Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 30 - May 1 2026

Subject: Condensed Matter

Date: April 15, 2026 - 3:30 PM

URL: https://pirsa.org/26040018

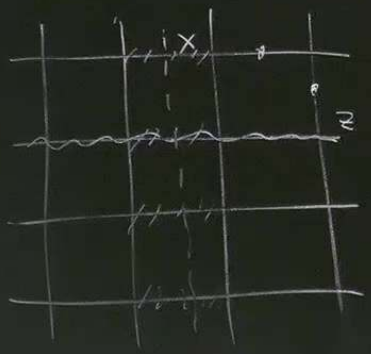
Toric code.

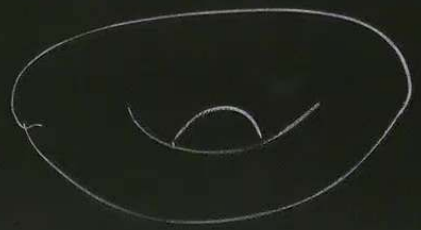
$$H = -\sum_{v} XXXX - \sum_{p} ZZZZ$$

Loop operators
(1-form symmetry)

$$Z_C = \prod_{e \in C} Z_e$$

$$X_{C'} = \prod_{e \in C'} X_e$$





GSD = 4. $\delta \sim e^{-O(L)}$

Consider open strings C, C'.

Z_C $(X_{C'})$ away from end pts.

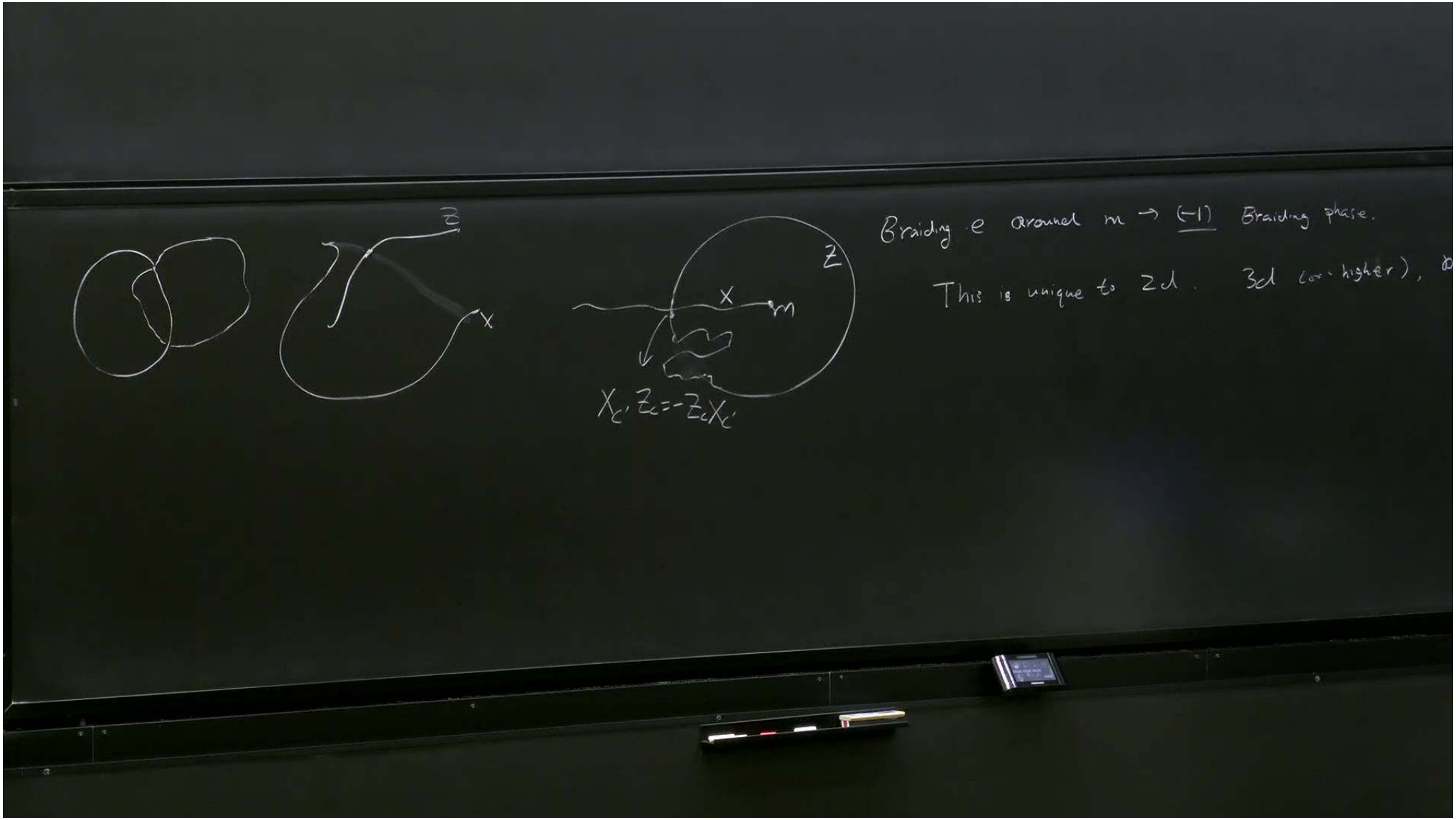
At end pts anticommute with $A_v = XXXX$ $(B_p = ZZZZ)$

