Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 30 - May 1 2026

Subject: Condensed Matter

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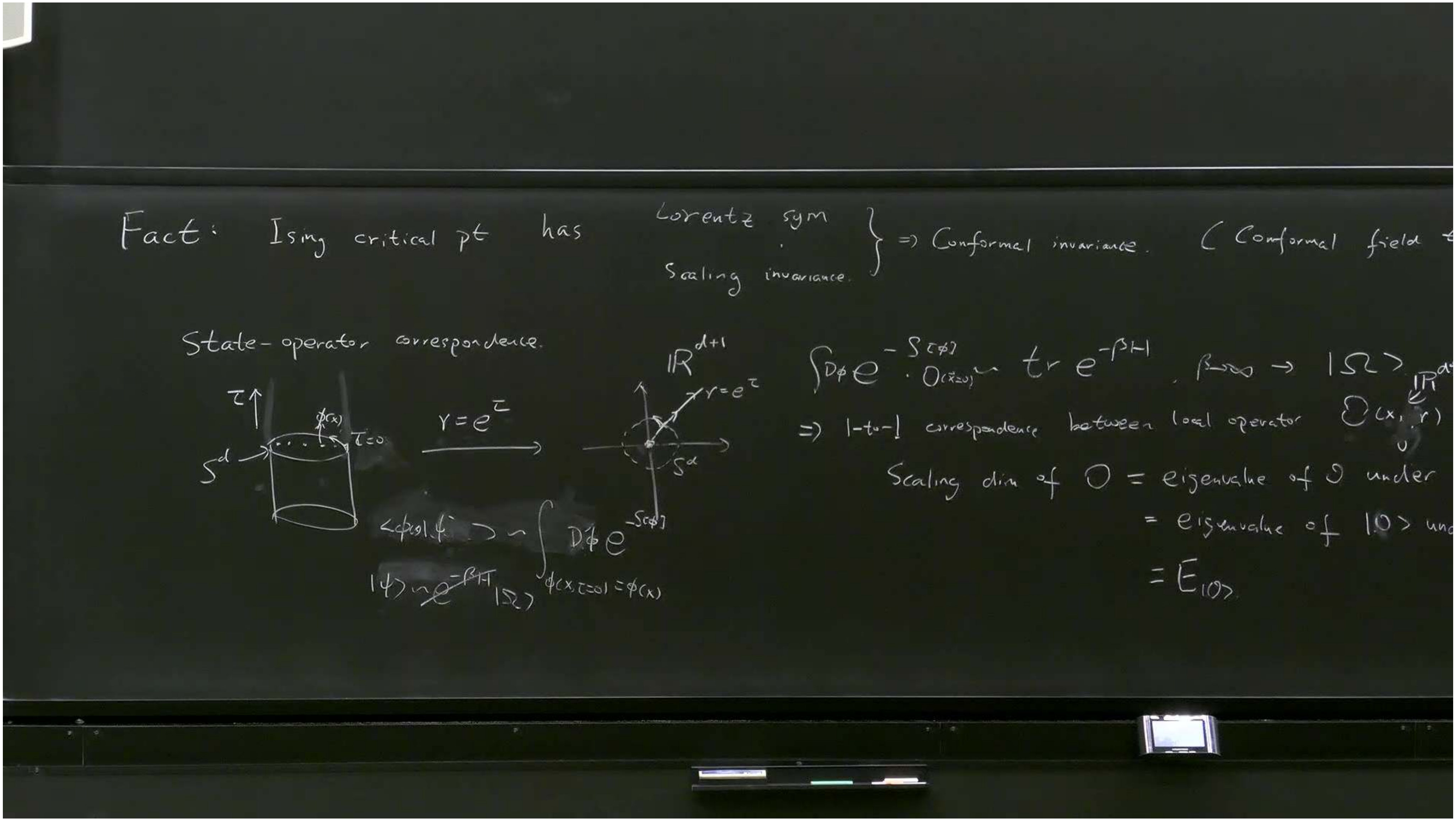

Fact: Ising critical pt has Lorentz sym
Scaling invariance
=> Conformal invariance. (Conformal field
State-operator correspondence.
τ
φ(x)
τ=0
S^d
r = e^τ
IR^{d+1}
r = e^τ
S^d
∫Dφ e^{-S[φ]} · O(x=0) ~ tr e^{-βH}, β→∞ → |Ω⟩
=> 1-to-1 correspondence between local operator O(x, r)
Scaling dim of O = eigenvalue of D under
= eigenvalue of |O⟩ und
= E_{|O⟩}
⟨φ(x)φ ⟩ ~ ∫ Dφ e^{-S[φ]}
φ(x, τ=0) = φ(x)
|ψ⟩ ~ e^{-βH} |Ω⟩

e. (Conformal field theory)

