Title: Lecture - Quantum Fields & Strings, PHYS 77

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Collection/Series: Quantum Fields & Strings (Elective), March 30 - May 1, 2026

Subject: Quantum Fields and Strings

Date: April 10, 2026 - 1:00 PM

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$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + D^{\mu}\phi^{\dagger} \underbrace{D_{\mu}\phi} - U(\phi)$$

$$\underbrace{\partial_{\mu}\phi + ie \underbrace{A_{\mu}}_{A^{a}_{\mu} T^{a}} \phi}$$

$U(\phi)$: gauge inv potential

$U(\phi) = U(g \cdot \phi)$

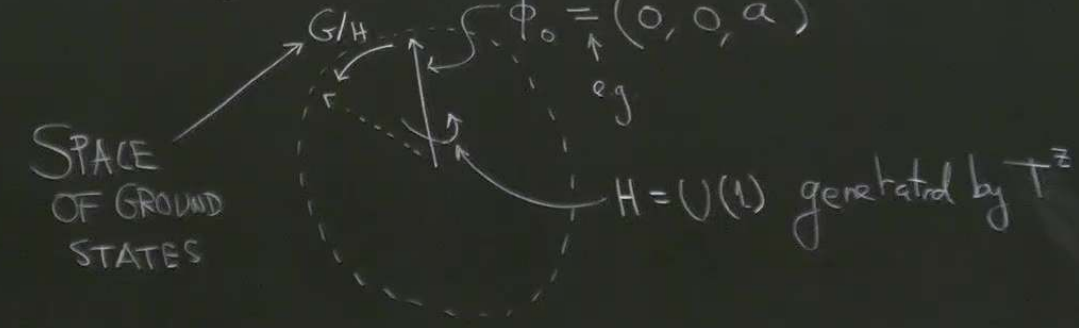
T^{a}: generators of gauge group G

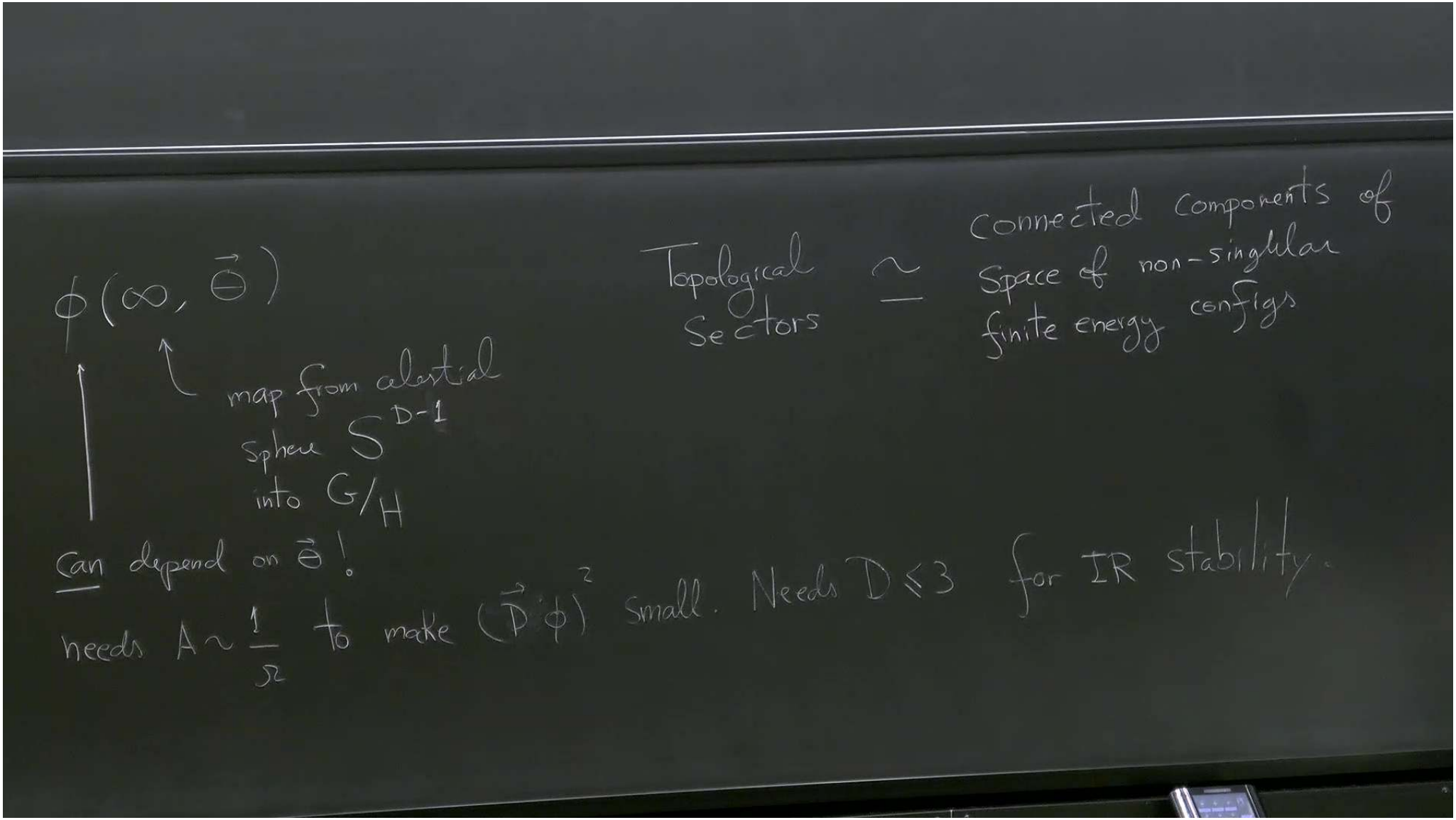
SPA
OF
ST

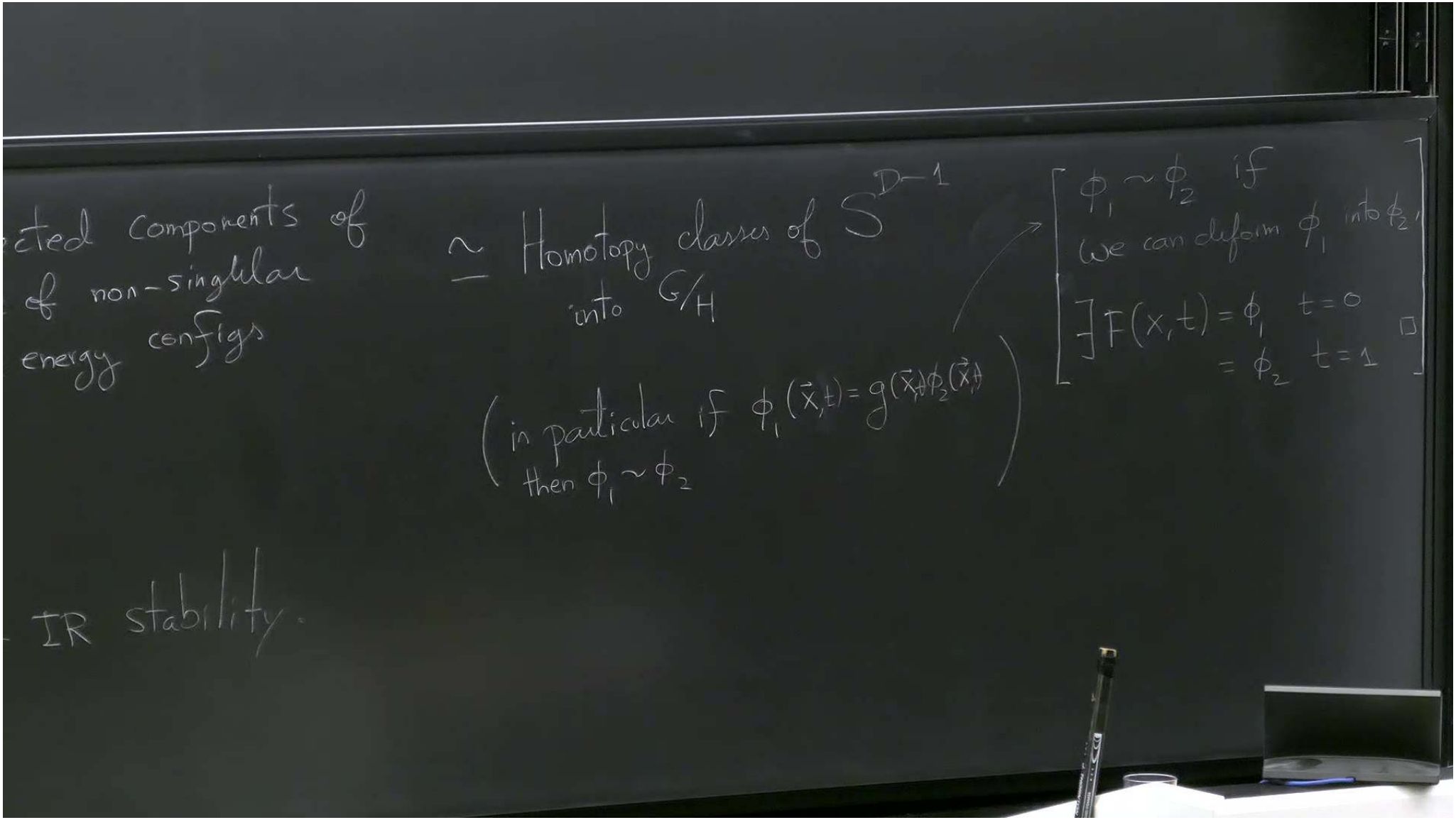
gauge inv potential

$U(\phi) = U(g \cdot \phi)$, e.g $U(\phi) = \frac{\lambda}{2}\left(\vec{\phi}^2 - a^2\right)$ for ϕ in the vector rep of $SO(3) = G$

of gauge group G



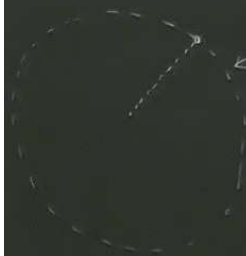

$\phi(\infty, \vec{\theta})$
map from celestial Sphere S^{D-1} into G/H
