Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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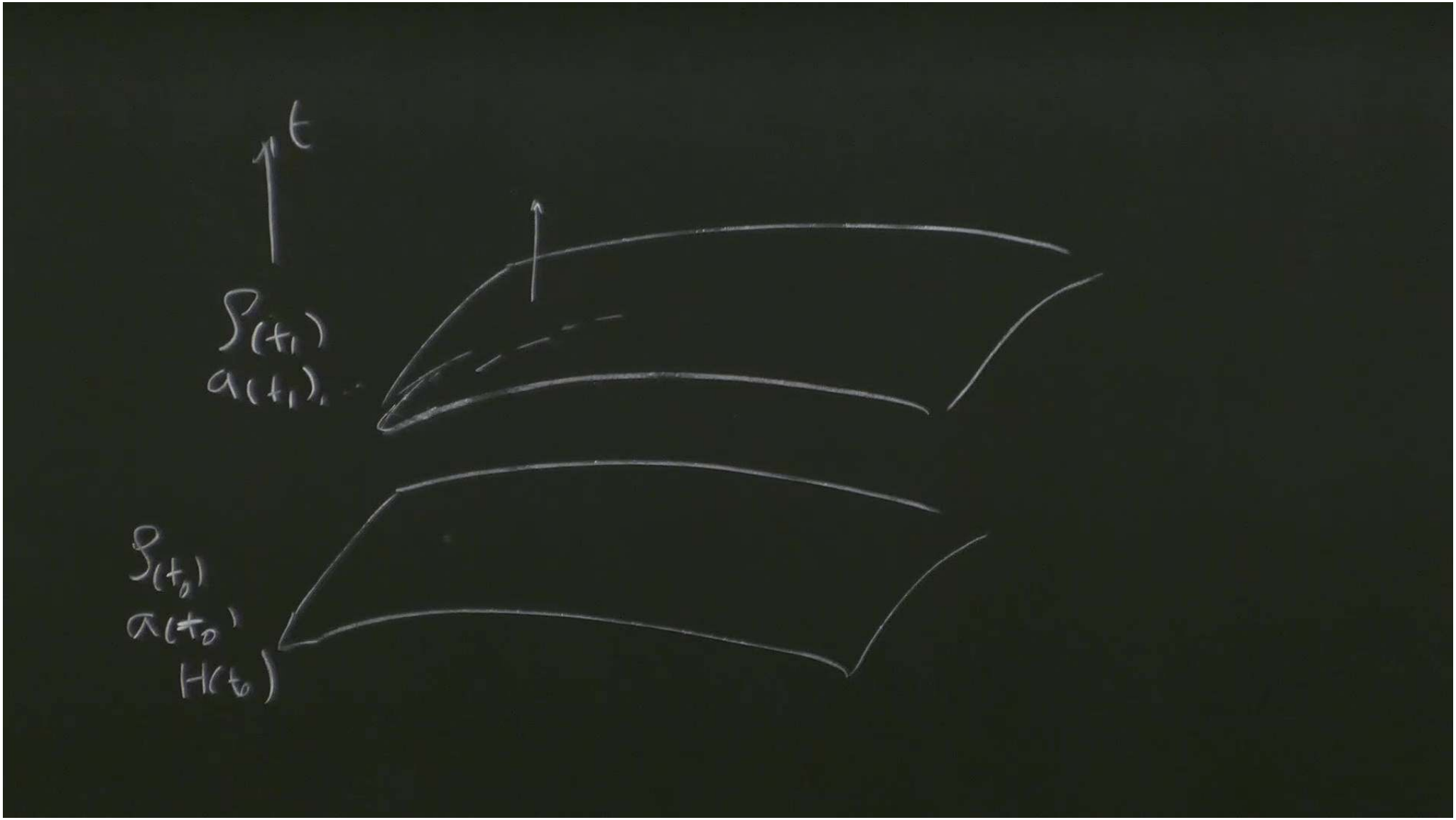
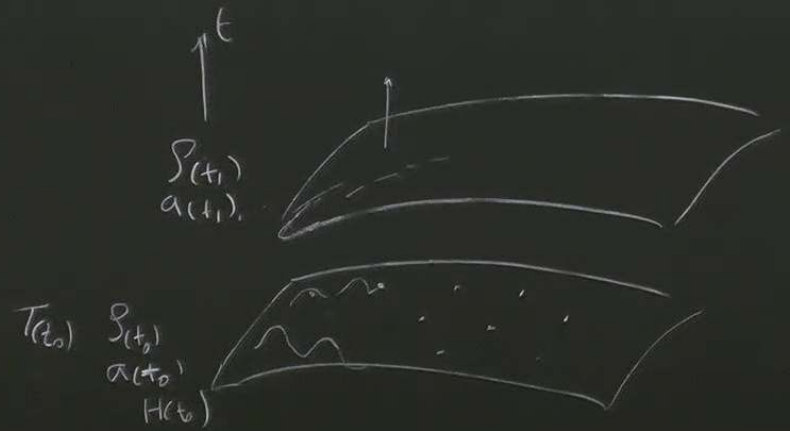

