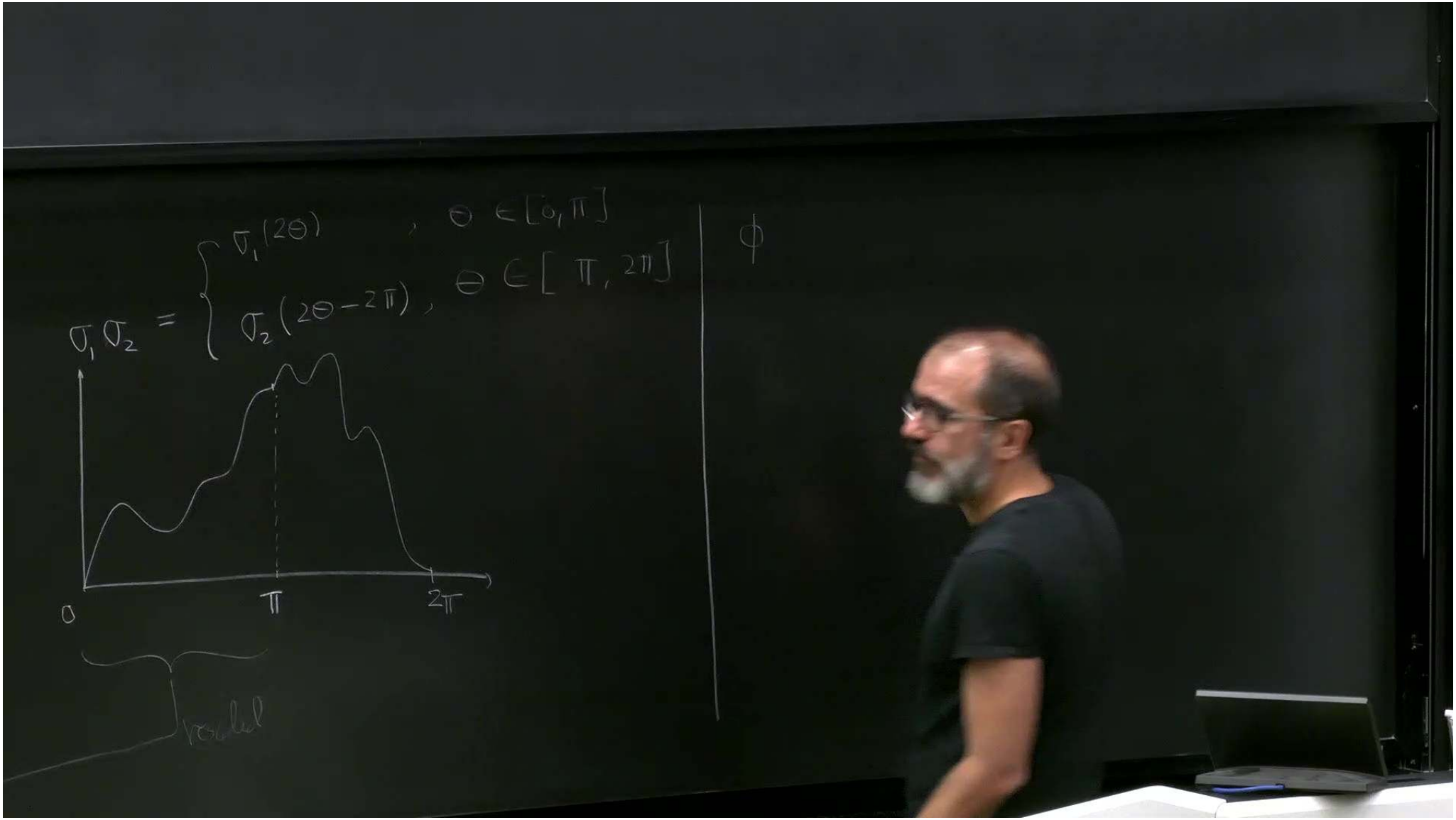
= homotopy classes $\neq$

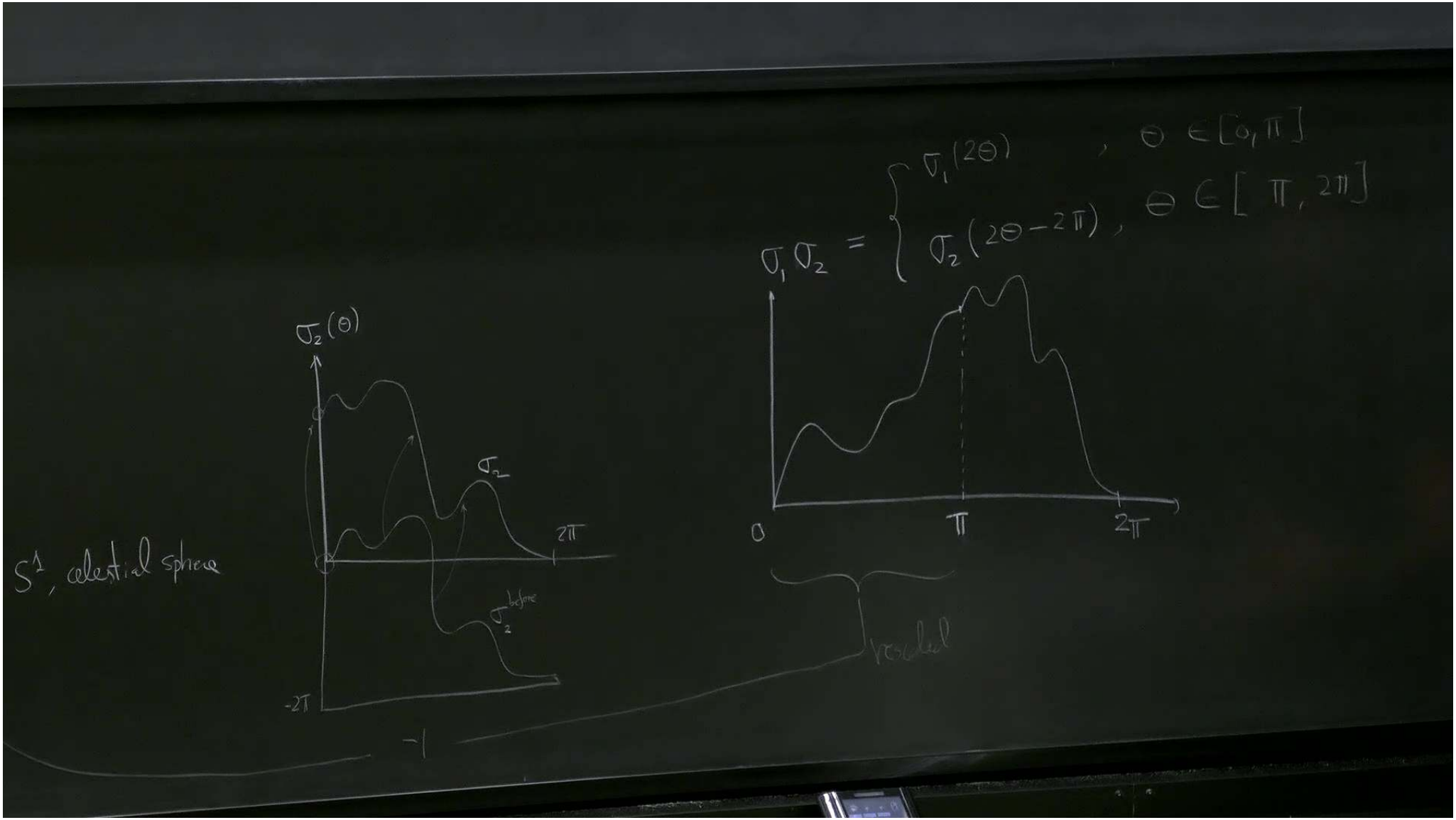
[= elements of the homotopy group $\Pi_n(G/H)$] ← Today we will flesh this out a bit more


D = 1
≡ 1
+
=
≡ 2 etc
≡ −1
≡ 0
ℤ for sme-gordon


for sine-gordon
D = 2 , G = U(1)
σ(θ) : S¹ ⟶ S¹
θ in S¹, the space of vacua
σ₁(θ)
0
2π
θ in S¹, celestial sphere
σ₂(θ)
2π
-2π
σ₁σ₂ = { σ₁(2θ) ; σ₂(2θ − 2π)
0
π
2π
θ
rescaled
+1
-1


$\sigma_1 \sigma_2 = \begin{cases} \sigma_1(2\theta) & , \theta \in [0, \pi] \\ \sigma_2(2\theta - 2\pi) & , \theta \in [\pi, 2\pi] \end{cases}$
$\sigma_2(\theta)$
2π
-2π
S^1, celestial sphere
0
π
2π
resultat


