Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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Beyond Equilibrium

$n_i a^3 \sim \text{Const.}$ $\quad \frac{dn_i}{dt} + 3\frac{\dot{a}}{a} n_i = 0 \quad \Longleftrightarrow \quad n_i \propto a^{-3}$

$1 + 2 \longleftrightarrow 3 + 4$ $\quad \frac{1}{a^3}\frac{d\, a^3 n_i}{dt} = C_i[\{n_i\}]$ Boltzman Equation

$\frac{d}{a^3 dt}(a^3 n_1) = -\alpha n_1 n_2 + \beta n_3 n_4$ $\quad \xrightarrow{\text{in Equil.}} \quad \beta = \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} \alpha$

$\Gamma_1 = n_1$

RHS = 0

$\Gamma_1 = \alpha n_2 = \langle \sigma v \rangle n_2$

PP

$$\frac{1}{a^3}\frac{d\,a^3 n_1}{dt} = -\langle\sigma v\rangle\left[n_1 n_2 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} n_3 n_4\right]$$

$$N_i \equiv \frac{n_i}{s} \propto n_i a^3$$

$$\frac{1}{N_1} \times \frac{1}{H} \qquad (H dt = d\ln a)$$

$$\frac{dN_1}{N_1 d\ln a} = \frac{d\ln N_1}{d\ln a} = -\frac{\langle\sigma v\rangle n_2}{H}\left[a^3\frac{n_1}{N_1} - \left(\frac{N_1 N_2}{N_3 N_4}\right)_{eq}\frac{n_3 n_4 a^3}{n_2 N_1}\right]$$

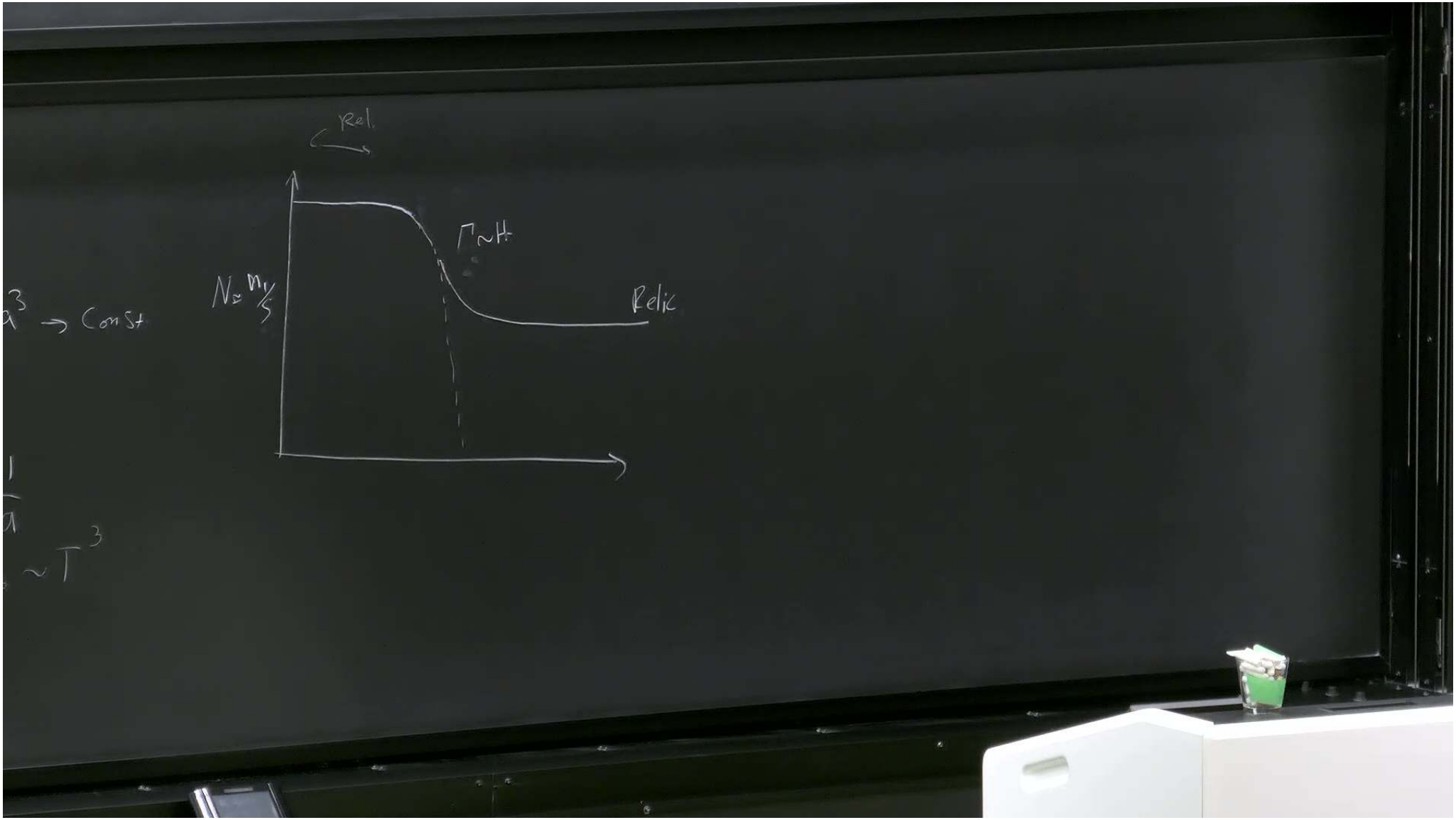
$$= -\frac{\Gamma_1}{H}\left[1 - \left(\frac{N_1 N_2}{N_3 N_4}\right)_{eq}\frac{N_3 N_4}{N_2 N_1}\right]$$