Can depend on $\vec{\theta}$!
needs $A \sim \frac{1}{\Omega}$ to make $(\vec{\nabla}\phi)^2$ small. Needs $D \leq 3$ for IR stability.
Topological Sectors $\simeq$ Connected Components of Space of non-singular finite energy configs


cted components of
of non-singular
energy configs
≅ Homotopy classes of S^{D-1}
into G/H
(in particular if $\phi_1(\vec{x},t) = g(\vec{x},t)\phi_2(\vec{x},t)$
then $\phi_1 \sim \phi_2$)
$\phi_1 \sim \phi_2$ if
we can deform ϕ_1 into ϕ_2
$\exists F(x,t)$ $= \phi_1$ $t=0$
$= \phi_2$ $t=1$
IR stability.

μ ~ generators of gauge group G

SPACE OF GROUND STATES

$G = U(1)$ in $D = 2$



$\sim \phi = a\, e^{i\sigma(\theta)}$

map $S^1 \to S^1$

can have winding

$\pi_1(S_1) = \mathbb{Z} \ni n = \int \frac{d\theta}{2\pi} \sigma'(\theta)$ = topological charge.

← needed $\lim_{r\to\infty} r \int d\theta\, A_\theta(r,\theta) = \frac{1}{e} 2\pi n$

[candl $\frac{d\phi}{d\theta}$

flux quantization!

$\# A\phi - \frac{d\phi}{d\theta} = 0$

$\theta) = \frac{1}{e} 2\pi n$

flux quantization!

