**Title:** Lecture - Quantum Fields & Strings, PHYS 77

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**Collection/Series:** Quantum Fields & Strings (Elective), March 30 - May 1, 2026

**Subject:** Quantum Fields and Strings

**Date:** April 20, 2026 - 1:00 PM

**URL:** https://pirsa.org/26040055

$$Z_{SG} = \int \mathcal{D}\phi \; \exp\left( - \int \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{\alpha}{\beta^2} \cos \beta\phi + \gamma \right)$$

$$Z_{\text{Thiring}} = \int \mathcal{D}\underset{\begin{pmatrix}\psi_R \\ \psi_L\end{pmatrix}}{\underset{\uparrow}{\psi}} \; \exp\left( - \int \bar{\psi} \not{\partial} \psi - m \bar{\psi}\psi - \frac{g}{2} \left( \bar{\psi} \gamma^\mu \psi \right) \right)$$

Windi

Current

Dictionary:

$$\beta^2 = \frac{4\pi}{1 + g/\pi}$$

$$\frac{\alpha_0}{\beta^2} \cos \beta\phi \longleftrightarrow -m_0 \bar{\psi}\psi$$

Winding current $\equiv -\frac{\beta}{2\pi}\varepsilon^{\mu\nu}\partial_\nu \phi \longleftrightarrow \bar{\psi}\gamma^\mu\psi \equiv$ fermion current

= soliton $\longleftrightarrow \psi$

= $\delta\phi \longleftrightarrow \psi\bar{\psi}$ bound state

* Lumps & anti-lumps attract for $\beta^2 < 4\pi$

* They form bound-states. Lightest bound-state = $\delta\phi$ naive particle 

nice in $g$ ($g > 0$, $g < 0$)

* Free @ $4\pi$

* Repel for $\beta^2 > 4\pi$ (only $\int$ exist!)

* Theory exists for $\beta^2 < 8\pi$

 $\dots$ finite. Only

 $\dots$ divergent

$\Downarrow$

Normal Order Works!

ren quantities

$\overleftarrow{a}^{\dagger}\ \overrightarrow{a}$

freedom!

$$\mathcal{H} = :\frac{1}{2}\hat{\Pi}^2 + \frac{1}{2}(\partial_x \hat{\phi})^2 + \frac{\alpha}{\beta^2}\cos\beta\hat{\phi} + \gamma :_{m}$$

$$\hat{\phi} = \int_{-\infty}^{+\infty} \frac{dk}{2\pi}\frac{1}{\sqrt{2\omega_k}}\left(\hat{a}_k e^{-ikx} + \hat{a}_k^{\dagger} e^{+ikx}\right), \quad \omega_k = \sqrt{k^2+m^2}$$

$$\hat{\Pi} = \dots \omega_k \to 1/\omega_k \quad + \to -$$

$$:\mathcal{H}:_m \; = \; :\mathcal{H}:_\mu$$

$$\alpha \to \alpha \left(\frac{\mu^2}{m^2}\right)^{\beta^2/8\pi}$$

finite

$$\gamma \to \gamma + \frac{\mu^2 - m^2}{8\pi}$$

Warm-up:

free massive scalar

$M, \mu, m$ all finite!

$$\mathcal{H} = :\frac{\hat{\Pi}^2}{2} + \frac{(\partial_x \hat{\phi})^2}{2} + \frac{M^2}{2}\hat{\phi}^2:_m$$