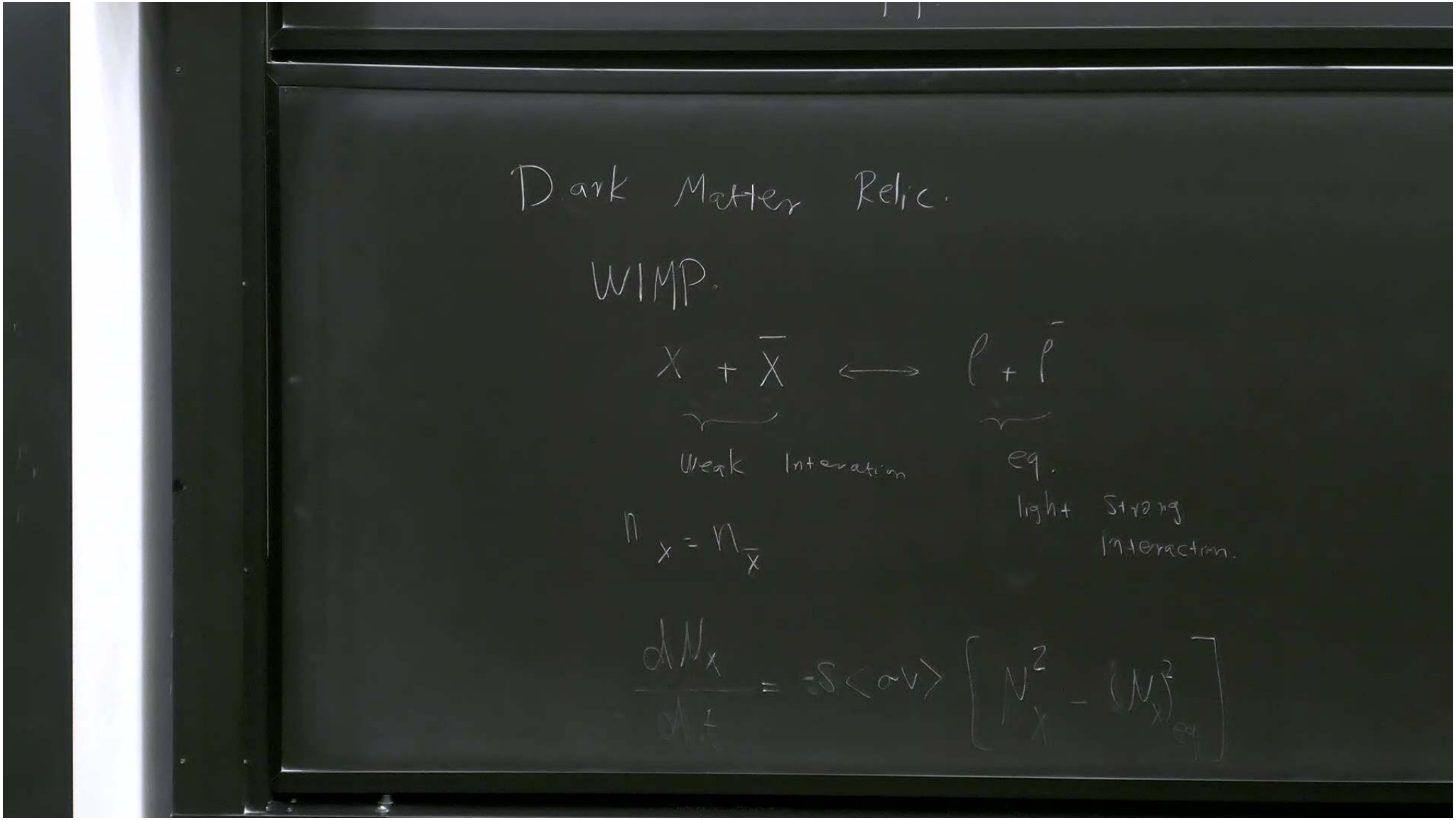
$\Gamma_1 \gg H$: System $\longrightarrow$ equil. quickly

