Title: Lecture - Cosmology, PHYS 621

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Collection/Series: Cosmology (Elective), PHYS 621, February 23 - March 27, 2026

Subject: Cosmology

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URL: https://pirsa.org/26030035

Thermal Equilibrium:

Relativistic Limit:

$$n_i = \frac{\zeta(3)}{\pi^2} g_i T^3 \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions} \end{cases}$$

$$\rho_i = g \int d^3p \, E(p) f(p) = \frac{\pi^2}{30} g T^4 \begin{cases} 1 & \text{bosons} \\ 7/8 & \text{fermions} \end{cases}$$

$$P_i = 1/3 \, \rho_i \quad (\text{"radiation"}) \qquad w = 1/3$$

CMB, $T_0 = 2.73\,K \Rightarrow \Omega_\gamma h^2 \simeq 2.5 \times 10^{-5}$

Non-Relativistic Limit $\left(x = m/T \gg 1 \right)$

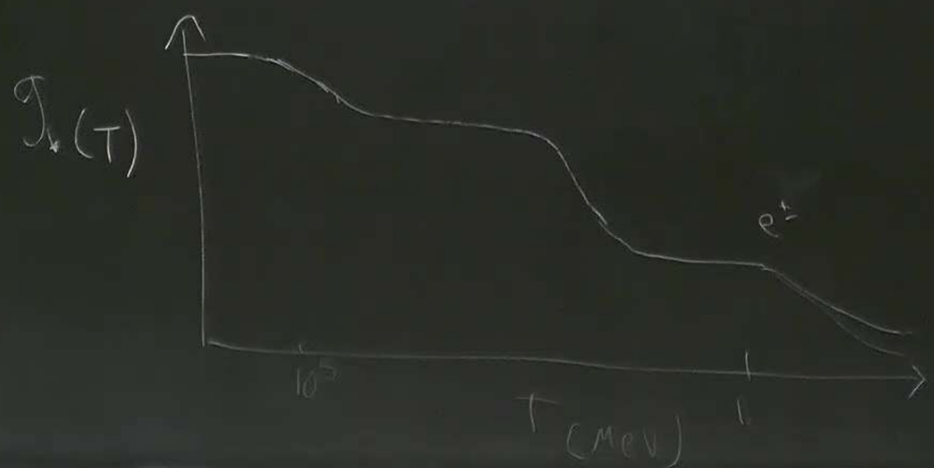
$$n_i = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

$$E(p) \simeq m \quad \rightarrow \quad \rho_i \simeq n\,m$$

$$\rightarrow \quad P_i = nT \ll \rho = mn \quad \Rightarrow \quad \boxed{w \simeq 0} \quad \text{Matter}$$

$$\rho_r = \sum_i{}' \rho_i \equiv \frac{\pi^2}{30} g_*(T)\, T^4$$

$$g_*(T) = \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=f} g_i \left(\frac{T_i}{T}\right)^4$$



$\rightarrow P_i = nT \ll \rho = mn \Rightarrow$

$-1/3$

Entropy is conserved

Thermo lemma. ☆ $\frac{\partial P}{\partial T} = \frac{\rho + P}{T}$ $f(p) = f(\frac{E}{T})$

$$dS = \frac{dU + PdV}{T} = \frac{1}{T}\left(d[(\rho+P)V] - VdP \right) = \frac{1}{T}\left(d[(\rho+P)V]\right) - \frac{V}{T^2}(\rho+P)dT$$

$U = \rho V$

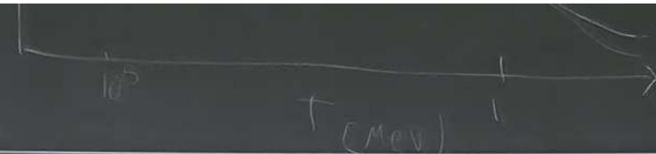
$$= d\left[\frac{\rho+P}{T}V\right]$$

$-\frac{dT}{T^2} = d\frac{1}{T}$

$$\frac{dS}{dt} = \frac{V}{T}\left[\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt}(\rho+P)\right] + \frac{V}{T}\left[\frac{dP}{dt} - \frac{\rho+P}{T}\frac{dT}{dt}\right] = 0$$

continuity → 0

☆ → 0



entropy density $s \equiv \dfrac{\rho + P}{T}$

$$s = \sum_i \frac{\rho_i + P_i}{T_i} \equiv \frac{2\pi^2}{45}\, g_{*s}(T)\, T^3$$