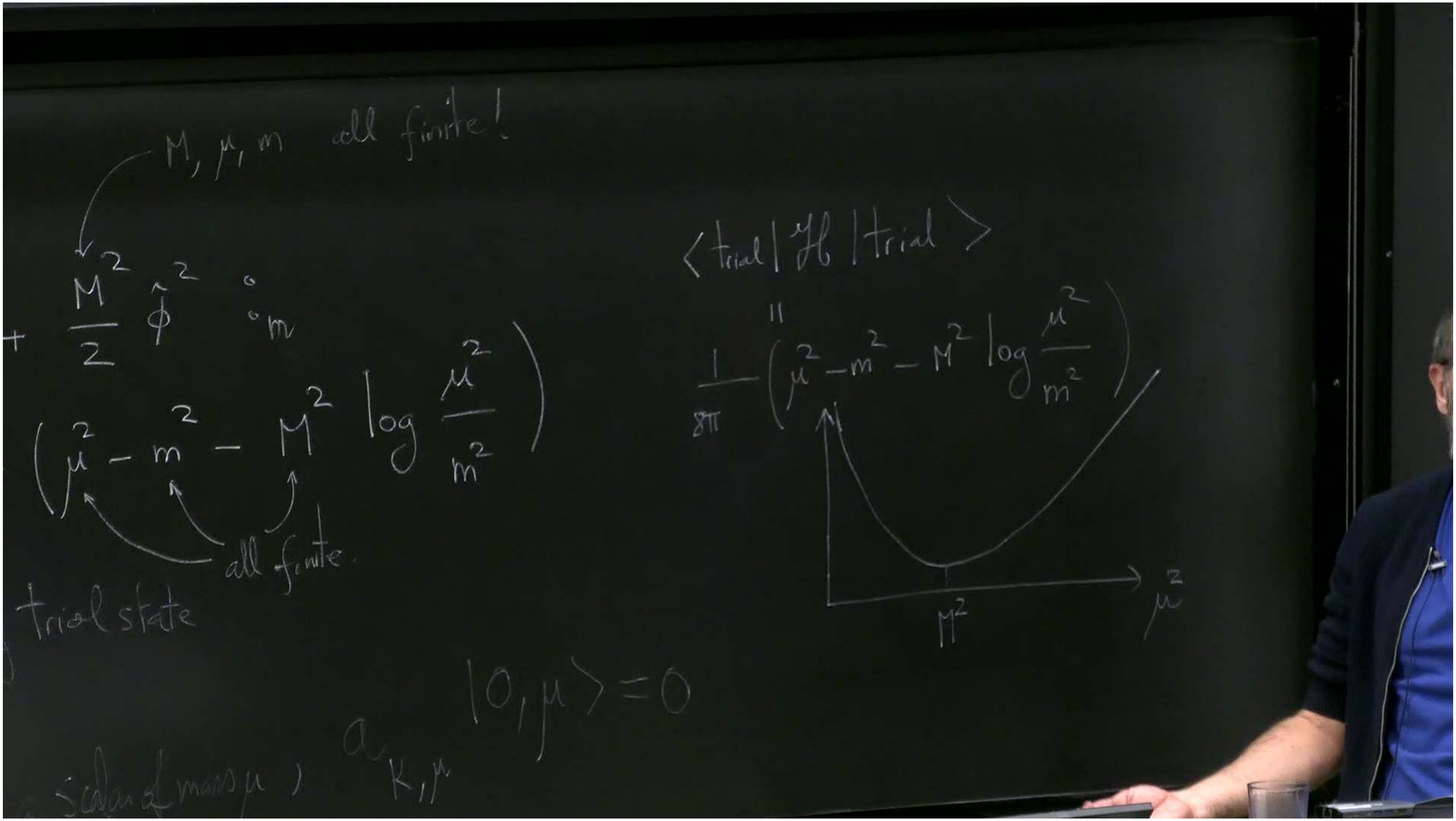
$$= :\text{same}:_\mu + \frac{1}{8\pi}\left(\mu^2 - m^2 - M^2 \log\frac{\mu^2}{m^2}\right)$$

all finite.

bound on GS energy using trial state

$$|\text{trial}\rangle = |0,\mu\rangle$$

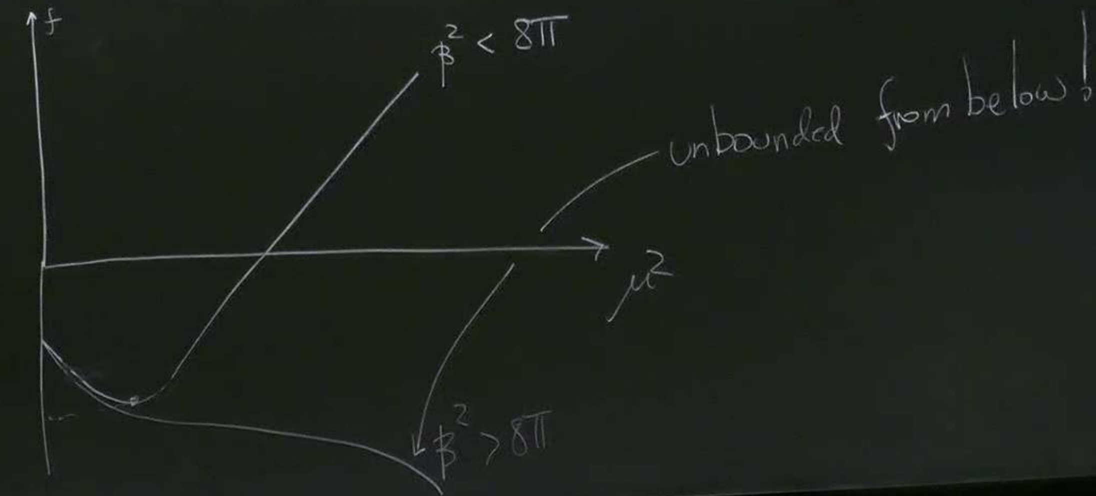
vacuum for a scalar of mass $\mu$, $a_{k,\mu}|0,\mu\rangle = 0$


$M, \mu, m$ all finite!
$+ \frac{M^2}{2} \hat{\phi}^2 \, {:}m$
$\left(\mu^2 - m^2 - M^2 \log \frac{\mu^2}{m^2}\right)$
all finite.
trial state
$|0, \mu\rangle = 0$
scalar of mass $\mu$, $a_{k,\mu}$
$\langle \text{trial} | \mathcal{H} | \text{trial} \rangle$
$= \frac{1}{8\pi}\left(\mu^2 - m^2 - M^2 \log \frac{\mu^2}{m^2}\right)$
$M^2$
$\mu^2$

vacuum-for

Same for SG:

$$\langle 0,\mu | :\mathcal{H}:_m |0,\mu\rangle = \underbrace{-\frac{\alpha}{\beta^2}\left(\frac{\mu^2}{m^2}\right)^{\beta^2/8\pi} + \frac{\mu^2-m^2}{8\pi} - \gamma}_{f}$$



from pt of view of SG = CFT + pert.