$\frac{d\phi}{d\theta}$

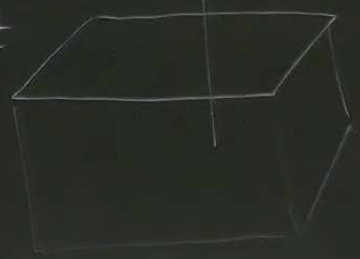
$- \frac{d\phi}{d\theta} = 0$

has physical implications in cond-mat

DRAW THESE
VORTICES ($n=1$),
ANTI-VORTICES ($n=-1$)

∞

things don't depend on z $\longrightarrow$ D=2



2) $\phi^* \phi \longrightarrow \vec{\phi} \cdot \vec{\phi}$ still in $D=2$

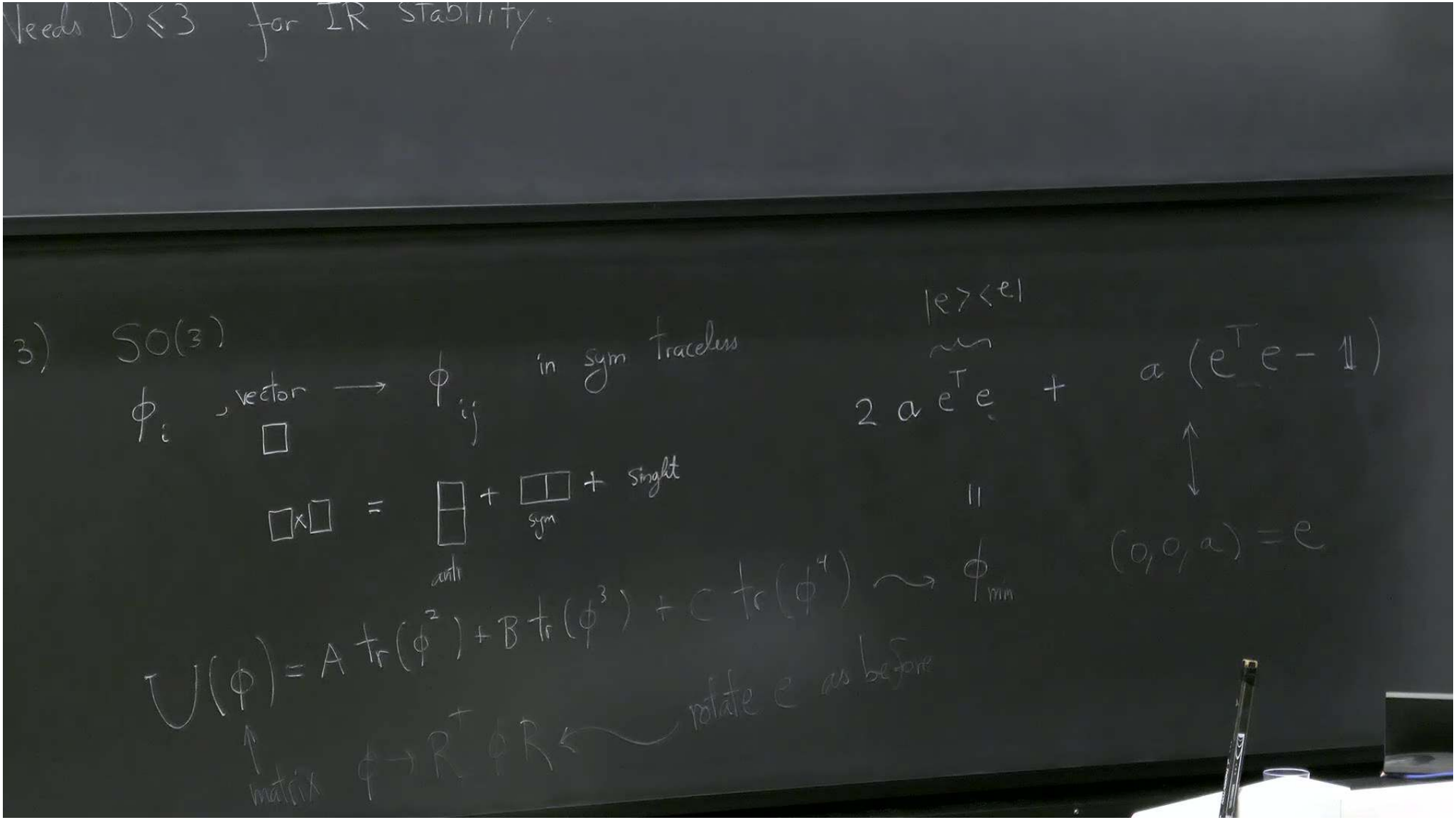
$U(1)$ $SO(3)$

3) $SO(3)$

ϕ_i, vector $\longrightarrow$ ϕ_{ij} $\square$ $\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$

$\pi_1(S^2) = \phi$

"can't lasso a sphere"


needs D≤3 for IR stability.
3) SO(3)
ϕ_i , vector $\longrightarrow$ ϕ_{ij} in sym traceless
□ × □ = □ + □ + singlet
sym
anti
$U(\phi) = A\,\mathrm{tr}(\phi^2) + B\,\mathrm{tr}(\phi^3) + C\,\mathrm{tr}(\phi^4) \rightsquigarrow \phi_{min}$
matrix $\phi \to R^T \phi R$ ⟵ rotate e as before
$|e\rangle\langle e|$
$2a\,e^T e + a(e^T e - \mathbb{1})$
$(0,0,a) = e$

2) $\phi^* \phi \longrightarrow \vec{\phi} \cdot \vec{\phi}$ still in $D=2$

$U(1)$ $SO(3)$

$\pi_1(S^2) = \phi$

"can't have a sphere"

no top sectors.