t
ρ(t1)
a(t1)
ρ(t0)
a(t0)
H(t0)

$\delta g_{\mu\nu}(\eta, x)$

$T_{\mu\nu}(\eta, x)$



$x^i dx^j - (\delta_{ij} + h_{ij})\, dx^i dx^j \Big]$

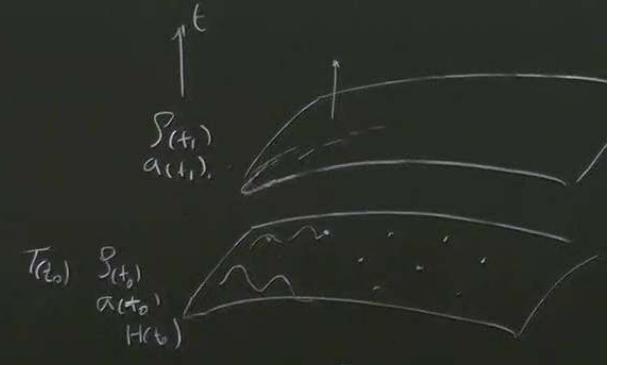
$h_{ij} = -2\phi\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}E + 2\partial_{(i}\hat{h}_{j)} + 2\hat{h}_{ij}$

$\partial^i = 0$ Scalar Scalar Vector $\partial^i \hat{h}_i = 0$ $\partial^i \hat{h}_{ij} = \hat{h}^i{}_i = 0$

Relativistic Perturbation Theory

$$g_{\mu\nu}(\eta, x) = \bar{g}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta, x)$$

$$T_{\mu\nu}(\eta, x) = \bar{T}_{\mu\nu}(\eta) + \delta T_{\mu\nu}(\eta, x)$$



$$ds^2 = a^2(\eta)\left[(1+2\underbrace{\Psi}_{\text{scalar}})\,d\eta^2 - 2B_i\,dx^i d\eta - (\delta_{ij} + h_{ij})\,dx^i dx^j\right]$$

$$B_i \equiv \partial_i B + \hat{B}_i$$

Scalar Vector $\partial^i \hat{B}_i = 0$

$$h_{ij} \equiv -2\phi\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}E$$

Scalar

$\partial_{\langle i}\partial_{j\rangle} E \equiv \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right) E$

$\partial_{(i}\hat{h}_{j)} \equiv \frac{1}{2}\left(\partial_i \hat{h}_j + \partial_j \hat{h}_i\right)$

Scalars : 4 , χ, Φ, E, B

vectors : 4

tensors : 2

10

$\partial^i \hat{h}_{ij} = \partial^i \hat{h}_{ji} = 0$

$T^0_{\ 0} = \bar{\rho} + \delta\rho = \bar{\rho}(1+\delta)$

$T^i_{\ 0} = (\bar{\rho} + \bar{p})\, v^i \equiv q^i$

$T^i_{\ j} = -(\bar{p} + \delta p)\delta^i_{\ j} - \Pi^i_{\ j}$

Anisotropic stress ≈ 0

$T^0_{\ 0} = \bar{\rho}_\varphi + \delta\rho_\varphi$

1 scalar

vector $\partial h_1 = 0$

ntinuity Equation: $\delta\rho' = -3\mathcal{H}((\delta\rho + \delta P) + 3\phi'(\bar{\rho} + \bar{P}) - \nabla\cdot q$