$\sigma_1\sigma_2 = \begin{cases} \sigma_1(2\theta), & \theta \in [0, \pi] \\ \sigma_2(2\theta - 2\pi), & \theta \in [\pi, 2\pi] \end{cases}$
ϕ
0
π
2π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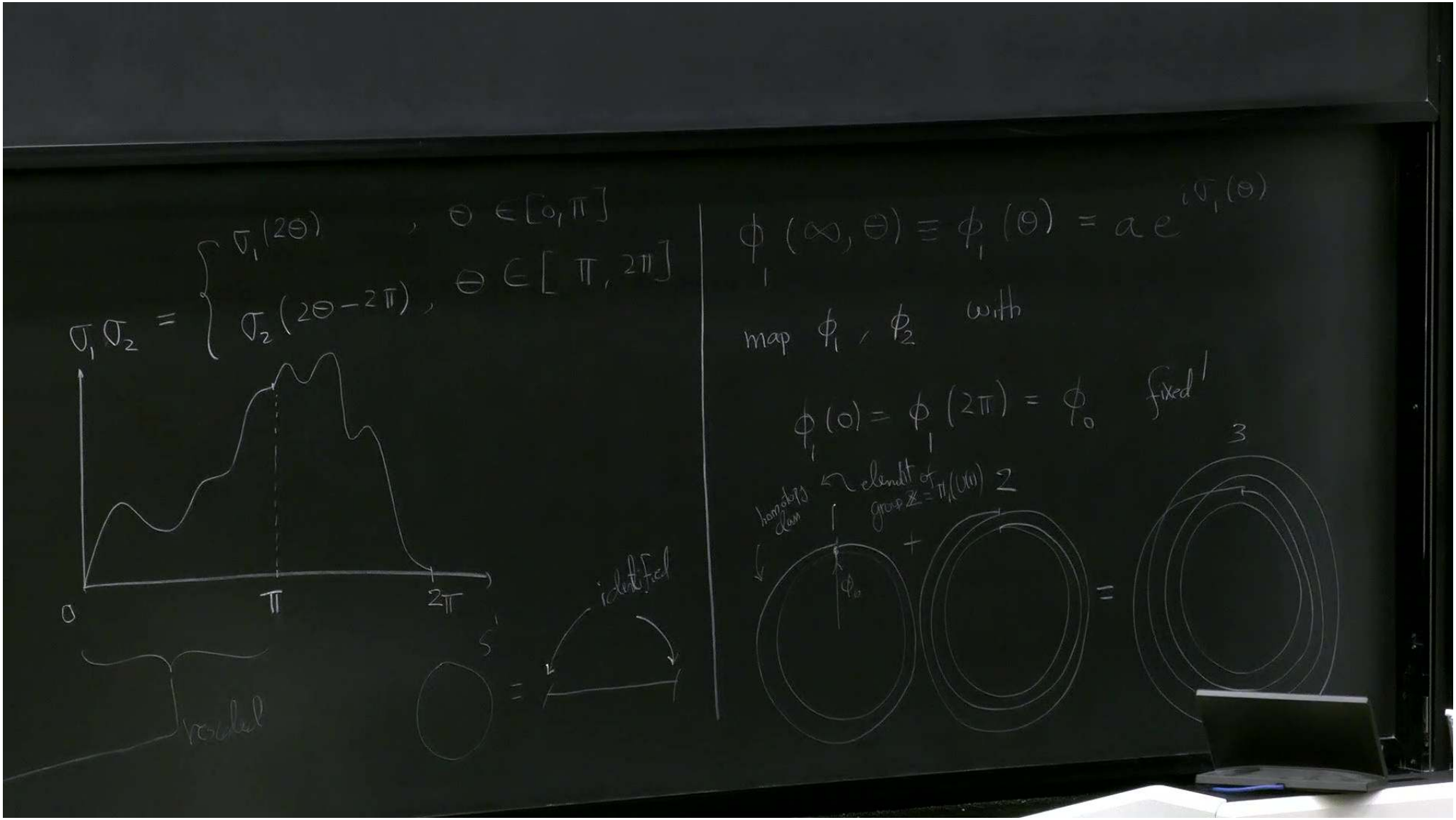
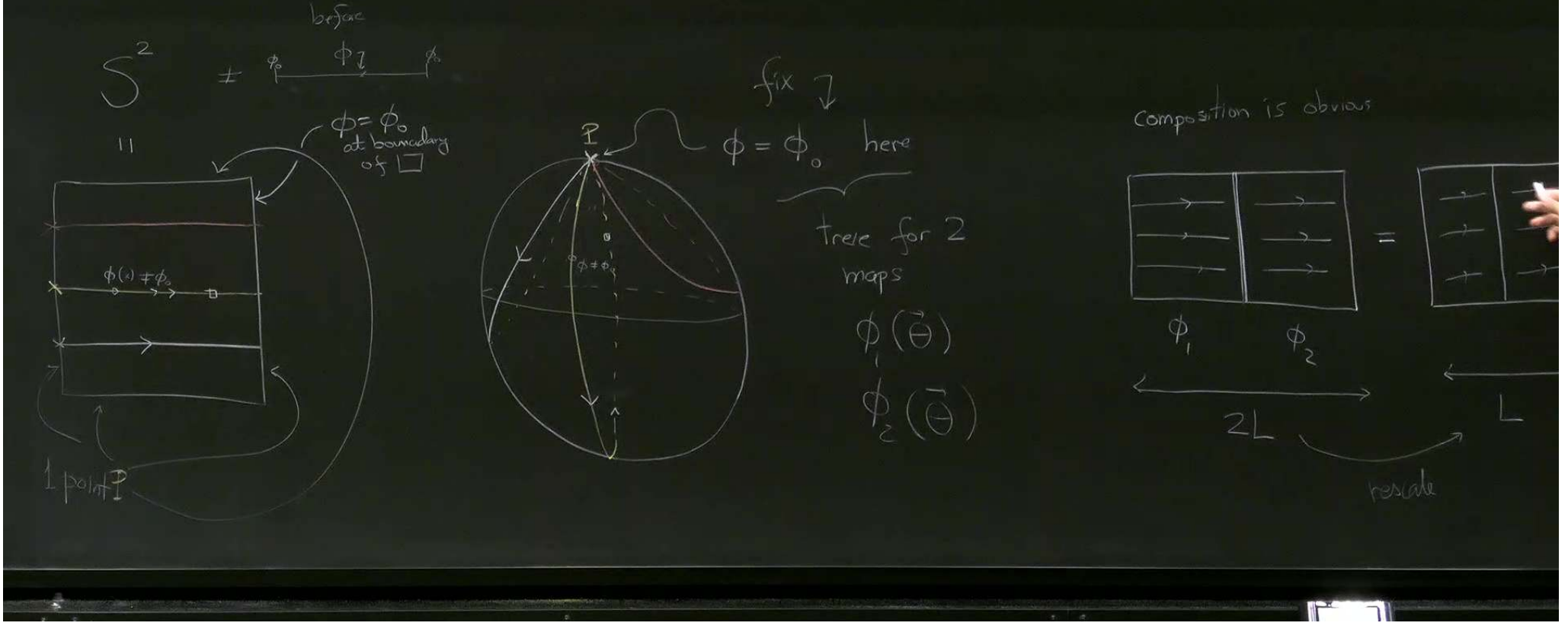
$\sigma_2(\theta)$
σ_2
2π
-2π
σ_2^{before}
S^1, celestial sphere
-1
0
π
2π
rescaled