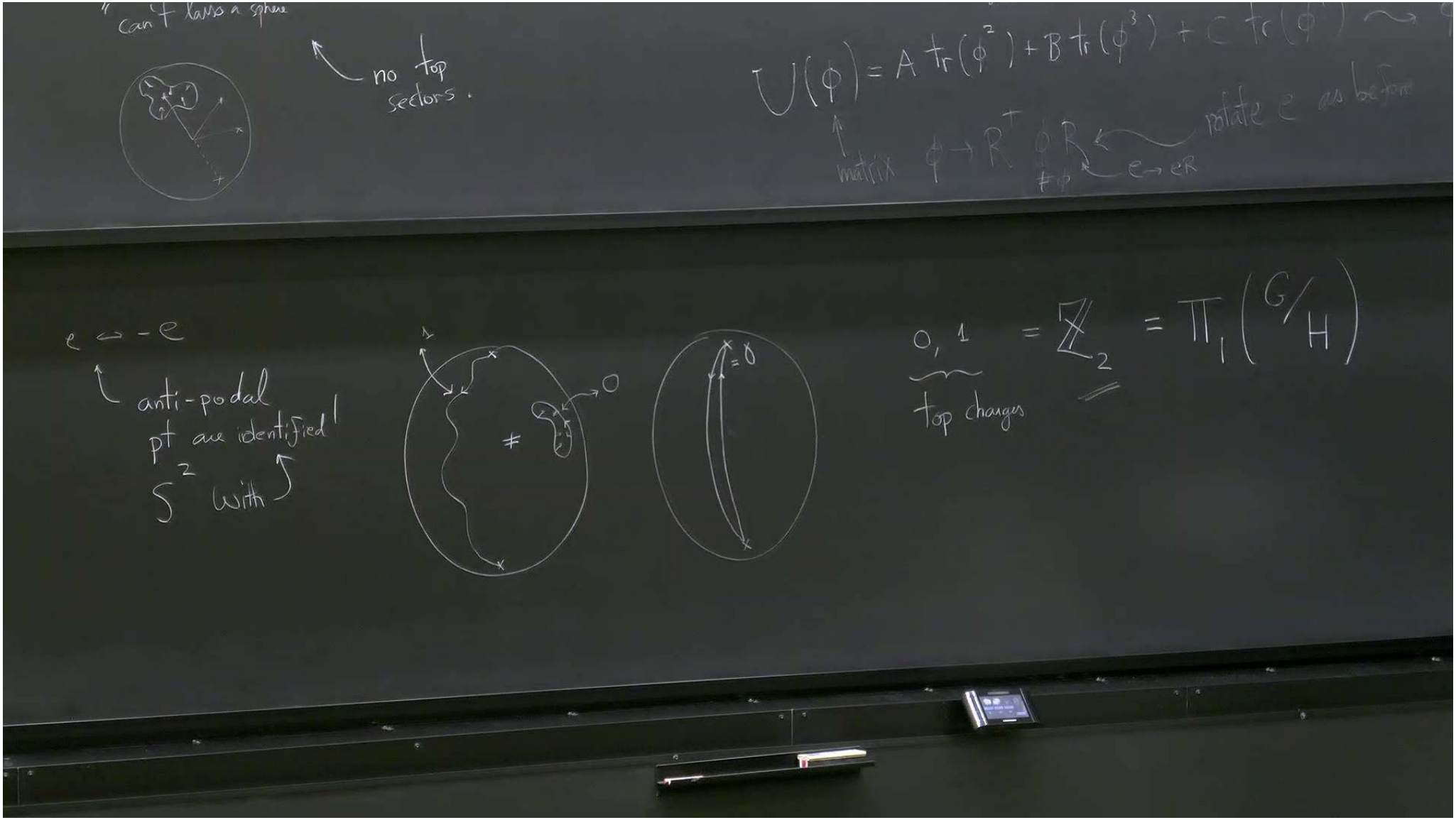
3) $SO(3)$

ϕ_i , vector $\longrightarrow$

$\square \times \square =$

$U(\phi) = A\, tr(\phi^2$

matrix $\phi \rightarrow$


can't have a sphere
no top sectors.
$U(\phi) = A\,\mathrm{Tr}(\phi^2) + B\,\mathrm{Tr}(\phi^3) + C\,\mathrm{Tr}(\phi^4)$
matrix
$\phi \to R^\dagger \phi R$
top
$e \to eR$
rotate e as before
$e \leftrightarrow -e$
anti-podal pt are identified
S^2 with
$\neq$
$= 0$
$0, 1 = \mathbb{Z}_2 = \pi_1\left(G/H\right)$
top charges

$\pi_1(S_1) = \mathbb{Z} \ni n = \frac{1}{2\pi}\int \dots$ = topological charge.

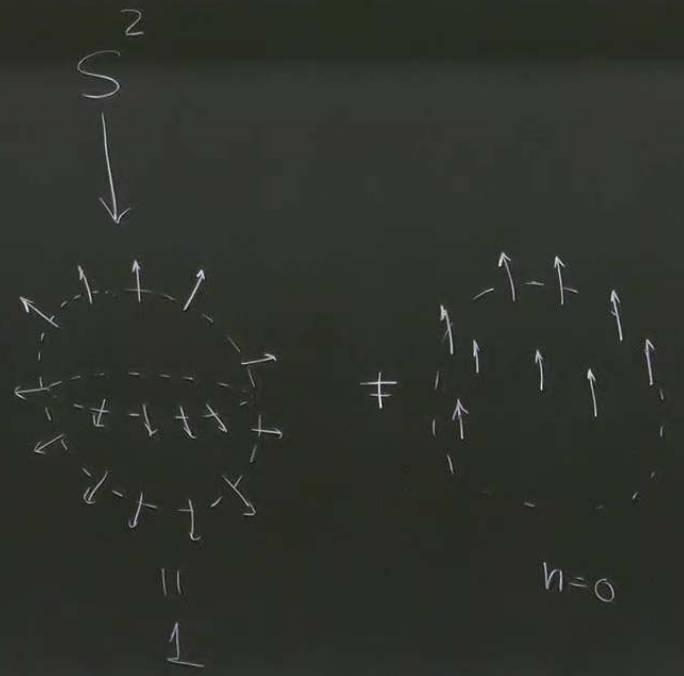
has physic

$D = 3$

$G = SO(3)$, ϕ vector, $G/H = S^2$

maps $\underbrace{S^2}_{\text{now}} \to S^2$, $\pi_2(S^2) = \mathbb{Z}$

We have winding again ⇝



$\pi_1(S_1) = \mathbb{Z} \ni n = \int \frac{d\theta}{2\pi} \sigma'(\theta)$ = topological charge.