$\Gamma_1 \lesssim H$: Species 1 $\longrightarrow$ freezes out $N_1 = \frac{n_1}{s} \propto n_1 a^3 \to$ Const

$N_1 \gg N_1^{eq}$, $N_i = N_i^{eq}$ $i \in \{2,3,4\}$

$$\frac{d \ln N_1}{d \ln a} = -\frac{\Gamma}{H}\left[1 - \frac{N_1^{eq}}{N_1}\right] \Rightarrow N_1 \downarrow$$


Rel.
$\Gamma \sim H$
$N \simeq n_\gamma$
Relic
$\rightarrow$ Const
$\sim T^3$


Dark Matter Relic.
WIMP.
$X + \bar{X} \longleftrightarrow l + \bar{l}$
Weak Interaction
eq.
light Strong
interaction.
$n_X = n_{\bar{X}}$
$\frac{dN_X}{dt} = -S\langle \sigma v \rangle \left[N_X^2 - (N_X)_{eq}^2 \right]$

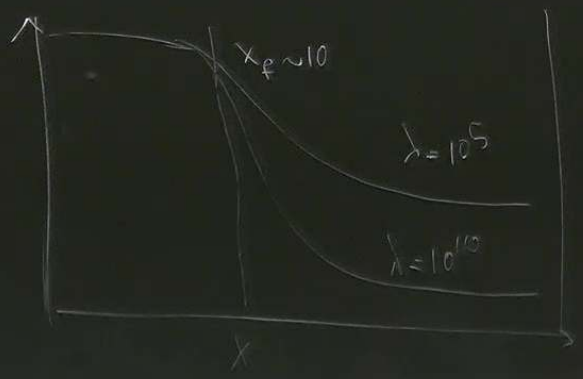
$$= -\frac{1}{H}\left[1 - \left(\frac{N_3 N_\chi}{N_3 N_\chi}\right)_{eq}\right]$$