$\mathcal{H} \to \frac{a'}{a}$

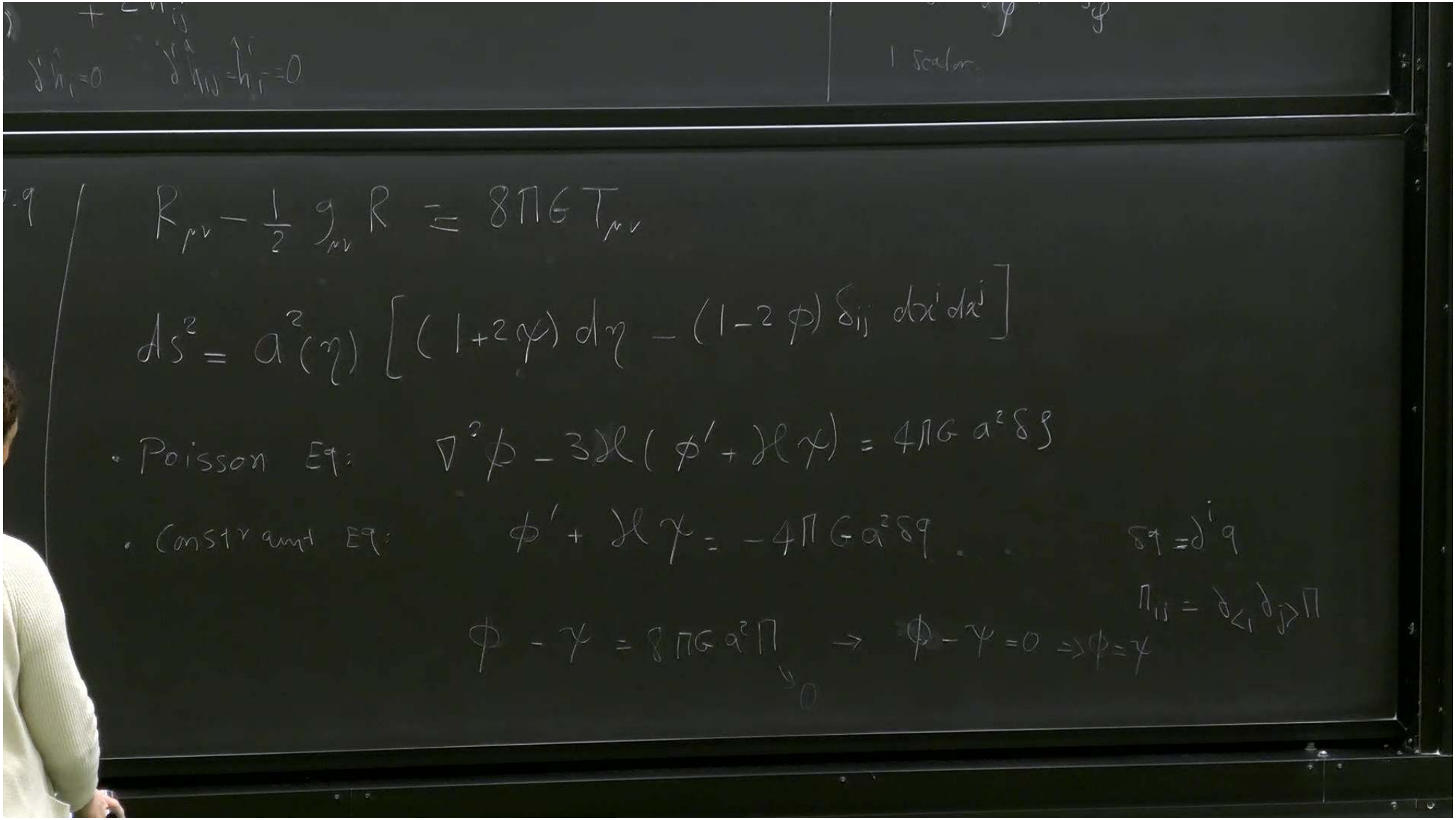
latter: $\delta_m' = -\nabla\cdot v_m + 3\phi'$