$$\left\langle e^{i\beta\phi(x_1)} \, e^{-i\beta\phi(x_2)} \right\rangle_{CFT} = \frac{1}{|x_1 - x_2|^{\beta^2/2\pi}}$$

$2\Delta_{\mathcal{O}=e^{i\beta\phi}}$

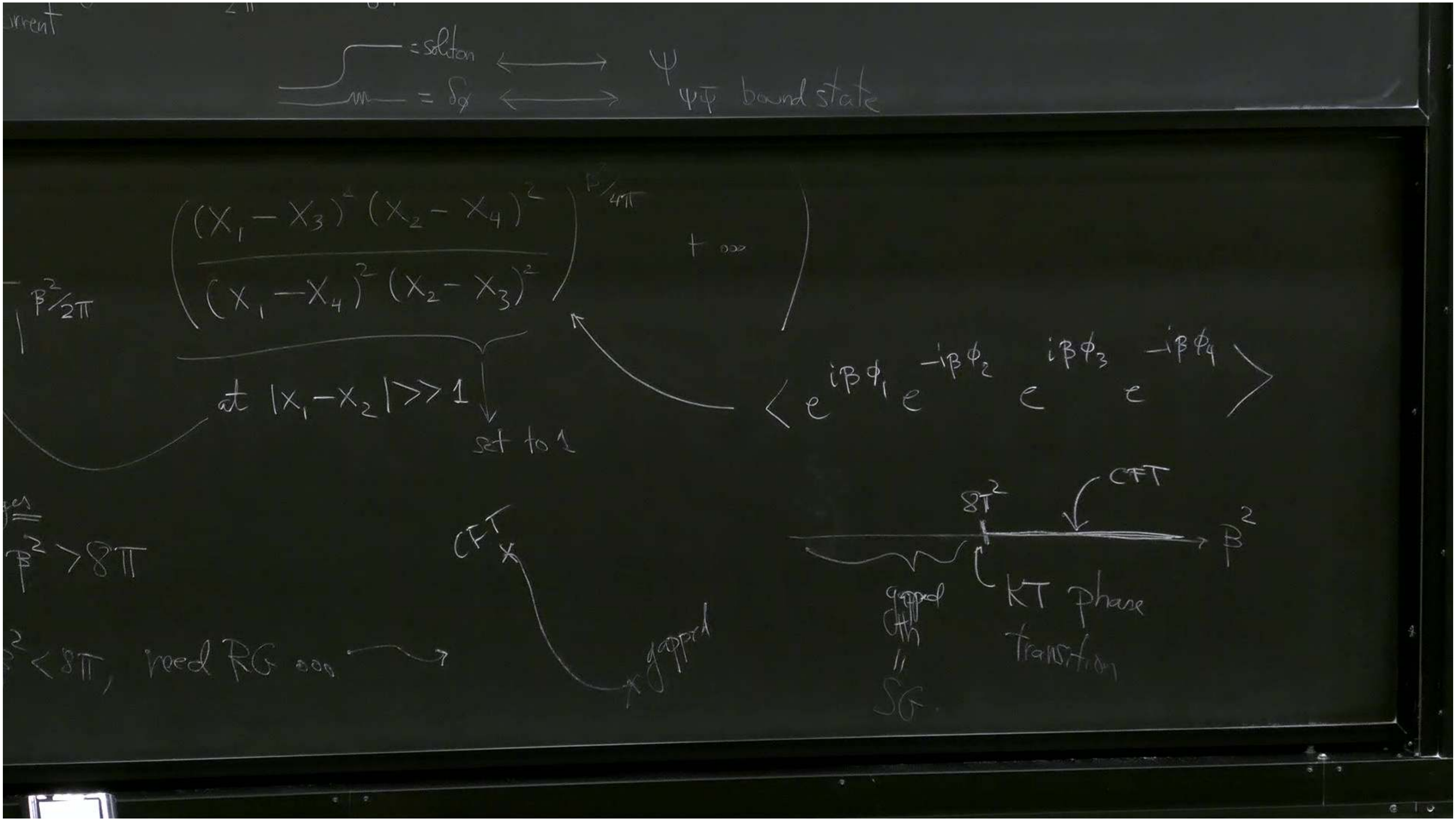
$> d = D+1$

irrelevant

$$\Delta_{\mathcal{O}} = \frac{\beta^2}{4\pi} \quad > 2 \text{ if } \beta^2 > 8\pi$$

$$< 2 \text{ if } \beta^2 < 8\pi$$

relevant


= soliton ⟷ Ψ
= δφ ⟷ ΨΨ̄ bound state
$\left( \frac{(x_1-x_3)^2 (x_2-x_4)^2}{(x_1-x_4)^2 (x_2-x_3)^2} \right)^{\beta^2/4\pi}$ + ...
$\beta^2/2\pi$
at $|x_1-x_2| \gg 1$
set to 1
$\langle e^{i\beta\phi_1} e^{-i\beta\phi_2} e^{i\beta\phi_3} e^{-i\beta\phi_4} \rangle$
$\beta^2 > 8\pi$
$\beta^2 < 8\pi$, need RG ...
CFT
gapped
$8\pi^2$
CFT
$\beta^2$
gapped
Gth
=
SG
KT phase transition

$$\left\langle e^{i\beta\phi(x_1)} e^{-i\beta\phi(x_2)} \right\rangle_{\substack{\text{CFT}\\ \text{+pert}}} = \underbrace{\frac{1}{|x_1-x_2|^{\beta^2/2\pi}}} \left( 1 + \# \int \frac{dx_3\, dx_4}{|x_3-x_4|^{\beta^2/2\pi}} \right.$$

