Title: Lecture - Cosmology, PHYS 621

Speakers: Ghazal Geshnizjani

Collection/Series: Cosmology (Elective), PHYS 621, February 23 - March 27, 2026

Subject: Cosmology

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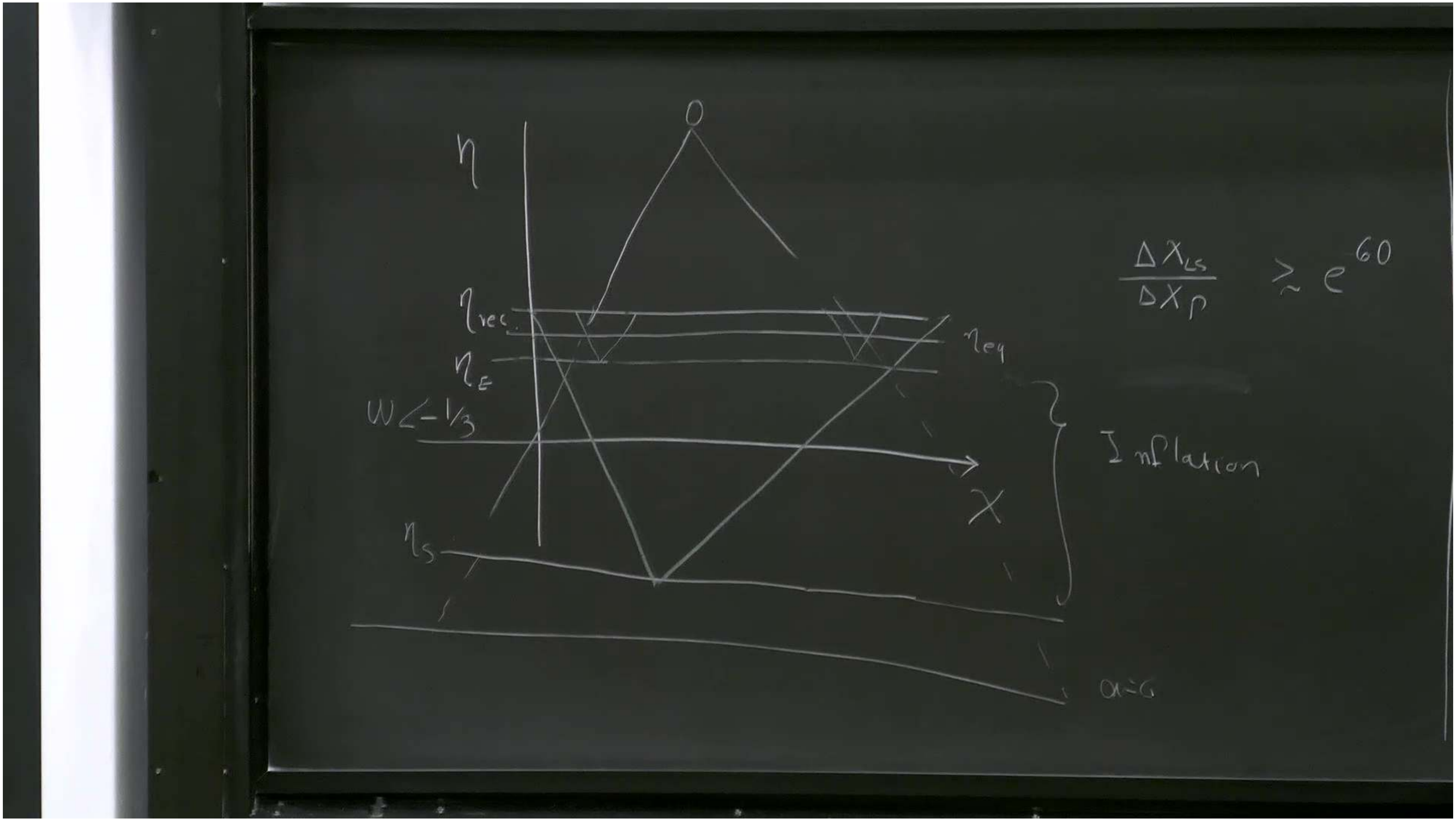