radiation $\delta_r' = -\frac{4}{3}\nabla\cdot v_r + 4\phi'$

$\tilde{a}^2 = a^2(1-2\phi_{(1,A)})$

$2\tilde{a}'\tilde{a} = 2a'a'(1-2\phi) - 2a^2\phi'$

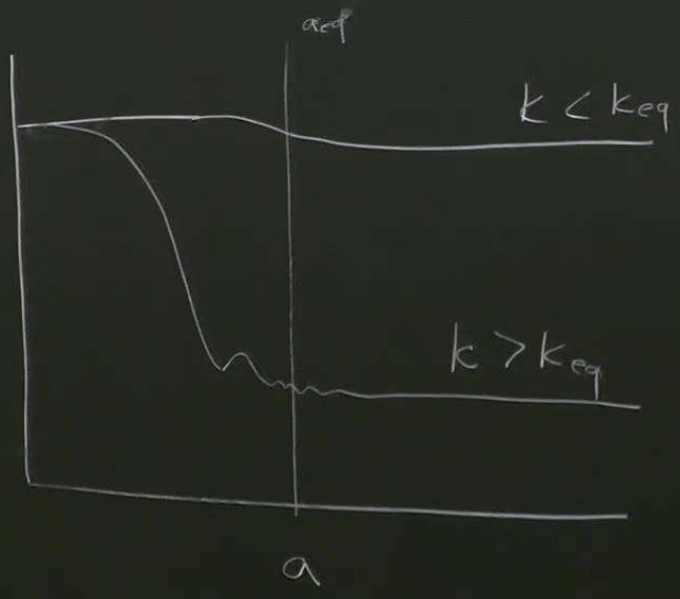
$\tilde{\mathcal{H}} = \frac{2a'(1-2\phi)}{a^2(1-2\phi)} - \frac{a^2\phi'}{a^2} = \mathcal{H} - \phi'$


1 scalar
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$
$ds^2 = a^2(\eta)\left[(1+2\psi)\,d\eta - (1-2\phi)\,\delta_{ij}\,dx^i dx^j\right]$
• Poisson Eq: $\nabla^2\phi - 3\mathcal{H}(\phi' + \mathcal{H}\psi) = 4\pi G a^2 \delta\rho$
• Constraint Eq: $\phi' + \mathcal{H}\psi = -4\pi G a^2 \delta q$
$\delta q = \partial^i q$
$\Pi_{ij} = \partial_{<i}\partial_{j>}\Pi$
$\phi - \psi = 8\pi G a^2 \Pi \to \phi - \psi = 0 \Rightarrow \phi = \psi$
$\Pi = 0$

$\phi' + (2\mathcal{H}' + \mathcal{H}^2)\phi = 4\pi G a^2 \delta P$

$\frac{1}{3} k^2 \phi$

$k^{3/2}\phi$

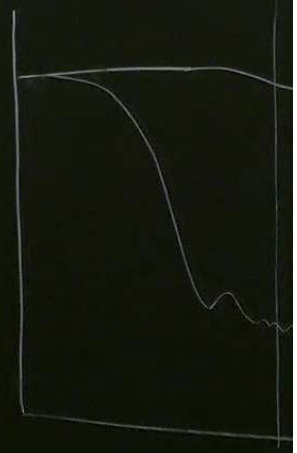


$k \gg R_H \sim \frac{1}{\mathcal{H}} \Rightarrow \phi = A \frac{\cos(\frac{1}{\sqrt{3}} k\eta)}{(k\eta)^2}$

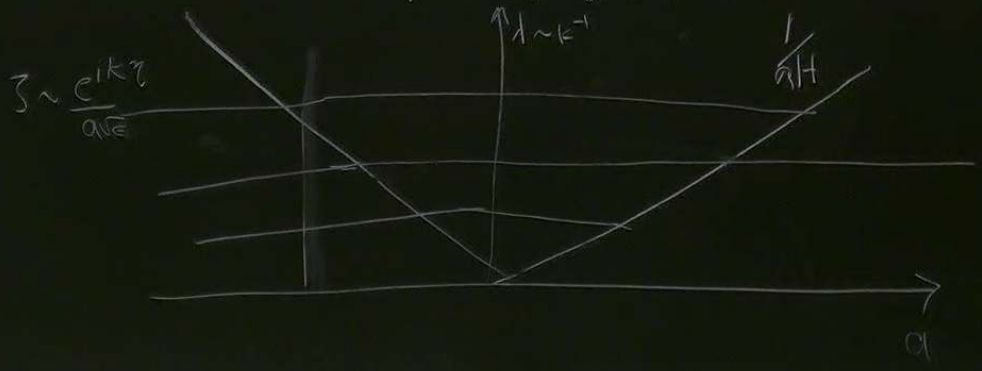
• Evolution: $\phi'' + 3\mathcal{H}\phi' + (2\mathcal{H}' + \mathcal{H}^2)\phi = 4\pi G a^2$

