Title: Lecture - Quantum Matter, PHYS 777

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 30 - May 1 2026

Subject: Condensed Matter

Date: April 09, 2026 - 3:30 PM

URL: https://pirsa.org/26040016

Office hour: Tomorrow 1 pm.

$$H_{\text{Ising}} = -J\sum Z_i Z_{i+1} - g\sum X_i$$

$$H_{\text{Majorana}} = -J\sum i\gamma_{2i}\gamma_{2i+1} - g\sum i\gamma_{2i-1}\gamma_{2i}$$

Majorana. $J < g$. trivial.

$J > g$. Topological.

Open bd. ($g=0$) missing $-J\, i\gamma_{2L}\gamma_1$

To close the ring, put back $-J' i\gamma_{2L}\gamma_1$

$J'=J$: periodic b.c for γ

$J'=-J$: anti-periodic b.c. for γ.

G.S. : $i\gamma_{2i}\gamma_{2i+1}=1$, $i\gamma_{2L}\gamma_1 = \mathrm{sgn}(J')$

$i\neq L$

Unique G.S. but with different:

$$(-1)^F = \prod_{i=1}^{L} i\gamma_{2i-1}\gamma_i = -\mathrm{sgn}(J')$$

ssing $-J : \gamma_{2L}\gamma_1$

back $-J' : \gamma_{2L}\gamma_1$

for γ

b.c for γ.

$\gamma_1 = \mathrm{sgn}(J')$

with different

$\gamma_1 = -\mathrm{sgn}(J')$

Total fermion charge $(-1)^F$ "responds" to flux (b.c.)

"Topological response"

Imagine gradually change $J' : J \longrightarrow -J$

must have degeneracy somewhere in between.

"Bulk-boundary correspondence"

Back to Ising.

① B.C. for γ $\sim -X$. $= \eta$.

② $(-1)^F = -\eta = X$.

↑

G.S. likes it. ⇒ Remain at g.s. after changing b.c.

⇒ Each b.c. gives a g.s. ⇒ 2-fold g.s. d.

ssing $-J : \gamma_{2L}\gamma_1$

back $-J' : \gamma_{2L}\gamma_1$

for γ

b.c. for γ.

$\gamma_1 = \mathrm{sgn}(J')$

with different

$\gamma_i = -\mathrm{sgn}(J')$

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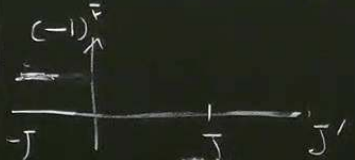
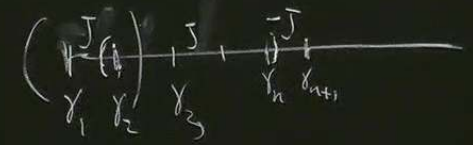


"Bulk-boundary correspondence"



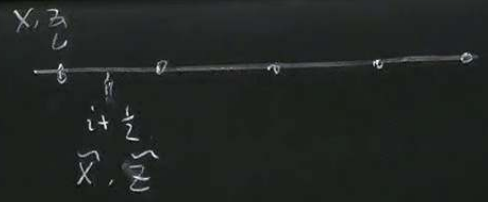
ramers-Wannier duality. (KW)

$$\tilde{X}_{i+\frac{1}{2}} = Z_i Z_{i+1}$$

$$\tilde{Z}_{i+\frac{1}{2}} = \prod_{0<j\le i} X_j$$

$$\tilde{X}_{i+\frac{1}{2}} \tilde{Z}_{j+\frac{1}{2}} = (-1)^{\delta_{ij}} \tilde{Z}_{j+\frac{1}{2}} \tilde{X}_{i+\frac{1}{2}}$$

$$H = -J \sum_i \tilde{X}_{i+\frac{1}{2}} - g \sum \tilde{Z}_{i-\frac{1}{2}} \tilde{Z}_{i+\frac{1}{2}}$$



$g \longleftrightarrow J.$

Paramagnet $\longleftrightarrow$ Ferromagnet

B. C. for $\tilde{Z}$: $\tilde{Z}_{L+i+\frac{1}{2}} = \left(\prod_{0<j\le L} X_j\right) \tilde{Z}_{i+\frac{1}{2}} = X \tilde{Z}_{i+\frac{1}{2}}$. dynamical b. c. $= X$.

$$\prod_i \tilde{X}_{i+\frac{1}{2}} = 1$$

Interpretation: start from Ferro. $|G.S.\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\uparrow\rangle$

$\tilde{Z}_i$ $\tilde{Z}_j$

$\rightarrow |\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$

$\tilde{Z}$: domain-wall creation

$\prod \hat{X}$: domain wall number mod 2.