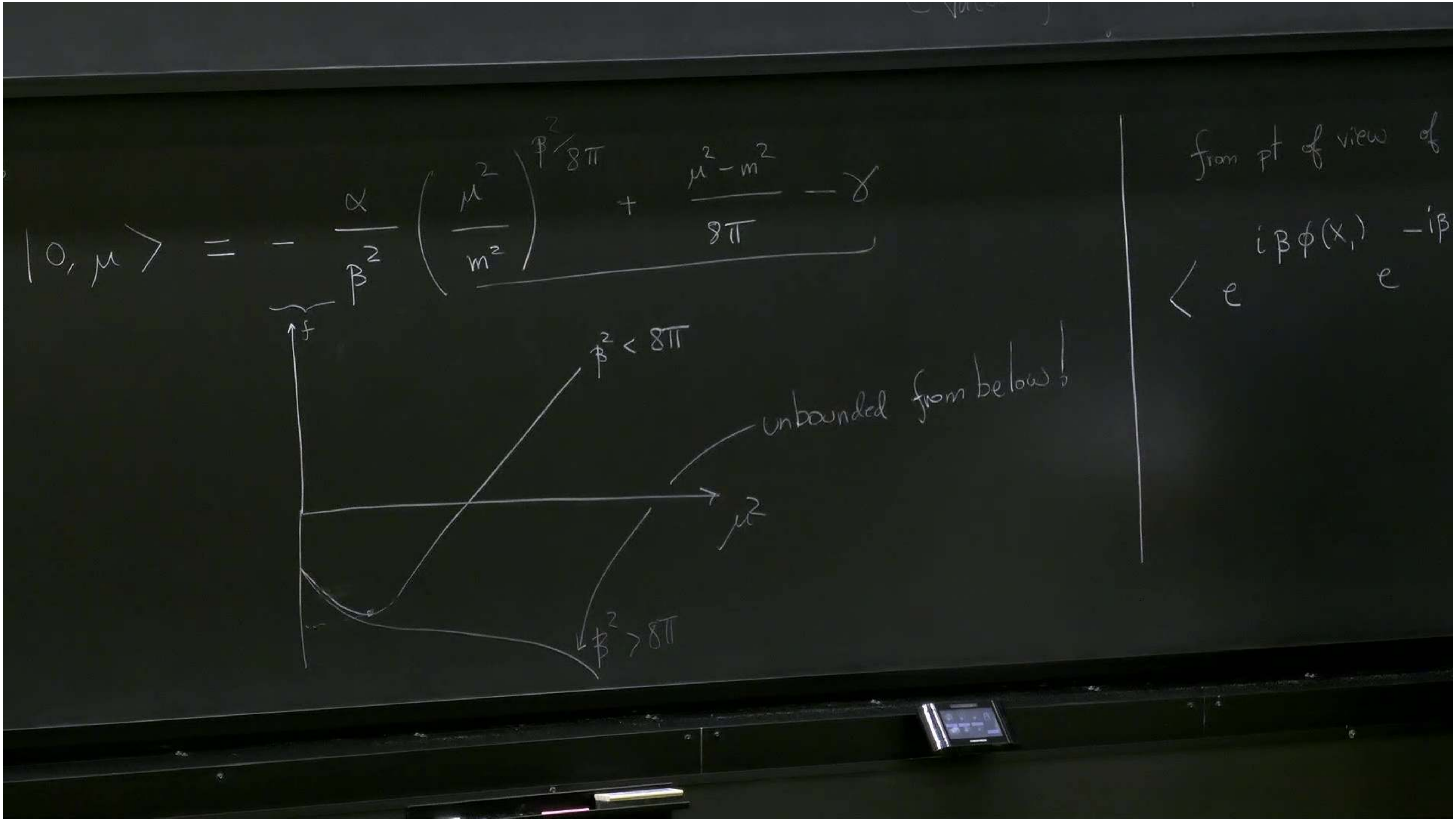
does nothing if $\beta^2 > 8\pi$

get this at $\beta^2 > 8\pi$

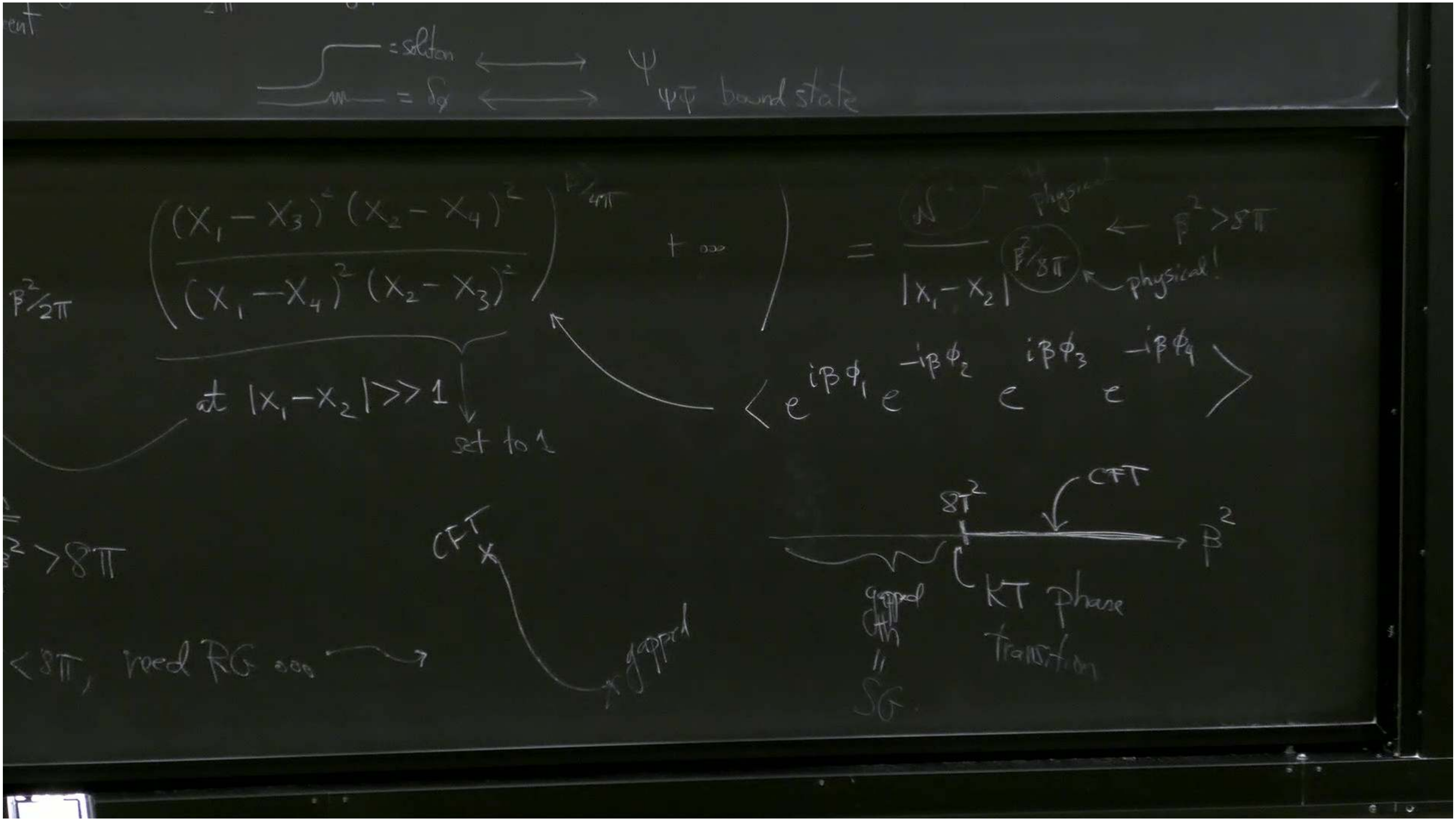
the pert. was irrelevant!

converges for $\beta^2 > 8\pi$

if $\beta^2 < 8\pi$,



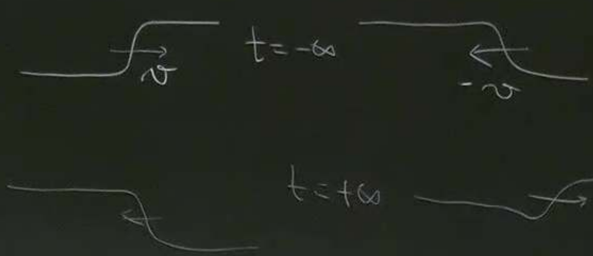
$|0,\mu\rangle = -\frac{\alpha}{\beta^2}\left(\frac{\mu^2}{m^2}\right)^{\beta^2/8\pi} + \frac{\mu^2 - m^2}{8\pi} - \gamma$
f
$\beta^2 < 8\pi$
unbounded from below!
$\mu^2$
$\beta^2 > 8\pi$
from pt of view of
$\langle e^{i\beta\phi(x_1)} e^{-i\beta}$


= soliton
$\Psi$
$\psi\bar{\psi}$ bound state
$\beta^2/4\pi$
$+ \cdots$
$\frac{(X_1 - X_3)^2 (X_2 - X_4)^2}{(X_1 - X_4)^2 (X_2 - X_3)^2}$
$= \frac{N}{|x_1 - x_2|^{\beta^2/8\pi}}$
$\leftarrow \beta^2 > 8\pi$
physical!
$\beta^2/2\pi$
at $|x_1 - x_2| >> 1$
set to 1
$\langle e^{i\beta\phi_1} e^{-i\beta\phi_2} e^{i\beta\phi_3} e^{-i\beta\phi_4} \rangle$
CFT
$8\pi^2$
$\beta^2$
gapped
KT phase transition
SG
$\beta^2 > 8\pi$
CFT
gapped
$< 8\pi$, need RG …

irrelevant!