H

$\beta \to \infty \to |\Omega\rangle$ $\mathbb{R}^{d+1}$

en local operator $\mathcal{O}(x, r) \longleftrightarrow |\mathcal{O}\rangle$ on S^d

Δ = eigenvalue of $\mathcal{O}$ under $r \to e^b r$: $\tau \to \tau + b$

= eigenvalue of $|\mathcal{O}\rangle$ under time-translation

$= E_{|\mathcal{O}\rangle}$

Ising S^1 ~ Free massless Majorana / $\mathbb{Z}_2$

$$E_{AP} \sim -\frac{1}{24} \cdot \frac{2\pi}{L}$$

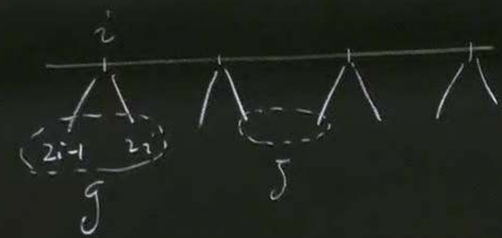
$$E_P \sim \frac{1}{12} \cdot \frac{2\pi}{L} + \mathcal{O}\left(\frac{1}{L^2}\right)$$

$$\Delta E = \frac{1}{8} \cdot \frac{2\pi}{L} \underset{\text{unit sphere}}{=} \frac{1}{8} = \Delta_{Z_i}$$

$$\langle Z_i Z_j \rangle \sim \frac{1}{|i-j|^{1/4}} + \frac{1}{|i-j|^{\#}}$$

$$Z_i \sim \underset{\phi}{a_1 \mathcal{O}_1} + \underset{\phi^3}{a_2 \mathcal{O}_2} + \underset{\phi \partial^2 \phi}{a_3 \mathcal{O}_3} + \ldots$$

Kitaev chain. $(H_{Ising} = -J\sum Z_i Z_{i+1} - g\sum X_i)$



Go back to "free fermion" theory.

$$H = -J\sum_i i\gamma_{2i}\gamma_{2i+1} - \underline{g\sum i\gamma_{2i-1}\gamma_{2i}}.$$

$J > g$ vs. $J < g$ what's difference?

① $J = 0$. g.s. $i\gamma_{2i-1}\gamma_{2i}|\Omega\rangle = |\Omega\rangle$. unique.

② $g = 0$.

$g=0$. g.s. $i\gamma_{2i}\gamma_{2i+1}=1$ p.b.c.

Try open boundary, $-J\, i\gamma_{2L}\gamma_1$ missing.

2-dim $\mathcal{H}$, label using
fermion parity $i\gamma_1\gamma_{2L} \sim 2c^\dagger c-1$.

H doesn't involve γ_{2L}, γ_1, $[\gamma_{2L,1}, H]=0$

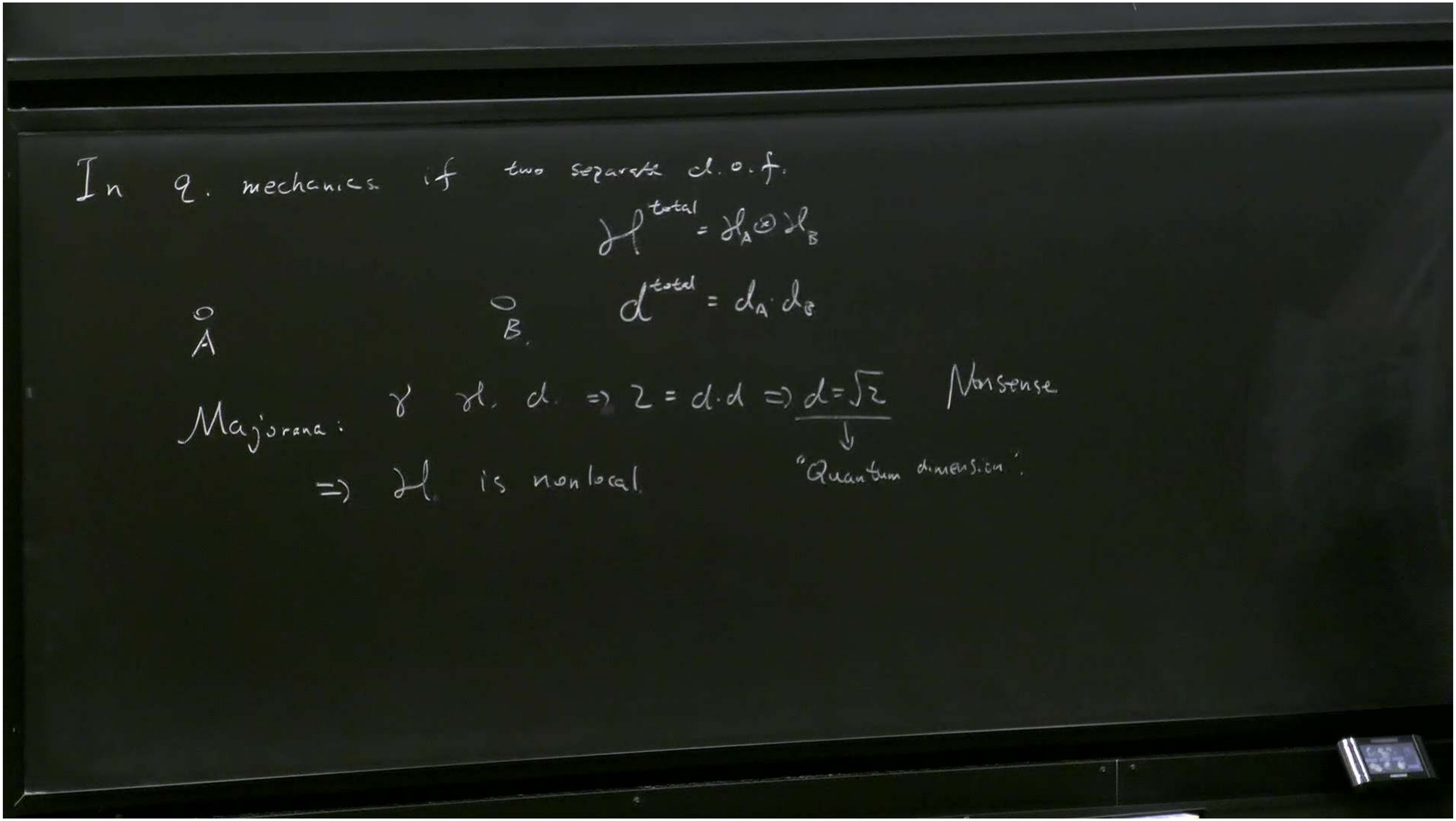
$|\psi\rangle$ vs. $\gamma_1|\psi\rangle$ degenerate.