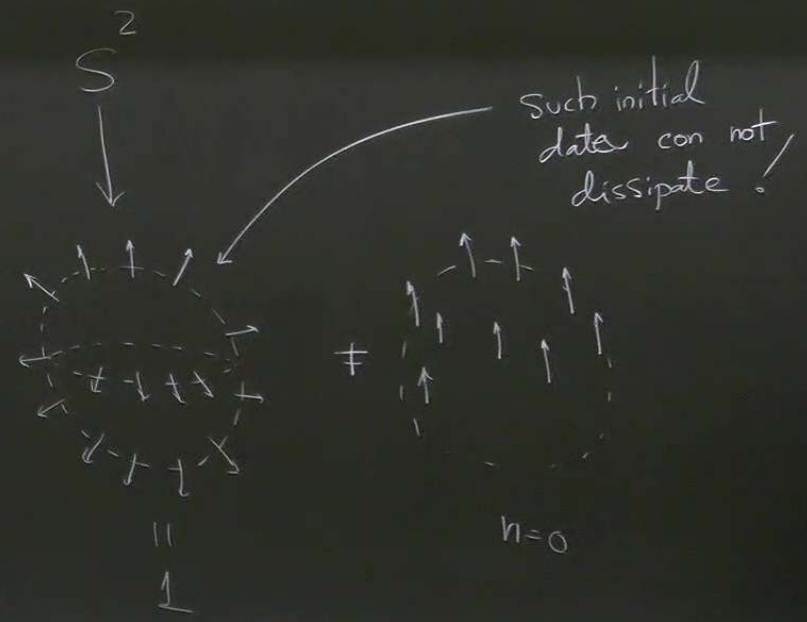
$\# A.\phi - \frac{d\phi}{d\theta} = 0$ ↕ has physical implications

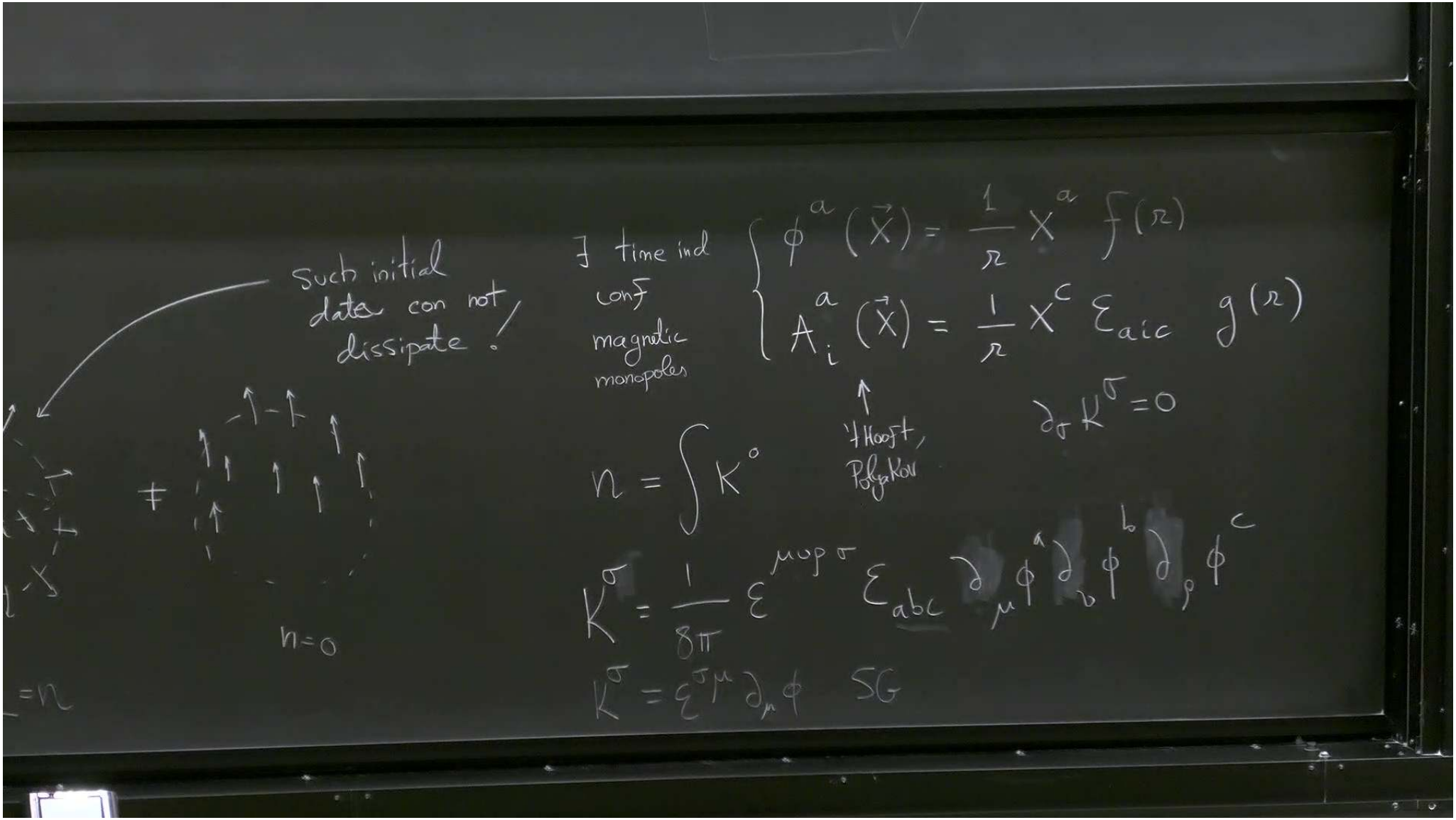
$D = 3$

$G = SO(3)$, ϕ vector, $G/H = S^2$

maps $\underbrace{S^2}_{\text{now}} \to S^2$, $\pi_2(S^2) = \mathbb{Z}$ = top charge

We have winding again ←




Such initial data can not dissipate!
∃ time ind conf magnetic monopoles
$\begin{cases} \phi^a(\vec{X}) = \frac{1}{r} X^a f(r) \\ A_i^a(\vec{X}) = \frac{1}{r} X^c \varepsilon_{aic}\, g(r) \end{cases}$
't Hooft, Polyakov
$\partial_\sigma K^\sigma = 0$
$n = \int K^0$
$K^\sigma = \frac{1}{8\pi} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abc} \partial_\mu \phi^a \partial_\nu \phi^b \partial_\rho \phi^c$
$K^\sigma = \varepsilon^{\sigma\mu} \partial_\mu \phi$ SG
n=0