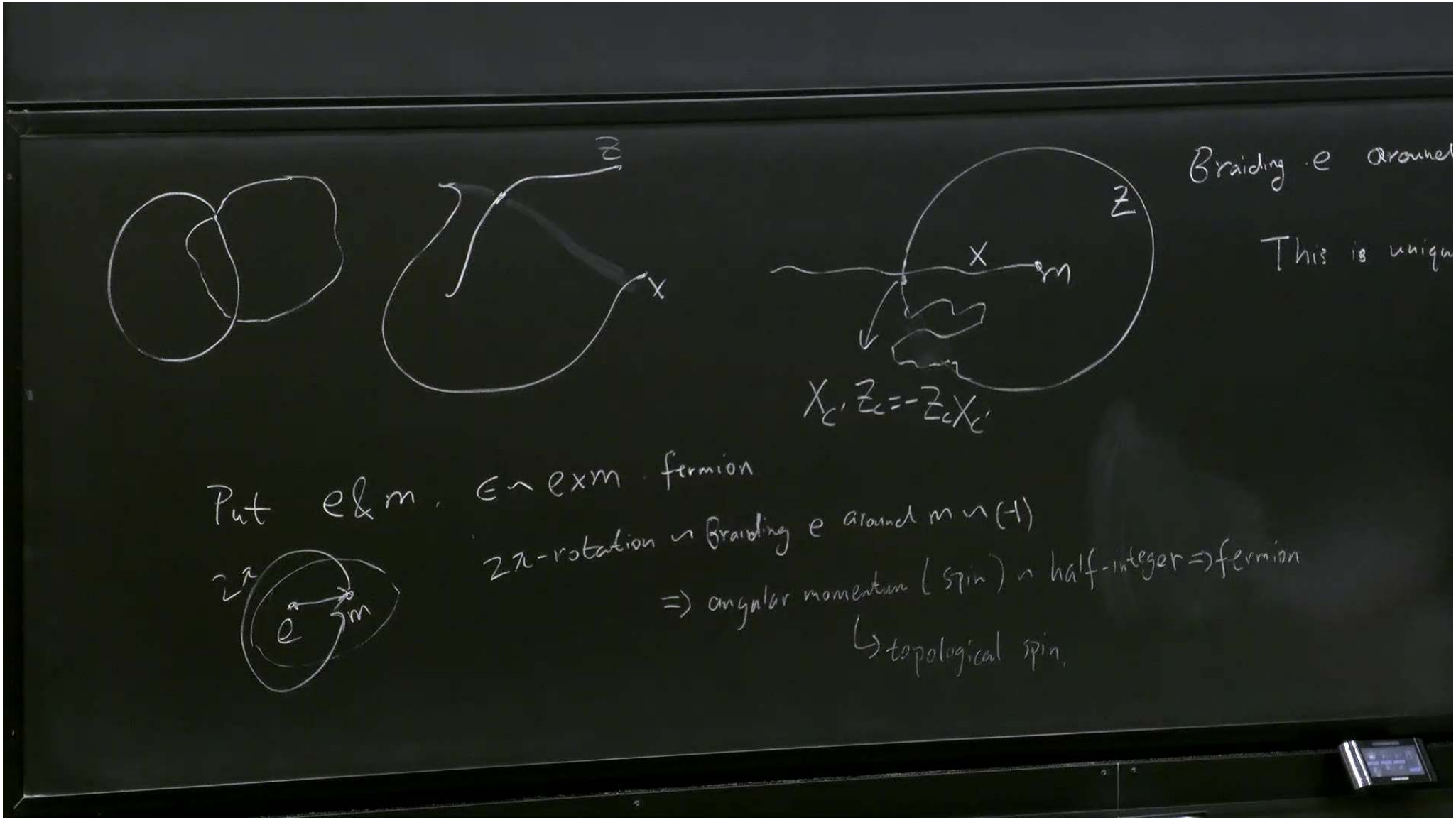
Open string: pair creation.