KW: particle $\leftarrow$ domain wall

$\uparrow$

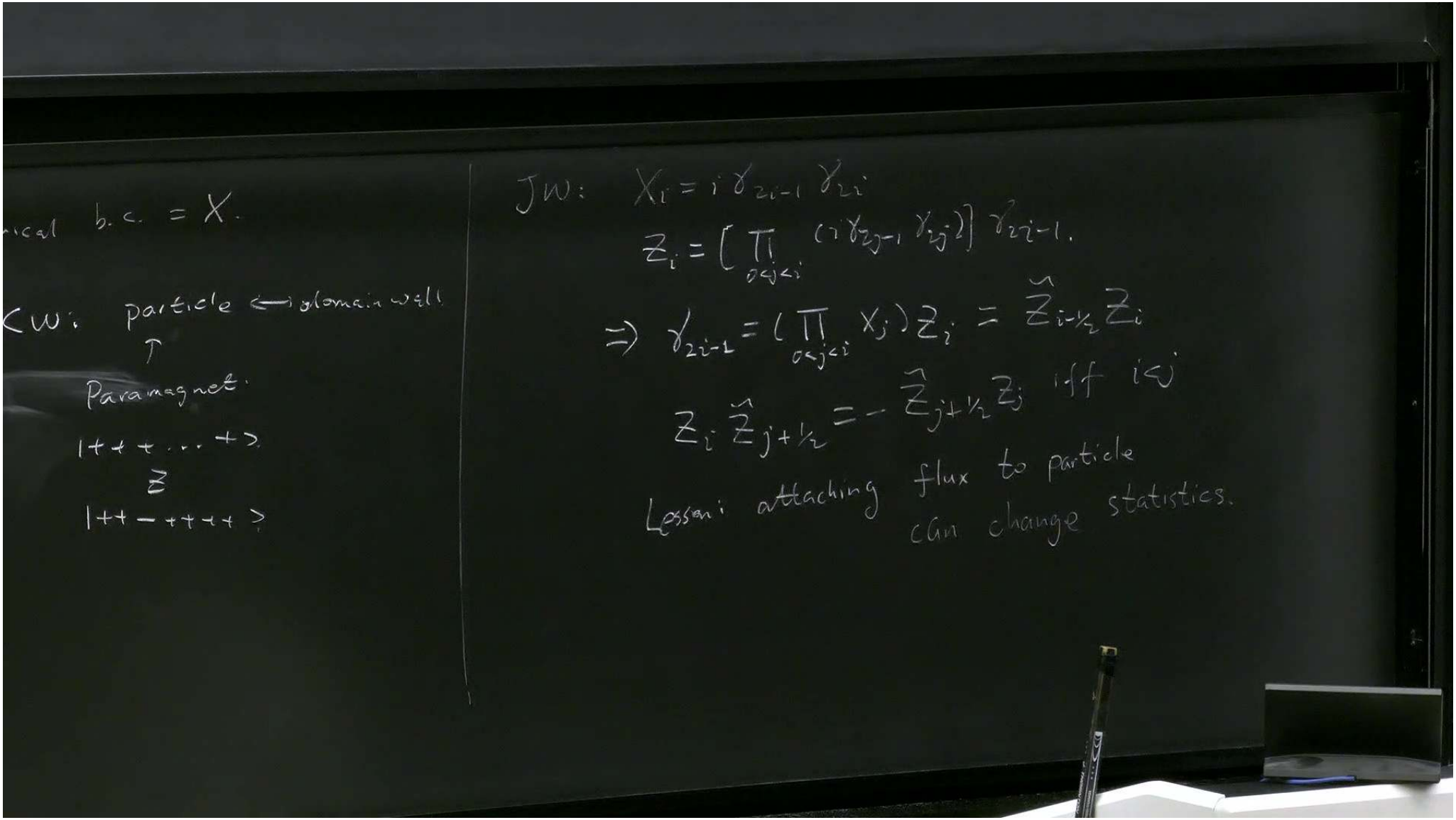
Paramagnet.