2 Lumps

$$\phi(x,t) = \frac{4}{\beta}\, \mathrm{arctg}\left( \eta \frac{\sin \omega t}{\cosh \eta \omega} \right)$$




$\sqrt{1 - v^2}$ likes $|v| < 1$

also $v = iV$

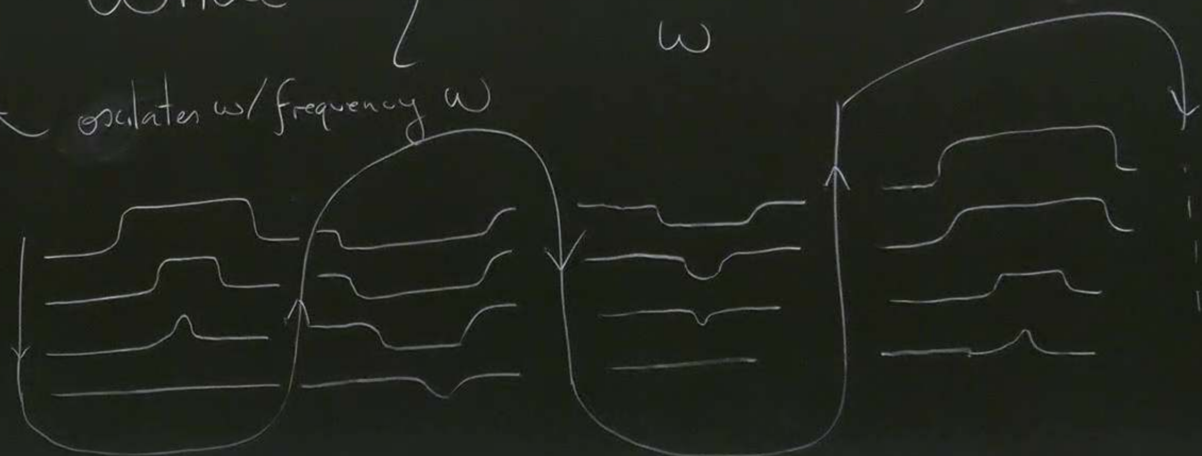
irrelevant!

if $\beta^2 < 8\pi$, need RG ...

gapped

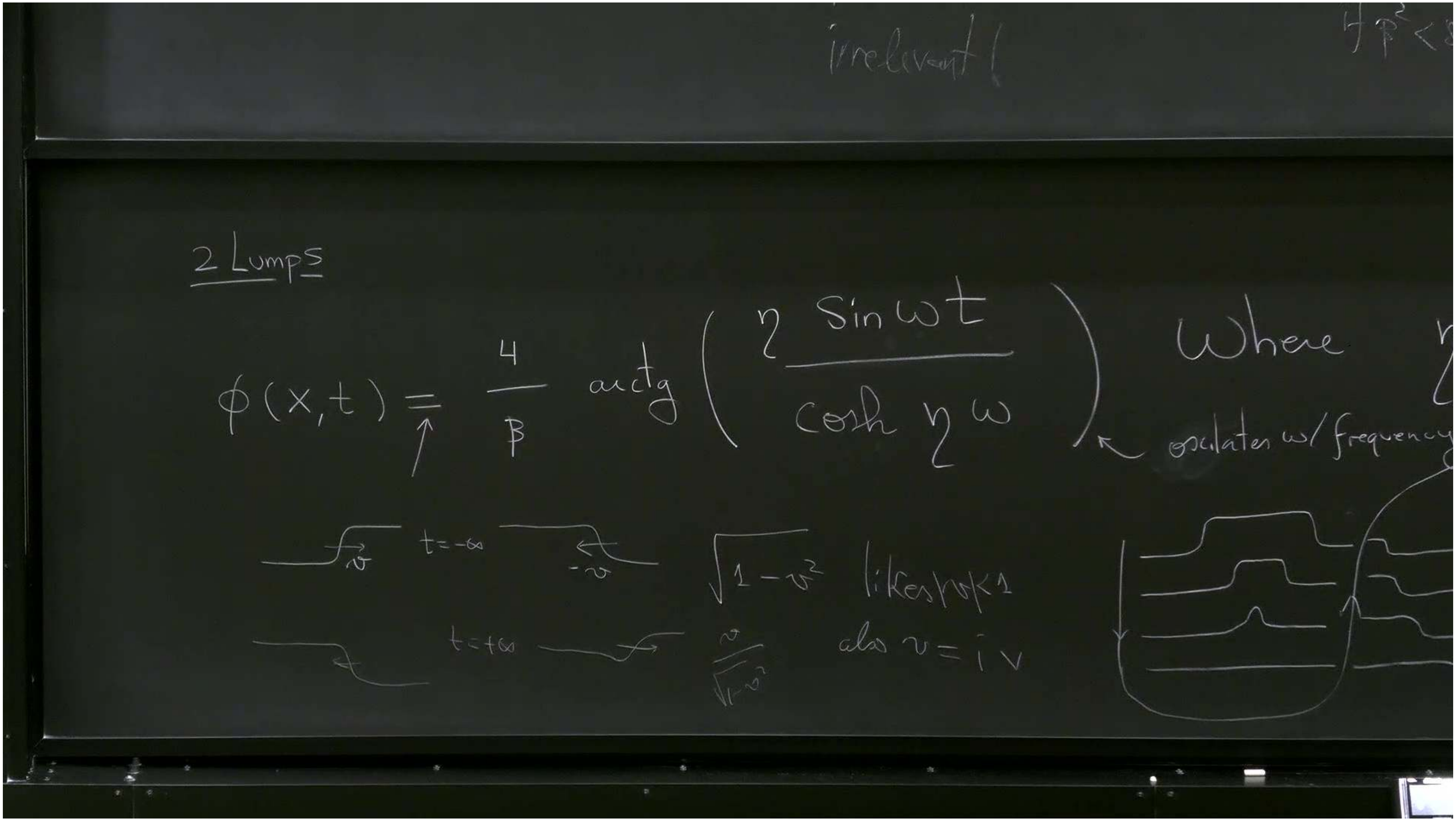
$$\ldots\mathrm{ctg}\left(\eta \frac{\sin \omega t}{\cosh \eta \omega}\right) \quad \text{Where} \quad \eta = \frac{\sqrt{\alpha^2-\omega^2}}{\omega}, \quad \omega \in [0,\alpha]$$