$\sigma_1 \sigma_2 = \begin{cases} \sigma_1(2\theta), & \theta \in [0, \pi] \\ \sigma_2(2\theta - 2\pi), & \theta \in [\pi, 2\pi] \end{cases}$
0
π
2π
S^1
identified
$\phi_1(\infty, \theta) \equiv \phi_1(\theta) = a e^{i\sigma_1(\theta)}$
map ϕ_1, ϕ_2 with
$\phi_1(0) = \phi_1(2\pi) = \phi_0$ fixed!
homotopy class
element of group $\mathbb{Z} = \pi_1(U(1))$
ϕ_0
3
2

$\phi = \phi_0$ using gauge freedom

2) $A = 0$ since monopoles are far

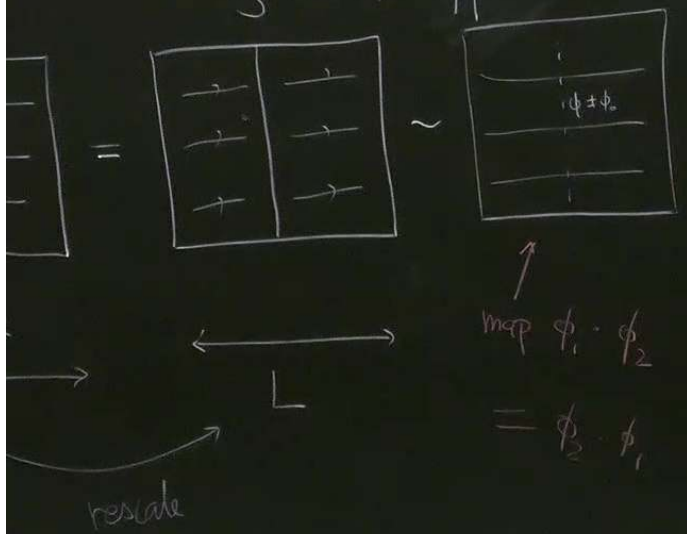
→ easy gluing!