$|+++\cdots+\rangle$

$\tilde{Z}$

$|++-++++\rangle$

JW


ical b.c. = X.
KW: particle ⟷ domain wall
↑
Paramagnet.
|+ + + ... +>
↕
|+ + − + + + +>
JW: $X_i = i\gamma_{2i-1}\gamma_{2i}$
$Z_i = \left[\prod_{0\le j<i} (i\gamma_{2j-1}\gamma_{2j})\right]\gamma_{2i-1}$.
$\Rightarrow \gamma_{2i-1} = \left(\prod_{0\le j<i} X_j\right) Z_i = \tilde{Z}_{i-1/2} Z_i$
$Z_i \tilde{Z}_{j+1/2} = -\tilde{Z}_{j+1/2} Z_j$ iff $i<j$
Lesson: attaching flux to particle
can change statistics.

Paramagnet $g > J$ Ferromagnet for $\tilde{Z}$.

$$|\tilde{\Omega}_\pm\rangle = |\uparrow\uparrow\uparrow\ldots\rangle \pm |\downarrow\downarrow\downarrow\ldots\rangle$$

$\Pi \tilde{X} = 1 \longrightarrow |\tilde{\Omega}_+\rangle$ unique g.s.

Anti-pbc for $\tilde{Z}$ $\Rightarrow$ Excitation energy $\sim g$.

$\Rightarrow$ Gauge d.o.f. gapped.

$\Rightarrow$ Baby version of Higgs mechanism

Ferromagnet

$\Rightarrow$

net. $|\Omega\rangle = |++++\ldots\rangle$.

$\Rightarrow$ Switching b.c. gives a different g.s. $\Rightarrow$ 2-fold g.s.d

$\Rightarrow$ A "free" $\mathbb{Z}_2$ gauge theory $\Longleftrightarrow$ A $\mathbb{Z}_2$ Ising ferromagnet.

Some intuition in terms of wavefunction.

1. Original spin. start from paramagnet

$J=0$. $|\Omega\rangle = |+++\ldots+\rangle$.

$J \nearrow$ $|\Omega\rangle$ have components of some particles
$|++-++-+\ldots\rangle$

$J>g$ $|\Omega\rangle$: Massive superposition of particles

$g=0$ $|\Omega\rangle = |\Upsilon\Upsilon\Upsilon\ldots\Upsilon\rangle = \sum_{\{S_i=\pm 1\}} \otimes |X_i = S_i\rangle$. $\Rightarrow$ Z charge particle

$|\Upsilon\rangle = |+\rangle + |-\rangle$

$\langle Z\rangle \neq 0$ ↑ condense".

2. Now

Ferromagnet. $g=0$. slightly breaks $\mathbb{Z}_2$ sym.

$$\delta H = -h \sum_i Z_i \quad , \quad 0 < h \ll J$$

$|\Omega\rangle = |\uparrow\uparrow\uparrow \cdots \uparrow\uparrow\uparrow\rangle$

Force in two domain walls.

$|\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\cdots\uparrow\rangle$

i j

Energy cost depends on $|i-j|$.

$\Delta E \sim h|i-j|$

If $J \to \infty$, keeps growing.

For $h \ll J < \infty$, growth stops when $h|i-j| \sim O(J)$

Energy cost depends on $|i-j|$.

$\Delta E \sim h|i-j|$

If $J \to \infty$, keeps growing.

For $h << J < \infty$, growth stops when $h|i-j| \sim O(J)$ } Confinement
what real quarks do.
↳ SU(N) YM in (3+1)d.

Domain wall = charged particle in Z_2 gauge theory.
gauge.

Instanton ⇒ Unique vacuum
Confinement of gauge charge.