Radiation

$$\phi'' + 4\mathcal{H}\phi' = -\frac{1}{3}k^2\phi$$



Matter $\phi'' + 3\mathcal{H}\phi' = 0$



$k \gg R_H \sim \frac{1}{\mathcal{H}}$

$$-\zeta = R = -\phi - \frac{\phi' + \mathcal{H}\phi}{\mathcal{H}\epsilon}$$

$R \longrightarrow \phi \longleftrightarrow \delta_m, \delta_r$

inflation — CMB + LSS

$$v = M_p Z(\eta) R \qquad \sqrt{2}\, a\epsilon$$

$$v_k'' - \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

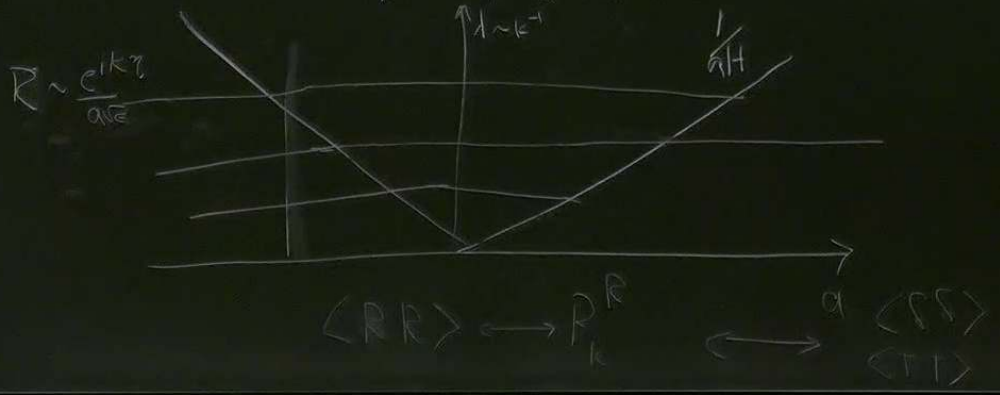
$v_k \sim z \qquad R \sim \frac{v}{z} \sim$ constant

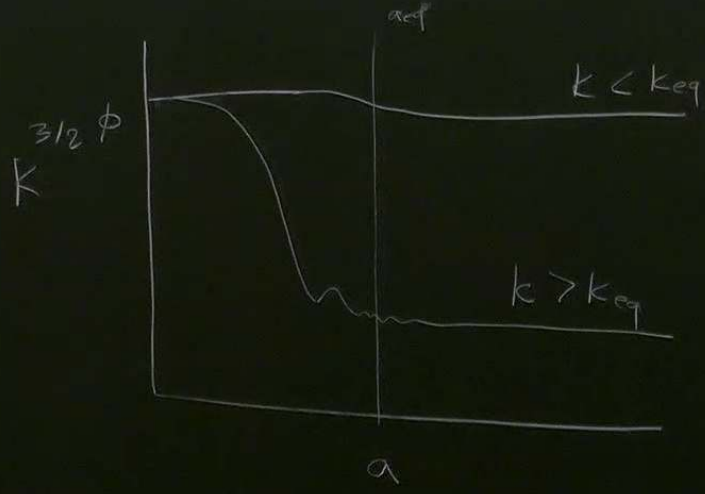
$$\frac{\rho(\frac{1}{\sqrt{3}} k\eta)}{(k\eta)^2}$$

• Evolution: $\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = 4\pi G a^2 \delta P$

Radiation $\Phi'' + 4\mathcal{H}\Phi' = -\frac{1}{3}k^2\Phi$

Matter $\Phi'' + 3\mathcal{H}\Phi' = 0$





$k \gg R_H \sim \frac{1}{\mathcal{H}} \Rightarrow \Phi = A$