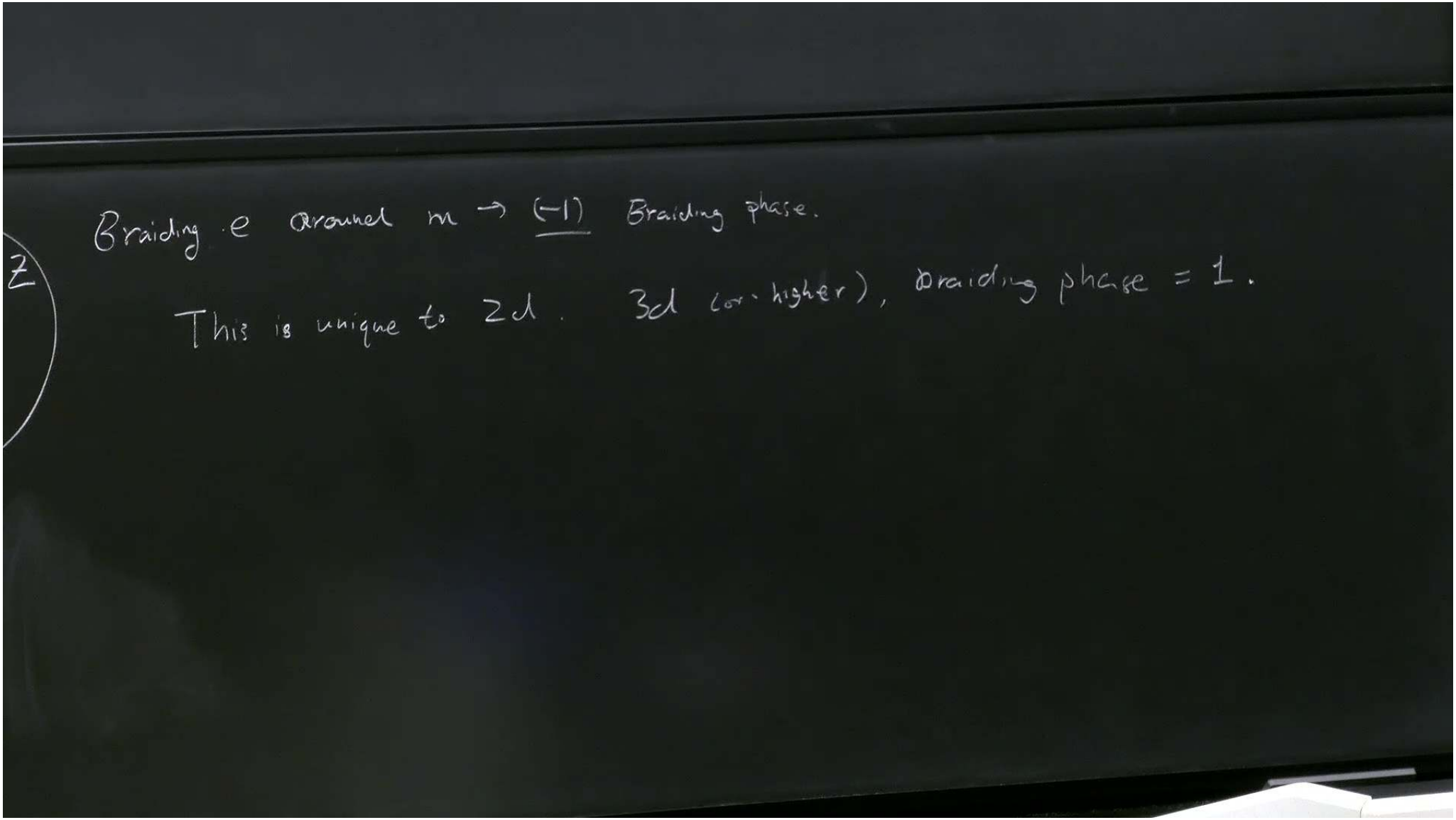
"parallel transport" a particle from 1 pt to another.