↖ dof in entropy

$$g_{*s}(T) = \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=f} g_i \left(\frac{T_i}{T}\right)^3$$

$$\frac{d}{dt} = \frac{V}{T}\left[\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt}(\rho + p)\right] + \frac{V}{T}\left[\frac{dp}{dt} - \frac{\rho + p}{T}\frac{dT}{dt}\right] = 0$$

continuity → 0

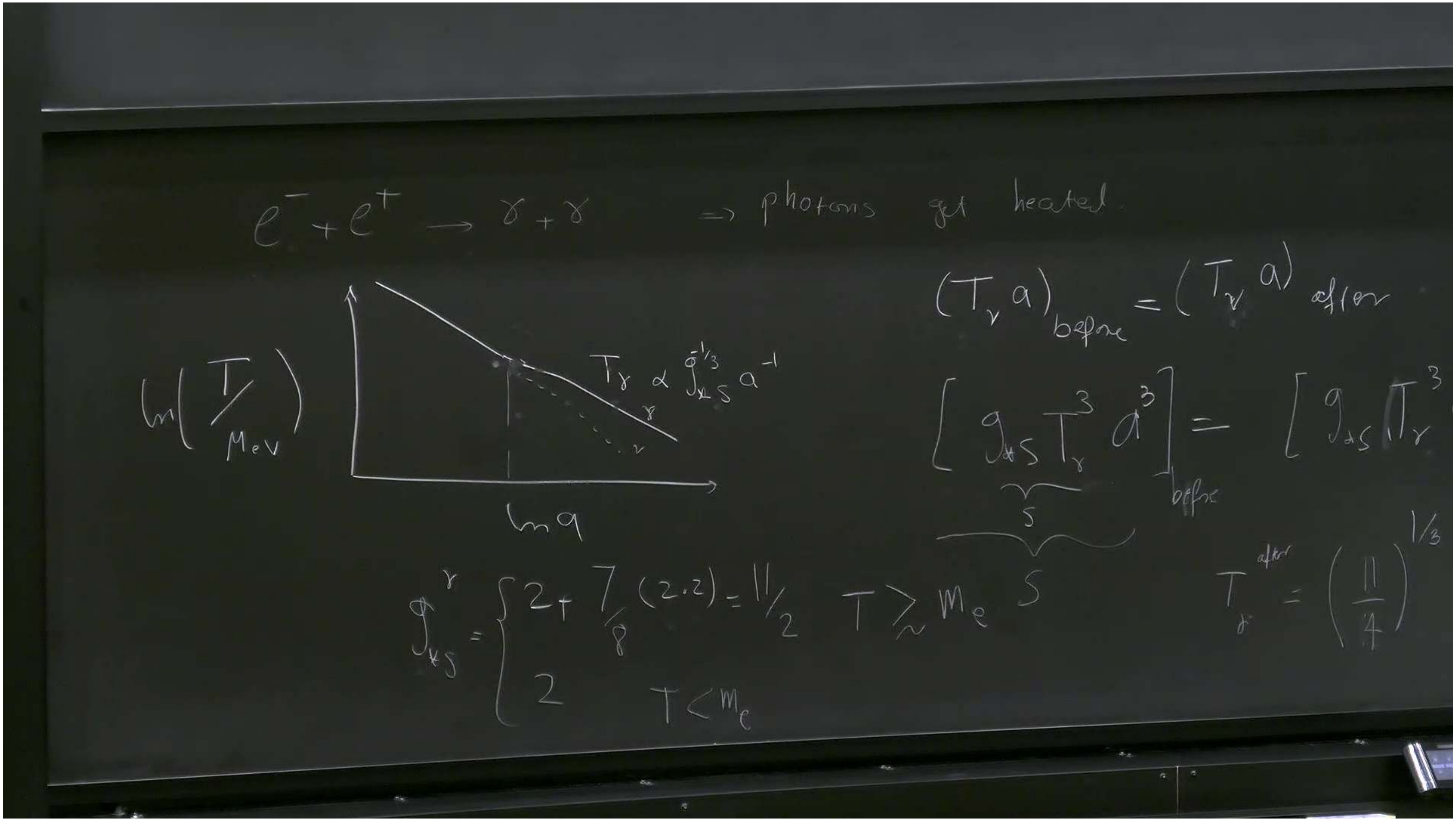
$$dS = 0 \Rightarrow d\, sV = 0 \Rightarrow s \propto a^{-3} \Rightarrow g_{*S}(T)\, T^3 \propto a^{-3}$$