$D = 3$

② $G = SO(3)$, ϕ vector, $G/H = S^2$

maps $S^2 \longrightarrow S^2$, $\Pi_2(S^2) = \mathbb{Z}$ = top charge

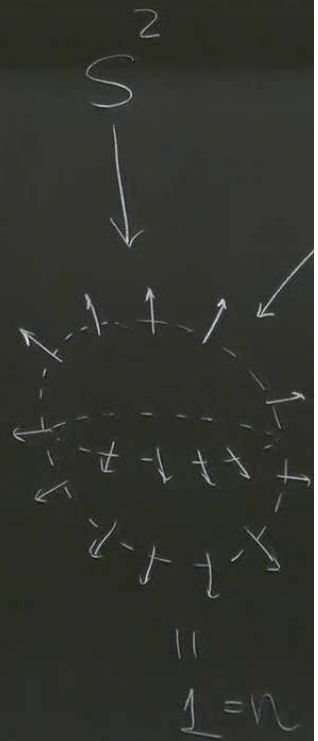
now

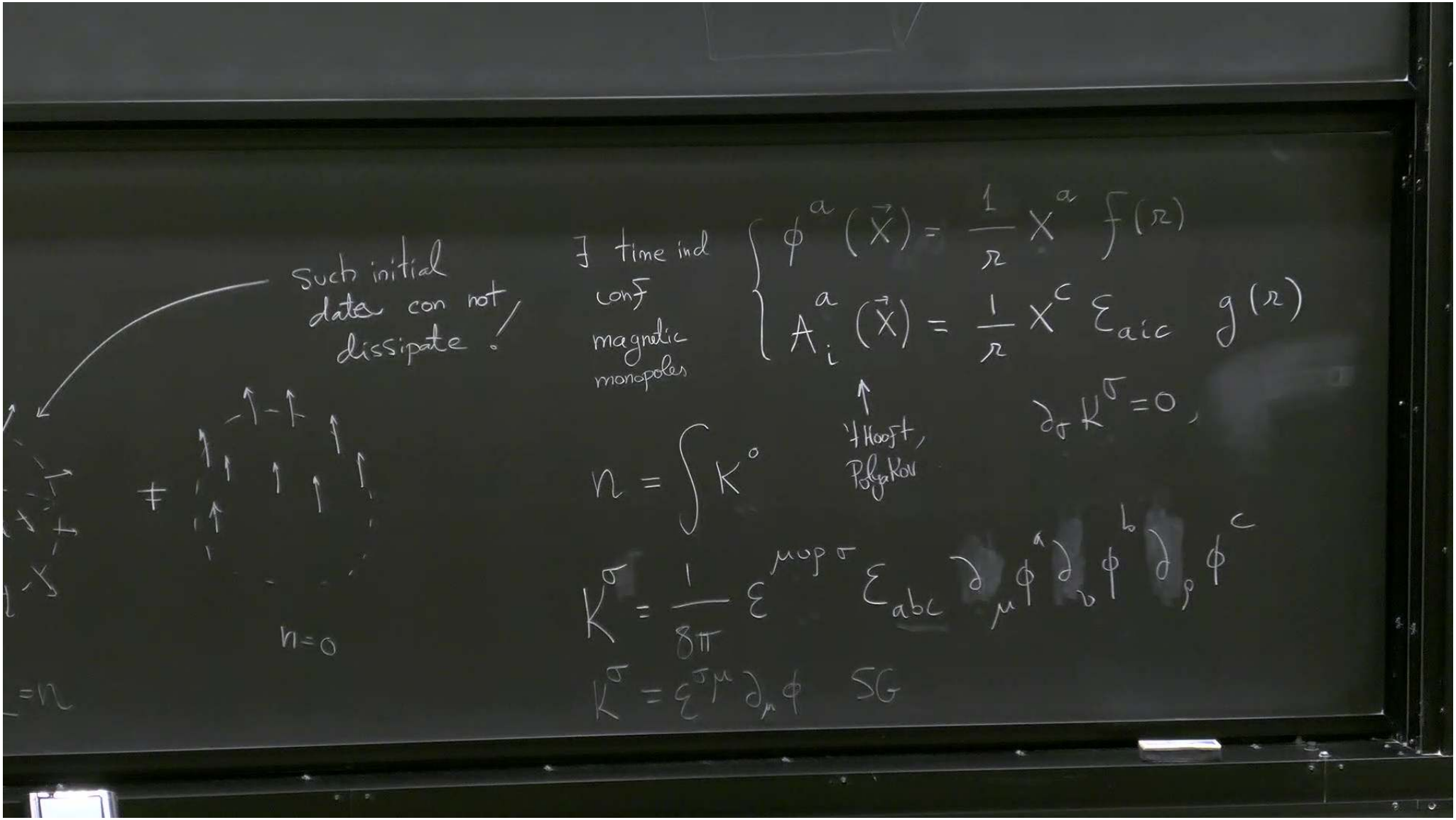
fixes $(0,0,0,a)$ (not here)

We have winding again ⇝

④ $G = SO(4)$, $SO(4)/SO(3) = S^3$

$S^2 \longrightarrow S^3$ (add a dim to previous picture) ⇝ no non-trivial config