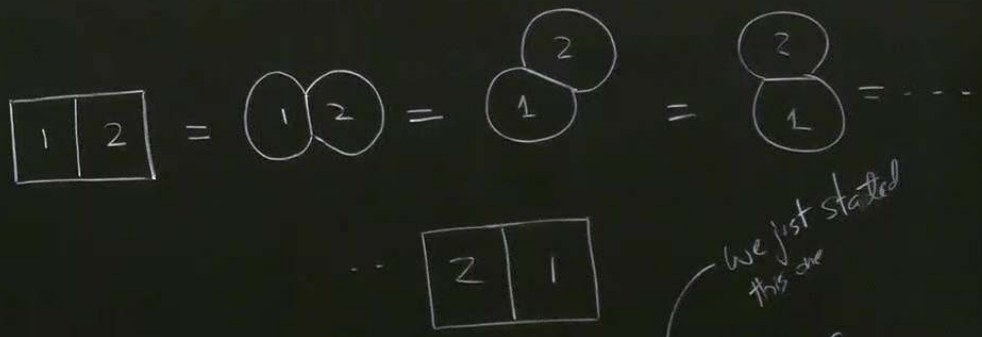
flesh this out a bit more

obvious

$S^2 \longrightarrow G/H$



ALWAYS ABELIAN

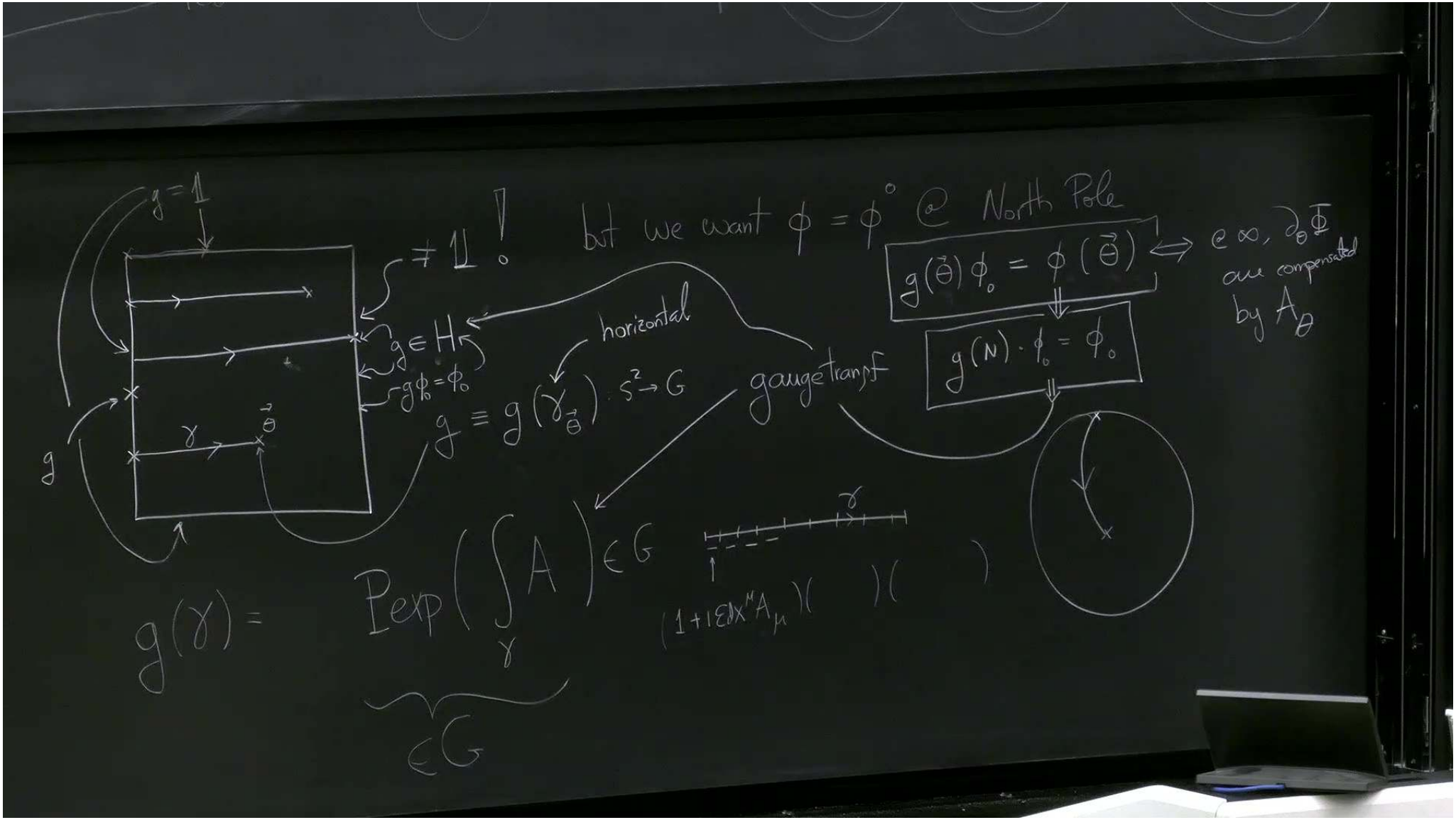