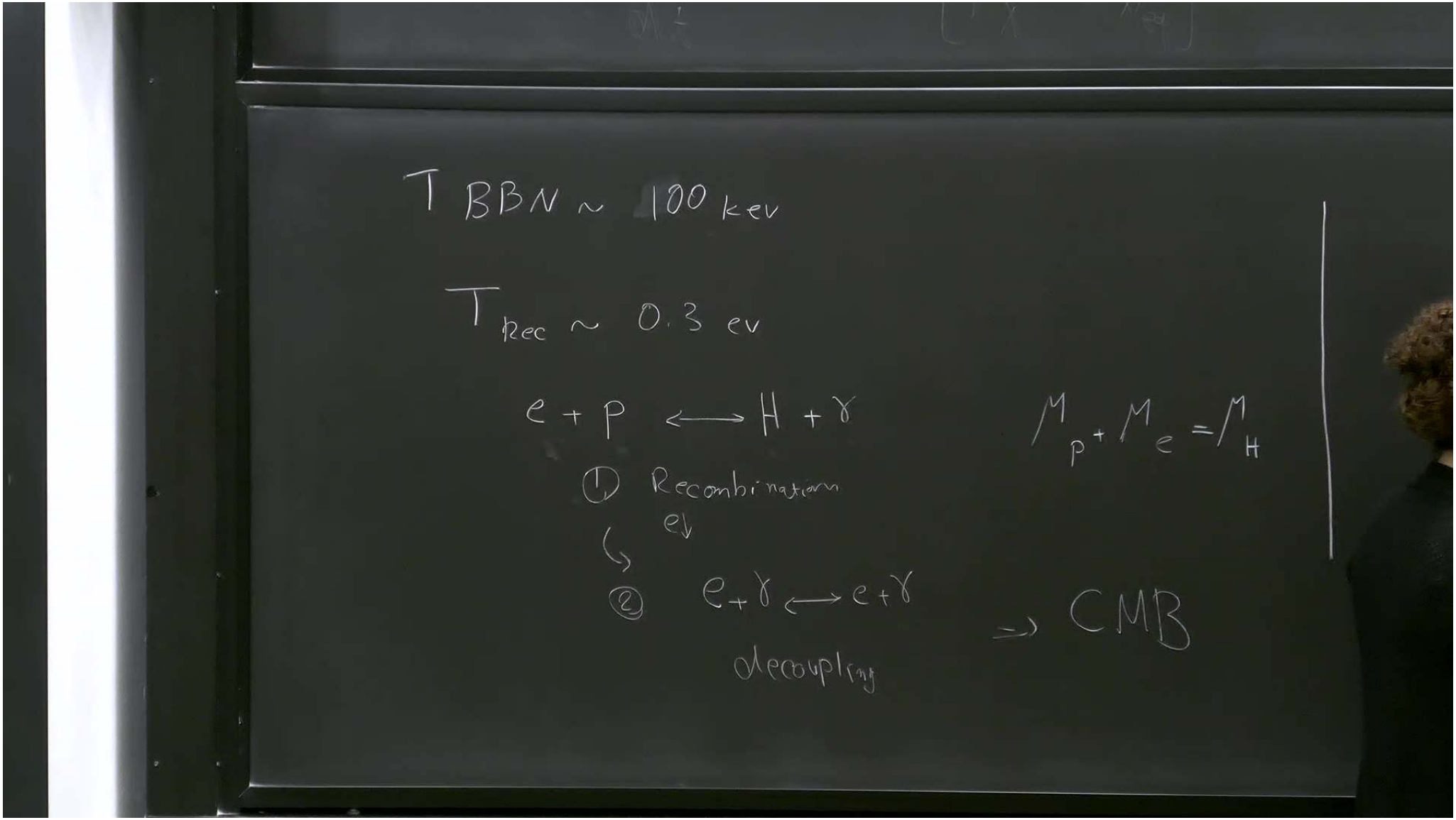
$$x = \frac{M_\chi}{T} \quad \Rightarrow \quad \frac{dN_\chi}{dx} = \frac{-\lambda}{x^2}\left[N_\chi^2 - (N_\chi^{eq})^2\right] \qquad \text{Ricatte eq.}$$

$$\lambda = (M_\chi, \langle\sigma v\rangle, H, T_0)$$

WIMP Miracle

obs. DM $\Rightarrow$ $\sqrt{\langle\sigma v\rangle} \sim 10^{-4}\,\text{GeV} \sim 0.1\sqrt{G_F}$




T BBN ~ 100 kev
T Rec ~ 0.3 ev
e + p ⟷ H + γ
M p+ M e = M H
① Recombination
ev
② e + γ ⟷ e + γ
decoupling
→ CMB