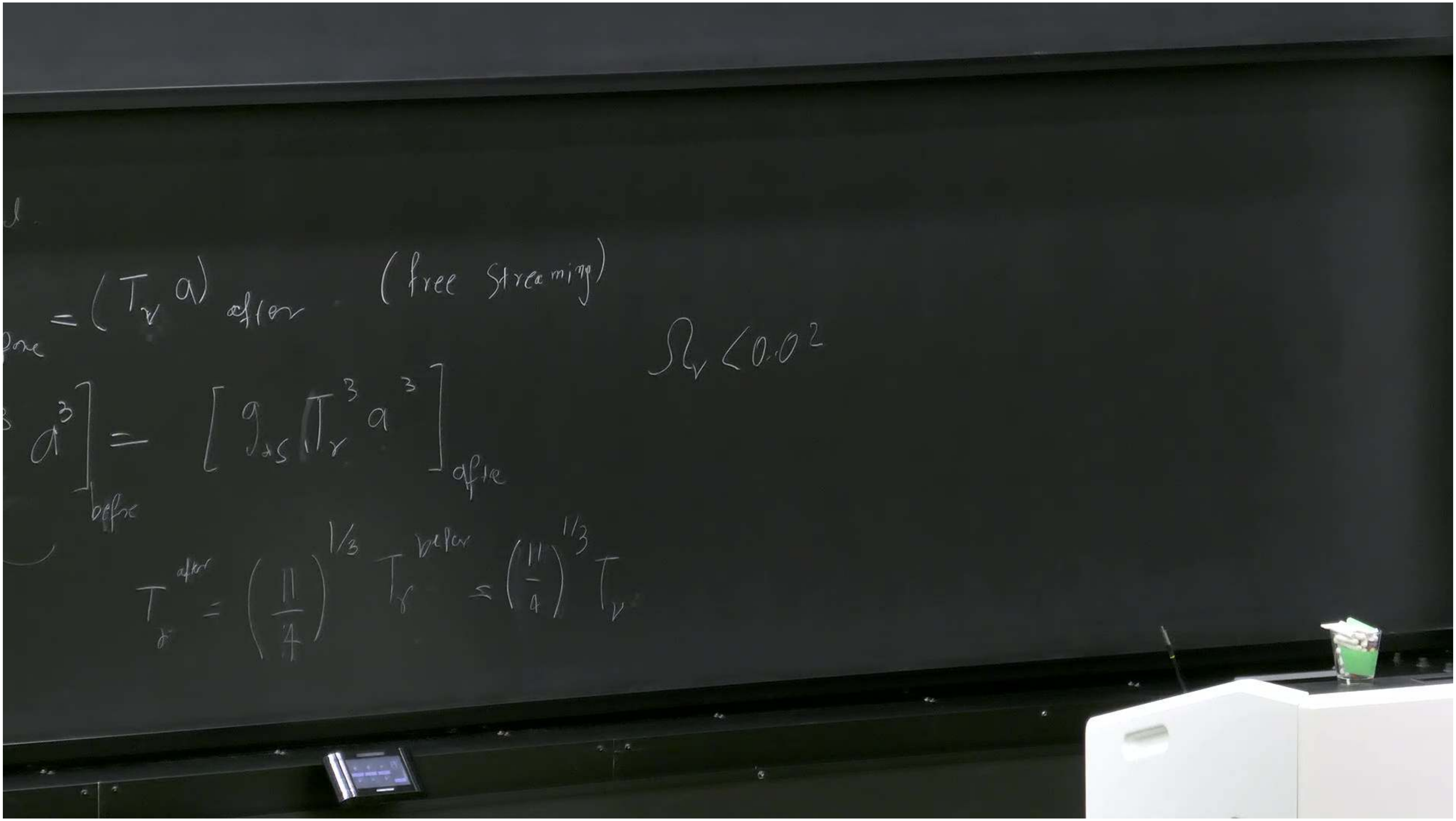
$$T \propto g_{*S}^{-1/3}\, a^{-1} \qquad \rho_r \propto T^4 \propto a^{-4}$$

$$\Gamma = |X|^2 + G_F^2 \times T^5 \qquad \nu + e^+ \longleftrightarrow \nu + e^+$$

G_F ↑ fermi const $\approx 10^{-5}\ \mathrm{GeV}^{-2}$

$$\frac{\Gamma}{H} = \frac{G_F^2 T^5}{T^2/M_p} \sim \left(\frac{T}{1\,\mathrm{MeV}}\right)^3 \sim 1 \qquad T_{dec} \sim 1\ \mathrm{MeV} \Rightarrow t_{dec} \sim 1\ \mathrm{sec}$$


$e^- + e^+ \to \gamma + \gamma$ ⇒ photons get heated
ln(T/MeV)
ln a
$T_\gamma \propto g_{*S}^{-1/3} a^{-1}$
$(T_\gamma a)_{before} = (T_\gamma a)_{after}$
$[g_{*S} T_\gamma^3 a^3]_{before} = [g_{*S} T_\gamma^3$
S
$g_{*S}^\gamma = \begin{cases} 2 + \frac{7}{8}(2\cdot 2) = 11/2 & T \gtrsim m_e \\ 2 & T < m_e \end{cases}$
$T_\gamma^{after} = \left(\frac{11}{4}\right)^{1/3}$


= (T_ν a) after (free streaming)
Ω_ν < 0.02
= [g_*s T_γ^3 a^3] after
before
T_γ^after = (11/4)^1/3 T_γ^before = (11/4)^1/3 T_ν