Particle excitation with $A_v = -1$: e-particle

$B_p = -1$: m-particle


Z
X
Z
X
m
$X_c, Z_c = -Z_c X_c$
Braiding e around m → (−1) Braiding phase.
This is unique to 2d. 3d (or higher),


Z
X
Z
X
m
$X_{c'} Z_c = -Z_c X_{c'}$
Braiding e around
This is uniqu
Put e & m. $\epsilon \sim e \times m$ fermion
2π-rotation ~ Braiding e around m ~ (-1)
2π
e
m
⇒ angular momentum (spin) ~ half-integer ⇒ fermion
↳ topological spin.


Braiding e around m → (−1) Braiding phase.
This is unique to 2d. 3d (or higher), Braiding phase = 1.

Phase transition

$$H = -\sum_v XXXX - \sum_p ZZZZ - h\sum_e Z_e$$

$h \ll 1$, topological order

$h \gg 1$, $|\Omega\rangle \sim |\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle$

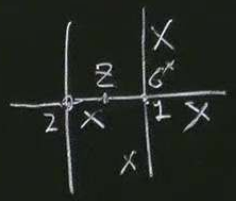
→ Map to a Z_2 gauge theory

cal spin.

On each vertex, introduce $G_v^{x,y,z}$ qubit, with constraint

Z_e

$G_v^x = \prod_{e \in v} X_e$, $G_v^x = \pm 1$ keeps track of e-excitation.



Interpret X_e as "electric field", G_v^x as "gauge charge".

Gauss' law.

Every physical (i.e. gauge-invariant) operator commute with $G^x \cdot \prod_{e \in v} X_e$

$Z_e \to G_1^z Z_e G_2^z$

GSD = 4 . $\delta \sim e$

$$H = -\sum_v G_v^x - h \sum_e G_i^z Z G_z^z - \sum_p \prod_{e \in p} Z_e \sim B^2$$

(gauge invariance: $G_i^z \to (-1)^{s_i} G_i^z$, $Z_{ij} \to (-1)^{s_i + s_j} Z_{ij}$)

Z: gauge connection $\sim e^{i \int_e \vec{A} \cdot d\vec{s}}$ $\xrightarrow{e}$

$G^z \sim b, b^\dagger$

$G^x \sim b^\dagger b$

$$B_p = \prod_{e \in p} Z_e \sim e^{i \int_p d^2x \cdot (\partial \times A)}$$

$\sim \cos[B]$

e : gauge charge.

m : gauge flux

ex m ~ fermion ~ $\gamma \sim \tilde{Z}$ z

uge invariance. $G_i^z \to (-1)^{s_i} G_i^z$. $Z_{ij} \to (-1)^{s_i + s_j} Z_{ij}$) Z_C still good sym.

e → $G^z \sim b,\ b^+$

$G^x \sim b^+ b$

Low energy. $zzzz = 1$, $Z_C = 1$

⇒ Can pick a gauge s.t. $z = 1$

$$\Rightarrow H' = -\sum_v G_v^x - h \sum_{\langle i,j \rangle} G_i^z G_j^z$$

$\langle i, j \rangle$: nearest-neighbor sites.

⇒ 2d T.C. ⟶ trivial transition $\overset{(2+1)d}{\simeq}$ Ising + Z_2 gauge field

1d: Ferro ⟶ trivial $\overset{(1+1)d}{\simeq}$ Ising + Z_2 gauge field

T. C. phase (ferro)

Gauge charge : trivial paramagnet.

G. S. D from Wilson loop $\prod_{e \in C_{x,y}} Z_e \sim e^{i \oint_{C_{x,y}} \vec{A} \, d\vec{s}} = \pm 1$.

Trivial phase:

Gauge charge. ferro.

Gauss' law : $\prod_v G_v^x = \mathbb{1}$ => unique g.s.

$|\uparrow\uparrow\uparrow\ldots\uparrow\rangle + |\downarrow\downarrow\downarrow\ldots\downarrow\rangle$

=> "Higgs phase"

What

$\sim \mathbb{Z} \quad \mathbb{Z}$

$\oint_{C_{xy}} \vec{A} \cdot d\vec{s} = \pm 1$

"Higgs phase"

What about m particle in Higgs phase

$h \to \infty$. $Z_e = 1$

$\sum_{\{S_i\}} |X_i = S_i\rangle$

m - created by $\prod_{C'} X$

Energy cost $\sim 2h \cdot |C'|$

$\Rightarrow 0$ (distance between m particles)

$\Rightarrow$ Condensing some particle (e) confines other particles with mutual braiding (m)

Ising: critical h_c. continuous transition

3d Ising (Wilson-Fisher). $\phi^2+\phi^4+(\partial\phi)^2$. ϕ is not gauge invariant.

ϕ is not a local operator.

ϕ^2 is fine

3d Ising/Z_2, "Ising*"