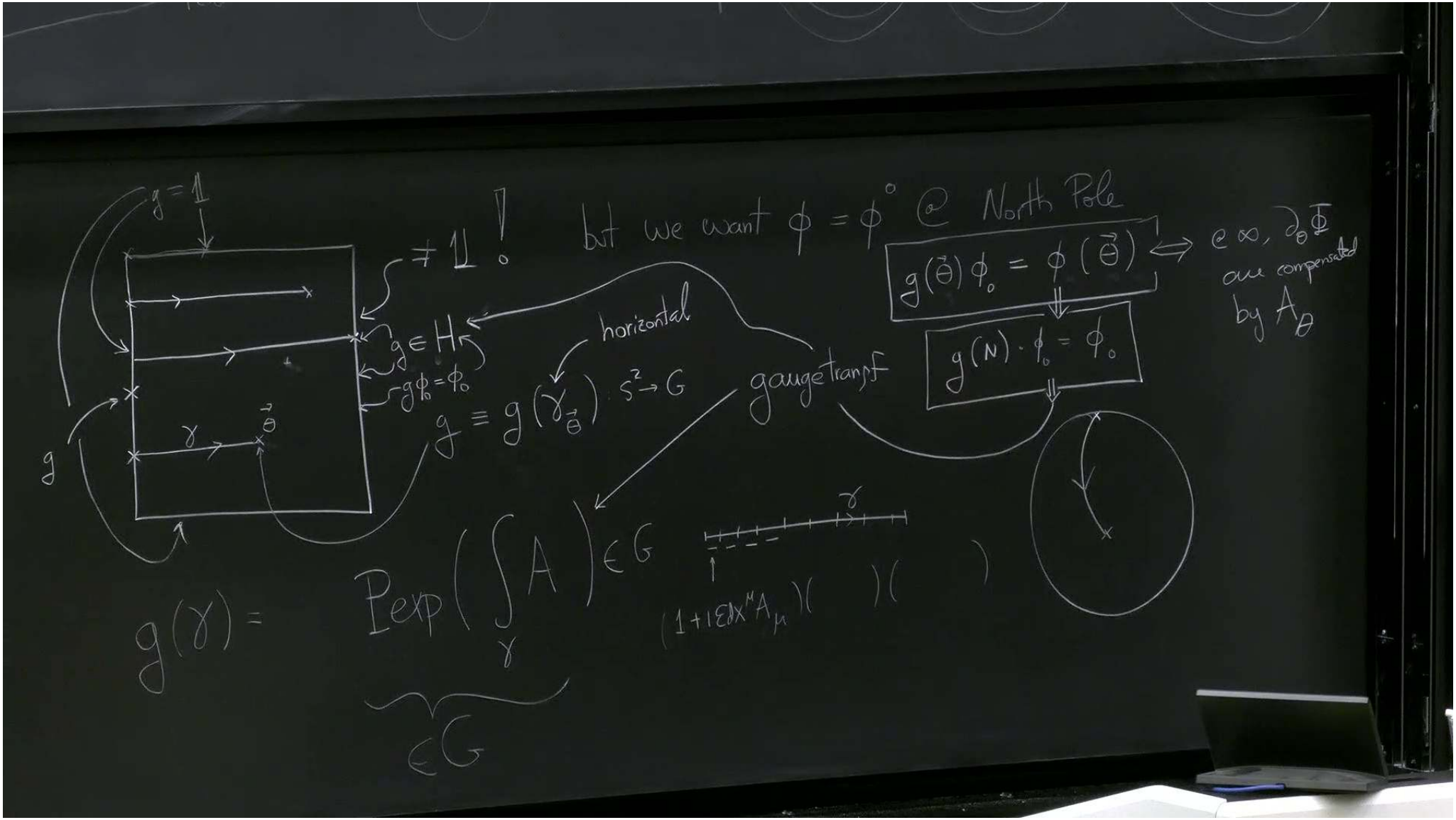
$\pi_2(G/H)$ is abelian, $\cong \mathbb{Z}$ for our $\eta = \omega\omega$ ← $G = SO(3)$ example