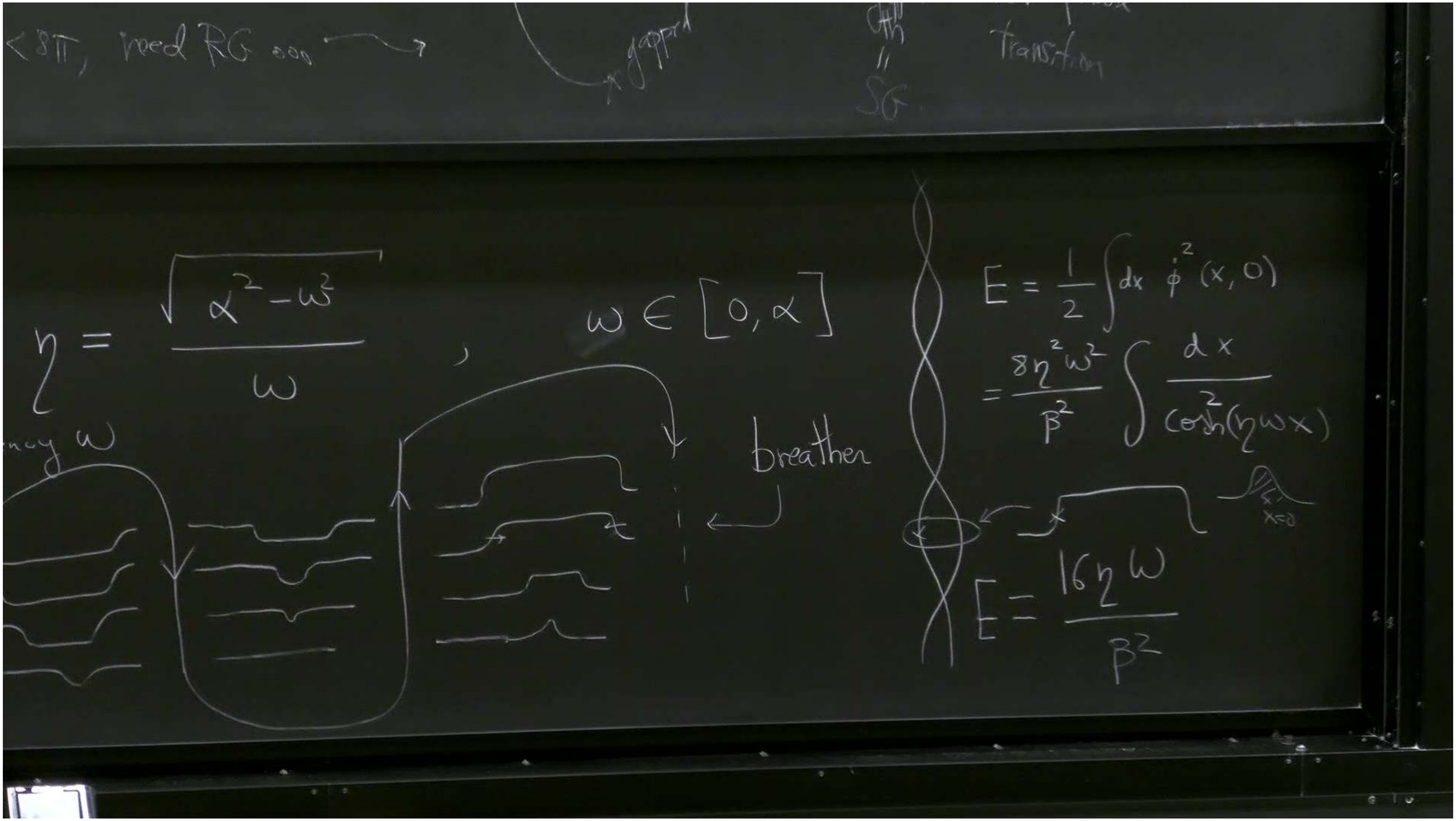
oscilates w/ frequency $\omega$

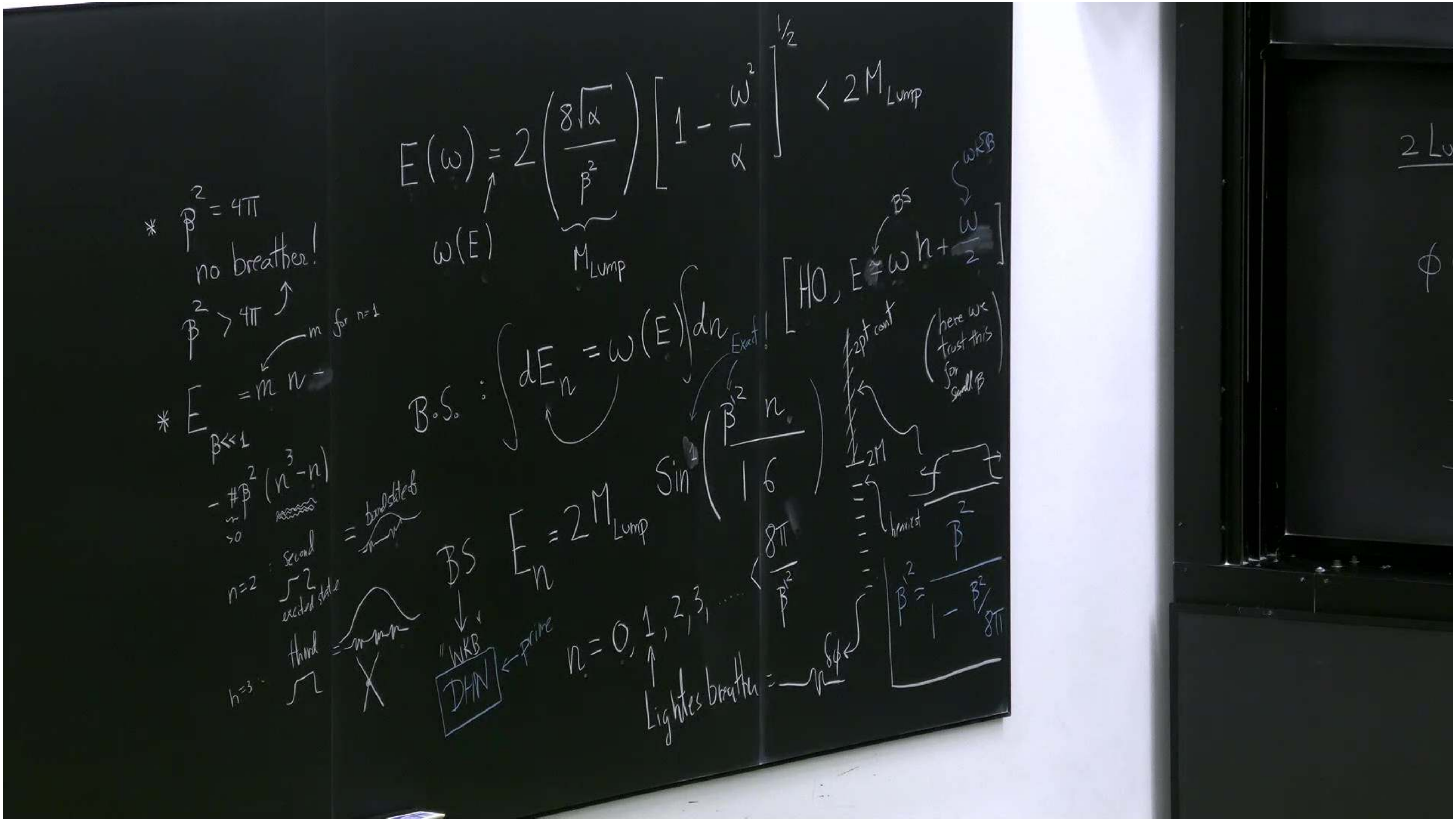


$\sqrt{1-v^2}$ likes $v<1$

also $v = i v$


irrelevant!
2 Lumps
$\phi(x,t) = \frac{4}{\beta} \operatorname{arctg}\left( \eta \frac{\sin \omega t}{\cosh \eta \omega} \right)$
Where
oscilates w/ frequency
$t=-\infty$
$v$
$-v$
$\sqrt{1-v^2}$
likes $\eta \ll 1$
$t=+\infty$
$\frac{v}{\sqrt{1-v^2}}$
also $v = iv$


$< 8\pi$, need RG ...
gapped
Gth
SG
transition
$\eta = \frac{\sqrt{\alpha^2 - \omega^2}}{\omega}$ ,
$\omega \in [0, \alpha]$
ncy $\omega$
breather
$E = \frac{1}{2}\int dx\, \dot{\phi}^2(x,0)$
$= \frac{8\eta^2\omega^2}{\beta^2}\int \frac{dx}{\cosh^2(\eta\omega x)}$
$x=0$
$E = \frac{16\eta\omega}{\beta^2}$


E(ω) = 2(8√α / β²)[1 − ω²/α]^{1/2} < 2M_Lump
ω(E)
M_Lump
* β² = 4π
no breather!
β² > 4π
m for n=1
* E_{β<<1} = m n −
− #β²(n³ − n)
>0
B.S. : ∫dE_n = ω(E)∫dn
Exact!
[HO, E = ω n + ω/2]
BS
WKB
2pt cont
here we trust this for small β
2M
heaviest
Sin⁻¹(β'² n / 16)
BS E_n = 2M_Lump
n=2 : second excited state
bound state of
third
n=3
8π/β'²
β'² = β² / (1 − β²/8π)
"WKB"
DHN ← prime
n = 0, 1, 2, 3, ....
Lightest breather
δφ