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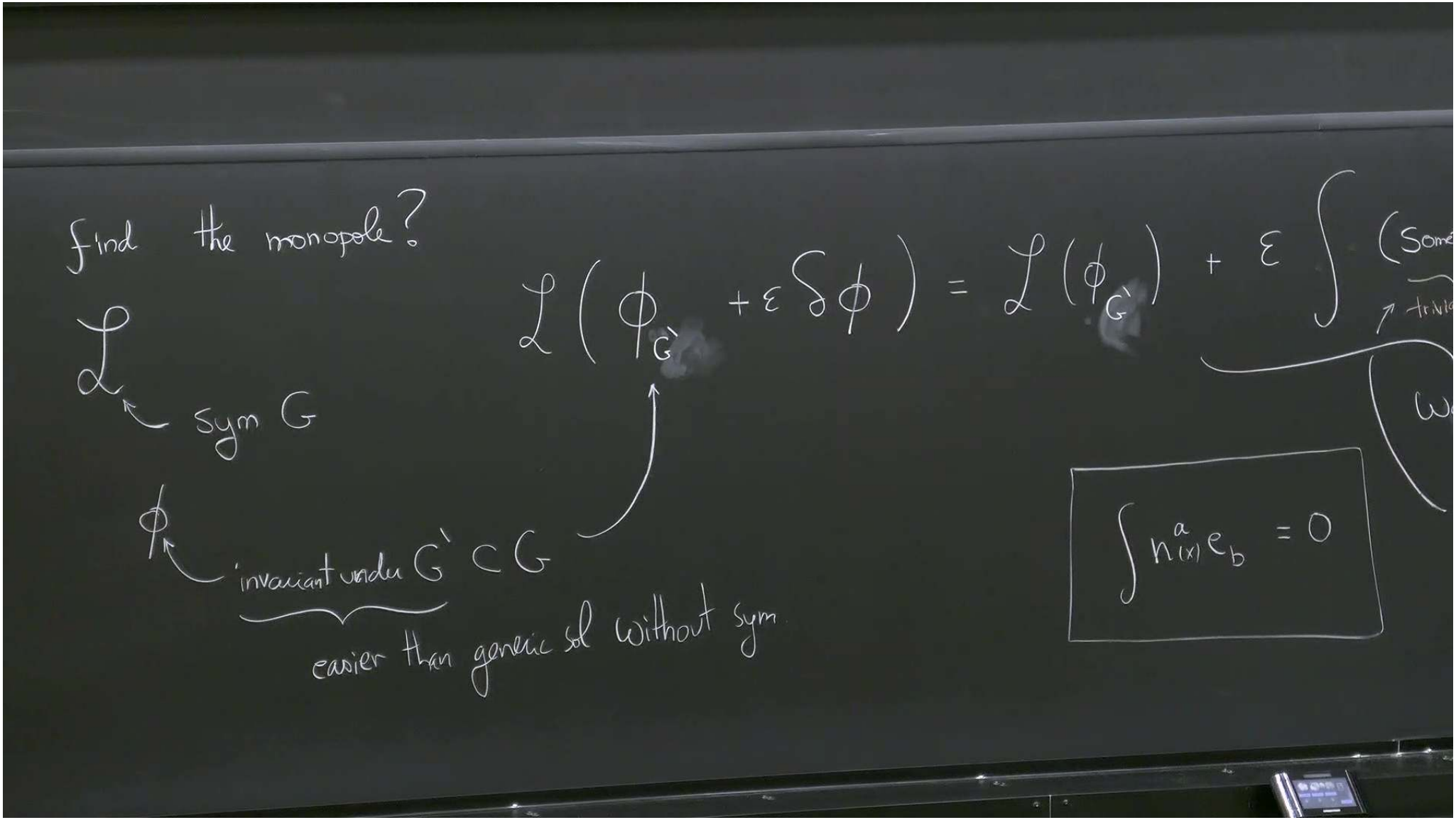
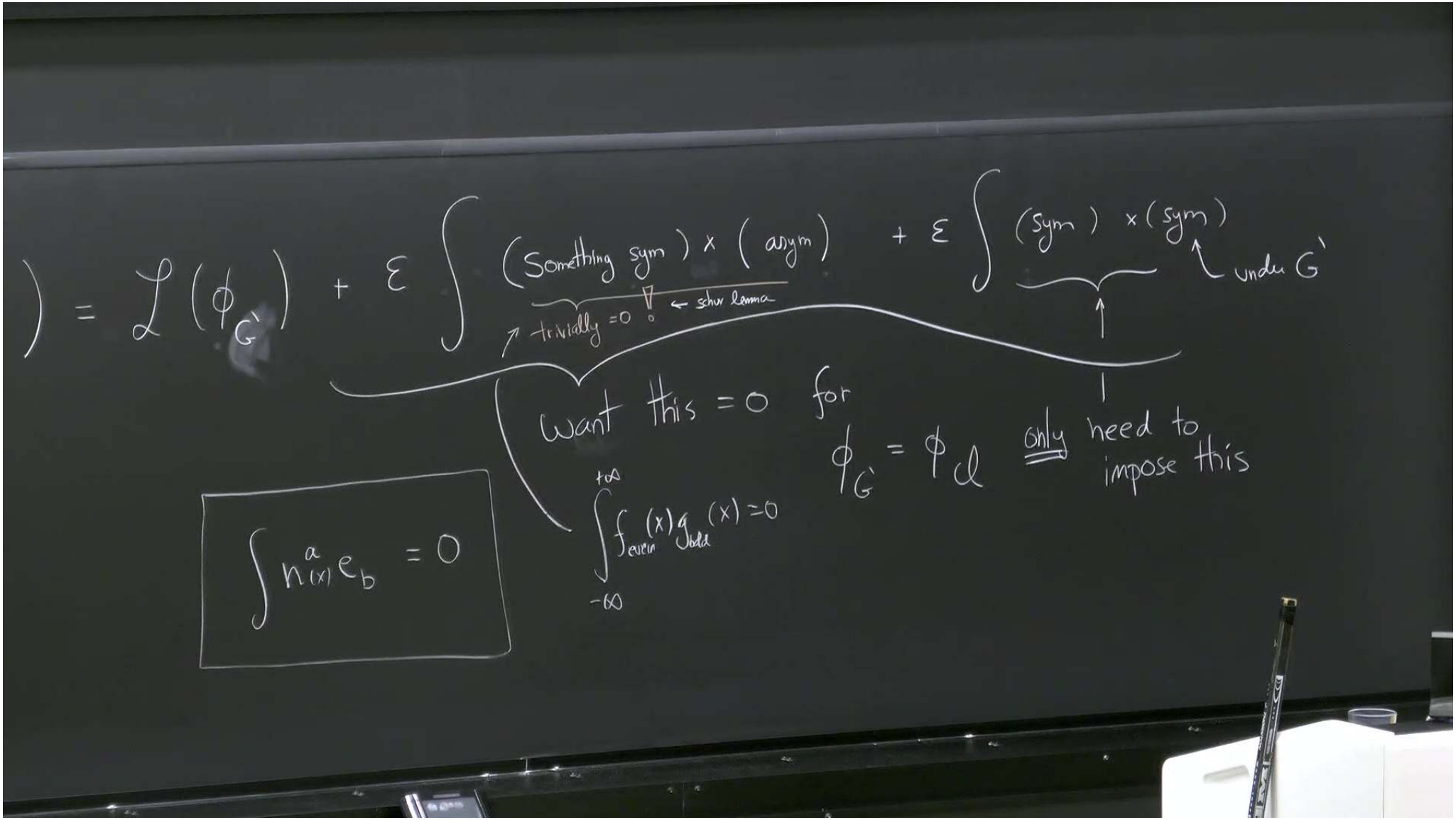
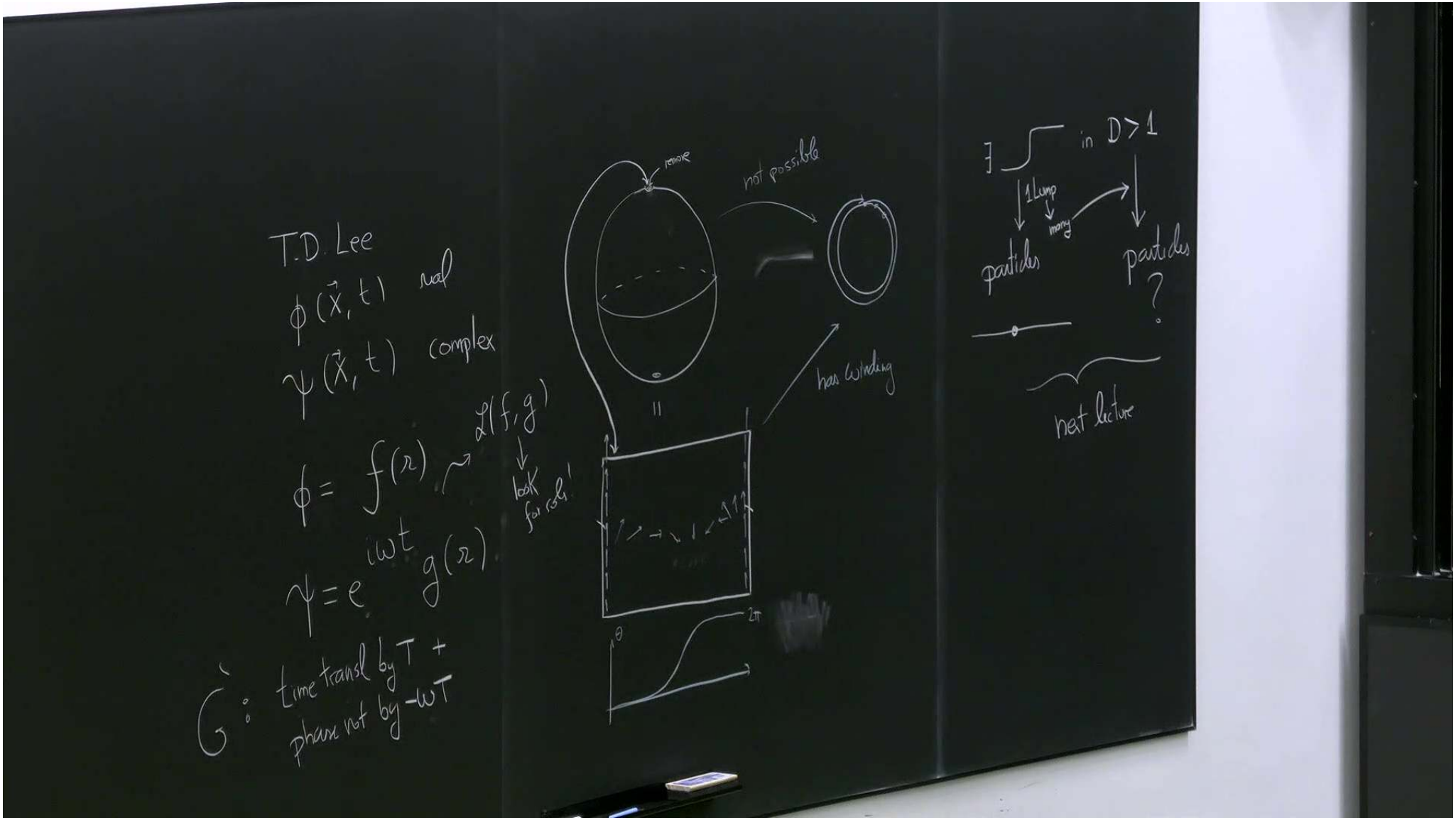
't Hooft, Polyakov
$\partial_\sigma K^\sigma = 0$,
$n = \int K^0$
$K^\sigma = \frac{1}{8\pi} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abc} \partial_\mu \phi^a \partial_\nu \phi^b \partial_\rho \phi^c$
$K^\sigma = \varepsilon^{\sigma\mu} \partial_\mu \phi$ SG
n=0


find the monopole?
$\mathcal{L}$
sym G
ϕ
invariant under $G' \subset G$
easier than generic sol without sym
$\mathcal{L}(\phi_{G'} + \varepsilon\delta\phi) = \mathcal{L}(\phi_{G'}) + \varepsilon \int$
$\int n^a_{i(x)} e_b = 0$


$\mathcal{L}(\phi_{G'})$ + ε ∫ (Something sym) × (asym) + ε ∫ (Sym) × (sym)
trivially = 0 ← Schur lemma
under G'
want this = 0 for $\phi_{G'} = \phi_{cl}$ only need to impose this
$\int n^a_{(x)} e_b = 0$
$\int_{-\infty}^{+\infty} f_{even}(x) g_{odd}(x) = 0$


T.D. Lee
$\phi(\vec{x},t)$ real
$\psi(\vec{x},t)$ complex
$\phi = f(r)$
$\psi = e^{i\omega t} g(r)$
$\mathcal{L}(f,g)$
look for f,g
G: time transl by T + phase rot by $-\omega T$
remove
not possible
has winding
θ
2π
in $D>1$
1 Lump
many
particles
particles
?
next lecture