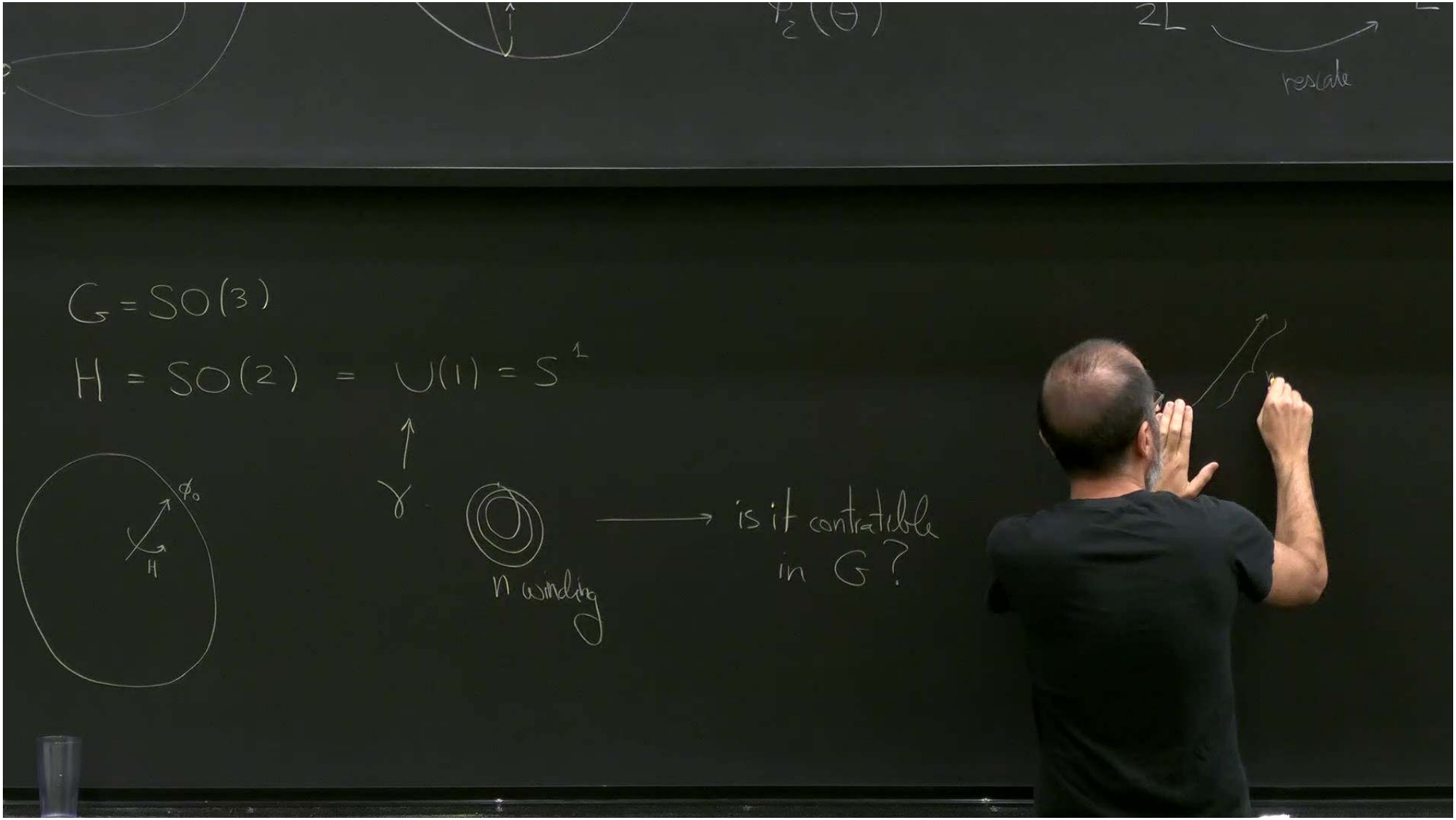
Theorem

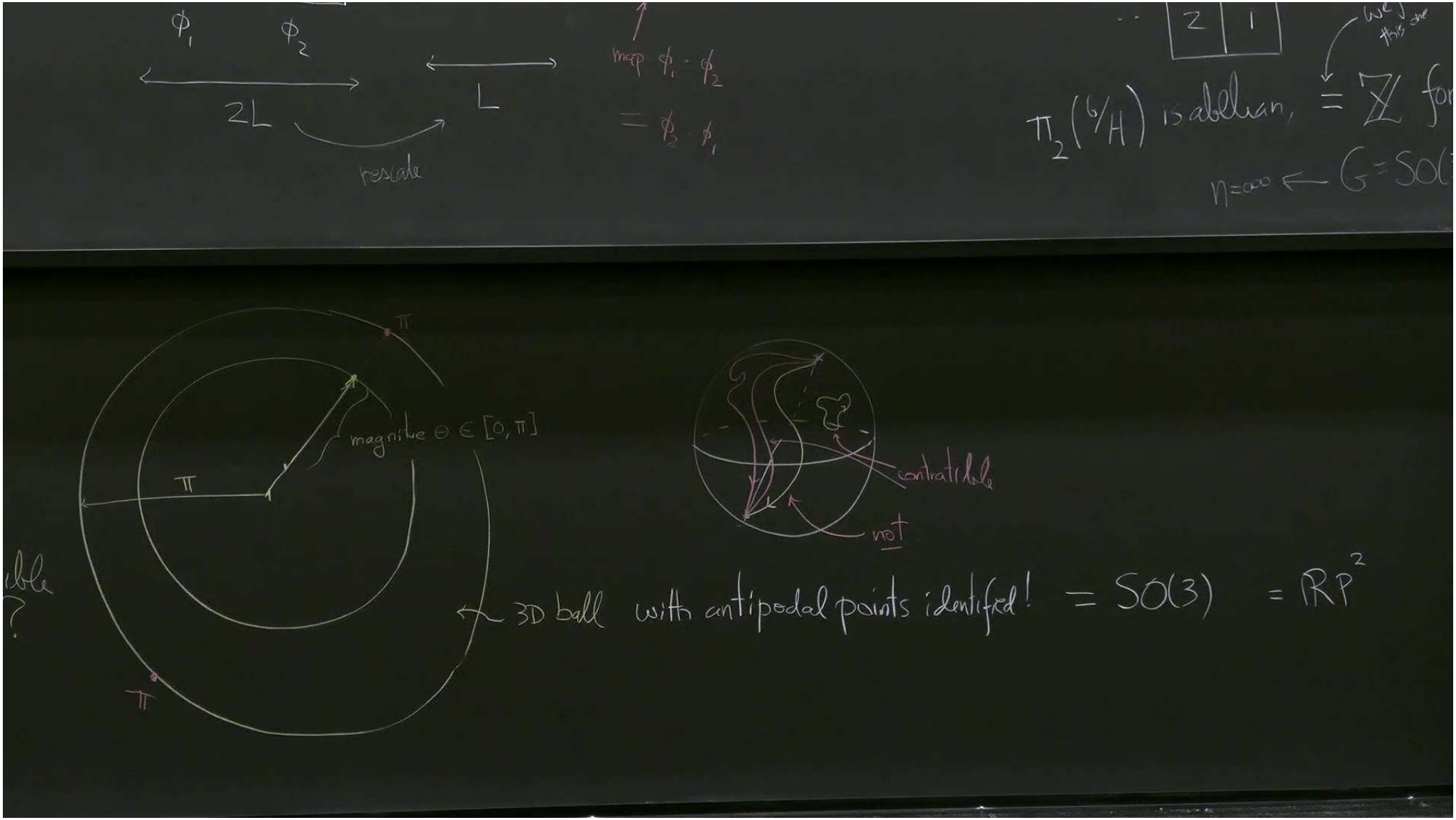
$$\pi_2\left(G/H\right) = \ker\left(\pi_1(H) \xrightarrow{\text{inclusion}} \pi_1(G)\right)$$