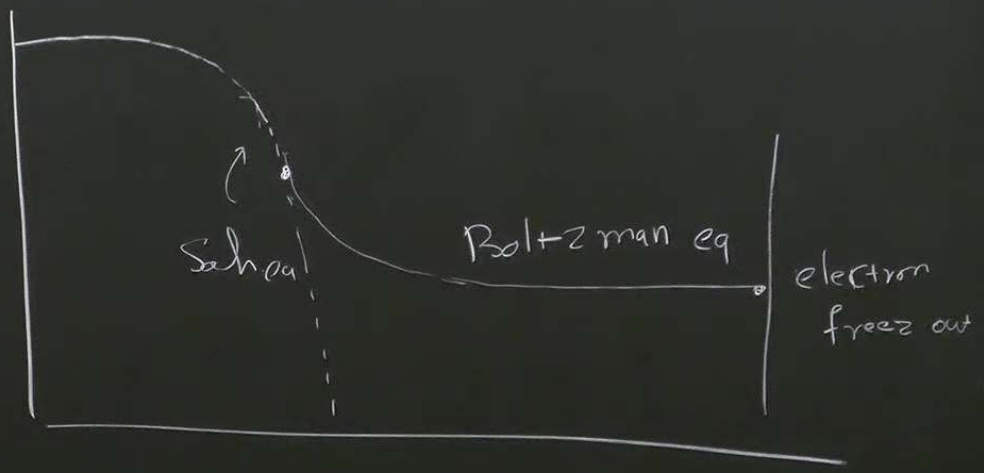
$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}}$$

$$\left(\frac{n_H}{n_e n_p}\right) = \frac{g_H}{g_e g_p} \left(\frac{m_H}{m_e m_p} \frac{2\pi}{T}\right)^{3/2} e^{\frac{m_p + m_e - m_H}{T}}$$

$$B_H = m_p + m_e - m_H = 13.6\ \text{eV}$$

$$X_e = \frac{n_e}{n_b} \rightarrow \text{Saha eq:} \quad \left(\frac{1 - X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2} \eta_b \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_E/T}$$

$$e^{\frac{m_p + m_e - m_H}{T}}$$



$$\left(\frac{1 - X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2}\eta_b\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_E/T}$$

$T_{BBN} \sim 100\ \text{kev}$

$T_{rec} \sim 0.3\ \text{ev}$, $z \simeq 1320 \ll z_{eq}$

$e + p \longleftrightarrow H + \gamma$ $\qquad M_p + M_e = M_H$

① Recombination ev

↳

② $e + \gamma \longleftrightarrow e + \gamma$ $\Rightarrow$ CMB

decoupling

$T_{dec} \simeq 0.27\ \text{ev}$ $\quad z \simeq 1100$

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2}$$

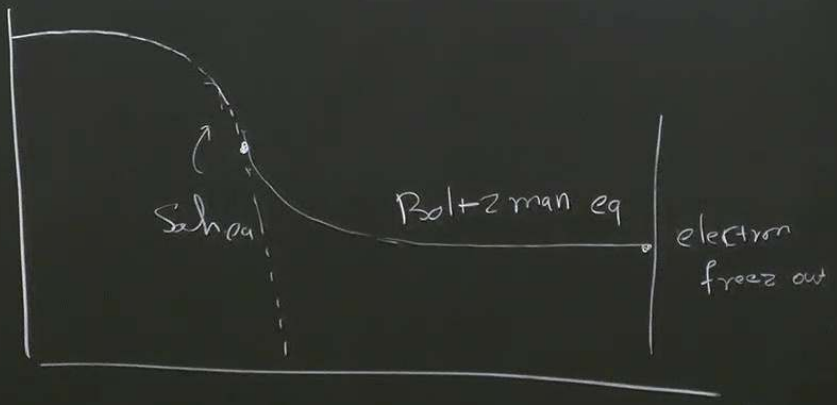
$$\left(\frac{n_H}{n_e n_p}\right) = \frac{g_H}{g_e g_p} \left($$

$B_H = m_p + m_e$

$$X_e = \frac{n_e}{n_b}$$

$$e^{\frac{m_p + m_e - m_t}{T}}$$

$\frac{n_e}{n_b}$



$$n_e^{\infty} \approx \eta^{-3} n_b$$

$$\left(\frac{1 - X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2} \eta_b \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_E/T}$$