G.S. at least 2-fold degenerate.

$\gamma_{2L}^2=\gamma_1^2=1$, $\gamma_{2L}\gamma_1=-\gamma_1\gamma_{2L}$

Complex basis $c=\frac{\gamma_1+i\gamma_{2L}}{2}$, $c^\dagger=\frac{\gamma_1-i\gamma_{2L}}{2}$, $c|0\rangle=0$

$c^\dagger|0\rangle$


In q. mechanics if two separate d.o.f.
$\mathcal{H}^{total} = \mathcal{H}_A \otimes \mathcal{H}_B$
$d^{total} = d_A \cdot d_B$
A
B.
Majorana: γ $\mathcal{H}.$ $d.$ $\Rightarrow 2 = d \cdot d \Rightarrow d = \sqrt{2}$
Nonsense
"Quantum dimension".
$\Rightarrow \mathcal{H}$ is nonlocal

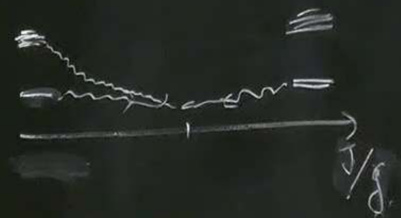
Nonsense

...tum dimension".

Perturbing $\delta H = \lambda \sum_{\gamma\gamma\gamma\gamma} \delta H_i \xrightarrow{?} i\gamma_1 \gamma_{2L} \cdot \delta \sim \lambda^{O(L)}$

$n \sim O(L)$-th order in perturbation.

δH^n

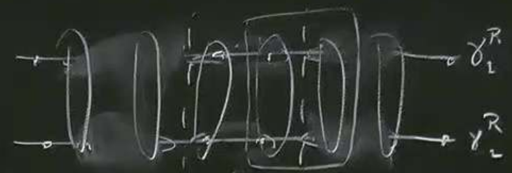


$g > J$ vs. $g < J$ belong to different phases.

$g > J$ "trivial phase".

$g < J$: Kitaev chain / Majorana chain $\leftarrow$ "Topological phase".

What if 2x Kitaev chain



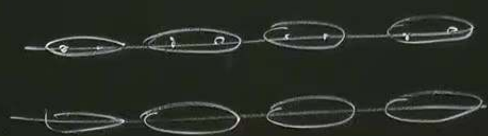
$\delta H^R = i\gamma_1^R\gamma_2^R \Rightarrow$ No robust g.s. degeneracy

Kitaev [redacted] ∪ Kitaeve = Trivial

$\|$

$(\text{Kitaeve})^{-1}$

$\gamma_1, \gamma_2, \gamma_3$



"Invertible phase": $|\psi\rangle \otimes |\tilde{\psi}\rangle \sim |\text{trivial}\rangle$

Invertible topological phase