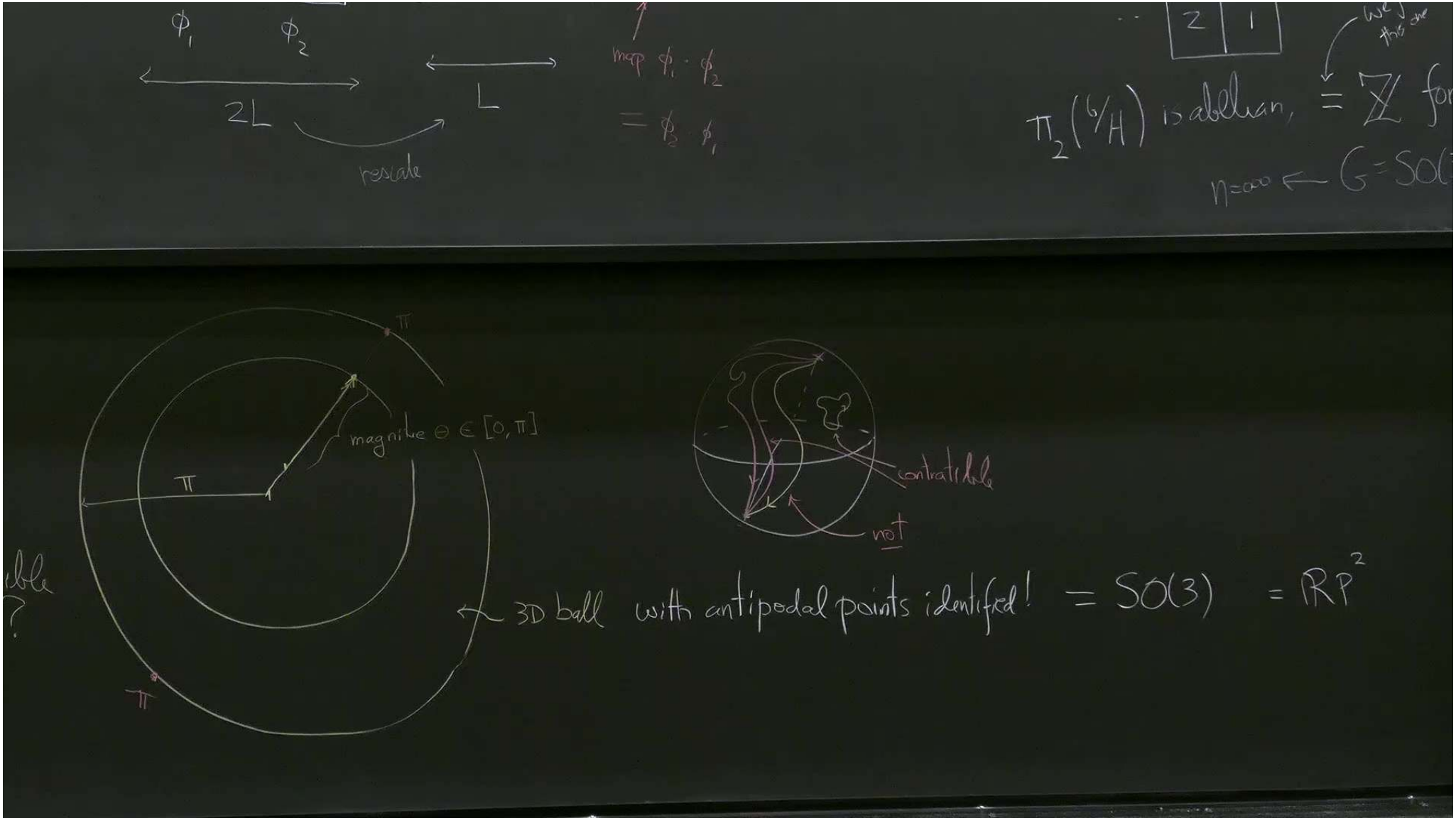
$\pi_2(G/H)$ = Set of 1D loops in H that are contractible when included in G


g=1
≠1 !
but we want φ = φ° @ North Pole
g(θ⃗) φ₀ = φ(θ⃗)
⇔ e∞, ∂₀Φ
are compensated
by A_θ
g(N) · φ₀ = φ₀
g ∈ H
gφ₀ = φ₀
horizontal
g ≡ g(γ_θ⃗) : S² → G
gauge transf
g
γ
θ⃗
g(γ) = P exp(∫_γ A) ∈ G
(1 + iεdx^μ A_μ)( )( )
∈ G


$g=\mathbb{1}$
$\neq \mathbb{1}$!
but we want $\phi = \phi_0$ @ North Pole
$g(\vec{\theta})\,\phi_0 = \phi(\vec{\theta})$
$\Leftrightarrow$ e ∞, $\partial_\theta \Phi$ are compensated by A_θ
$g(N)\cdot\phi_0 = \phi_0$
$g \in H$
$g\phi_0 = \phi_0$
horizontal
$g = g(\gamma_{\vec{\theta}}) : S^2 \to G$
gauge transf
g
γ
$\vec{\theta}$
$g(\gamma) = P\exp\left(\int_\gamma A\right) \in G$
$\in G$
$(1 + i\varepsilon \dot{x}^\mu A_\mu)$


$\gamma_2(\theta)$
2L
rescale
G = SO(3)
H = SO(2) = U(1) = S^1
ϕ_0
H
γ
n winding
is it contractible in G?


ϕ_1
ϕ_2
2L
L
rescale
map $\phi_1 \cdot \phi_2$
$= \phi_2 \cdot \phi_1$
2
1
(we) this one
$\pi_2(G/H)$ is abelian, $= \mathbb{Z}$ for
$\eta = \infty$ ← $G = SO($
π
magnitude $\theta \in [0, \pi]$
π
π
contractible
not
3D ball with antipodal points identified! $= SO(3) = RP^2$


ϕ_1
ϕ_2
2L
L
rescale
map $\phi_1 \cdot \phi_2$
$= \phi_2 \cdot \phi_1$
2
1
$\pi_2(G/H)$ is abelian, $= \mathbb{Z}$ for
$\eta = \infty$ ← $G = SO($
π
magnitude $\theta \in [0, \pi]$
π
π
contractible
not
3D ball with antipodal points identified! $= SO(3) = \mathbb{R}P^2$