O
η
η_rec
η_E
η_eq
W < -1/3
χ
η_s
Inflation
ΔX_LS / ΔX_P ≳ e^60
a≃0

Hubble $\quad H = \frac{\dot{a}}{a} \qquad [H] = [t^{-1}] = [L^{-1}] = [m]$

$$R_H^{phys} := \frac{1}{H}$$

$$R_H^{c} := \frac{1}{aH}$$

geometry $\leftarrow$ $\epsilon := -\frac{\dot{H}}{H^2} \overset{K=0}{=} -\frac{3}{2}(1+w)$

$$w = P/\rho$$

$$w \leq -\frac{1}{3} \Longleftrightarrow$$

$H = \frac{\dot{a}}{a}$ $\quad [H] = [t^{-1}] = [L^{-1}] = [m]$

$= \frac{1}{H}$

$= \frac{1}{aH}$

$= -\frac{\dot{H}}{H^2} \overset{K=0}{=} +\frac{3}{2}(1+w)$

P/ρ

NEC

$w \leq -\frac{1}{3} \iff 0 \leq \epsilon \leq 1$

$\epsilon = 0 \iff \dot{H} = 0 \iff H = \text{const.}$

$\epsilon = -\frac{dH}{H(Hdt)} = -\frac{d\ln H}{d\ln a} = -\frac{1}{2}\frac{d\ln\rho}{d\ln a}$

- $w < -\frac{1}{3}$ or $\epsilon < 1 \iff \ddot{a} > 0$

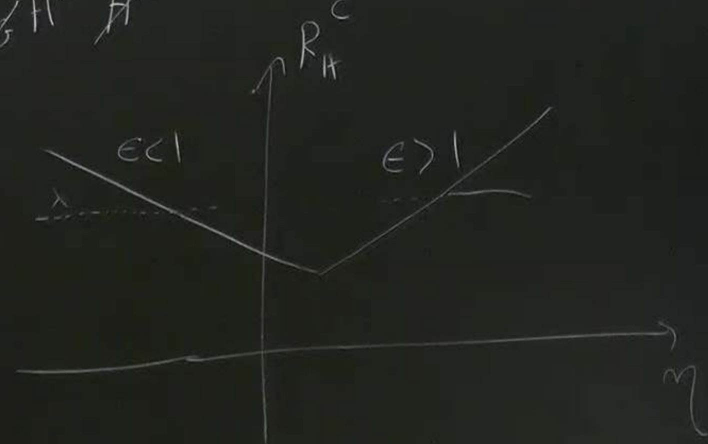
$$\dot\rho = -3H(\rho+p) \Rightarrow \frac{\dot\rho}{\rho H} = -3(1+w)$$

$$\Rightarrow \frac{\cancel{\frac{3}{4\pi G}}\,\cancel{H}\,\dot H}{\cancel{\frac{3}{8\pi G}}\,H^2\,\cancel{H}} = -3(1+w) \Rightarrow -\frac{\dot H}{H^2} =$$

$$H^2 = \frac{8\pi G}{3}\rho \Rightarrow \dot\rho = \left(\frac{3}{8\pi G}\right)2H\dot H$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho+3p) = -\frac{4\pi G}{3}\rho(1+3w)$$

$$\frac{dR_H^c}{dt} = \frac{d\,\frac{1}{aH}}{dt} = +\frac{1}{a}(\epsilon-1)$$



$\gtrsim e^{60}$

Hubble $\quad H = \frac{\dot a}{a} \qquad [H] = [t^{-1}] = [L^{-1}] = [m]$

$R_H^{phys} := \frac{1}{H}$

$R_H^{c} := \frac{1}{aH}$

geometry $\leftarrow \quad \epsilon := -\frac{\dot H}{H^2} \overset{K=0}{=} +\frac{3}{2}(1+w)$

$w = P/\rho$

e-folding number $\quad dN \equiv H dt = \frac{\dot a}{a} dt = d\ln a$

$w \leq -\frac{1}{3}$

$\epsilon = 0 \quad <$

$\epsilon = \frac{-dH}{H(Hdt)}$

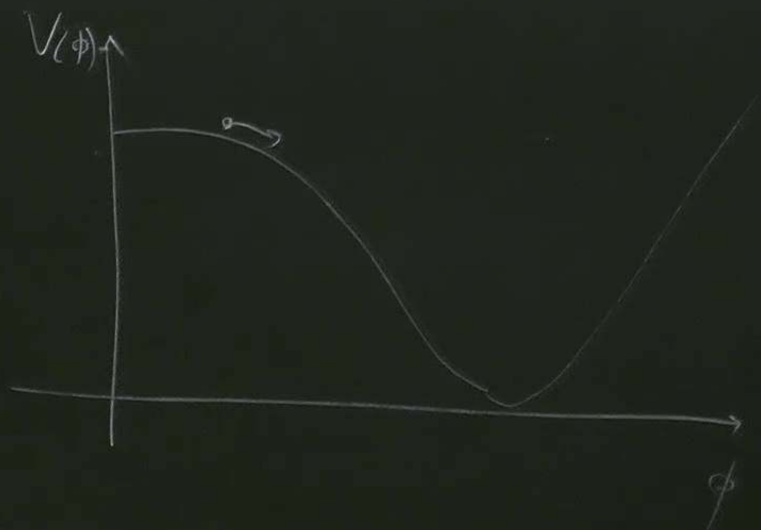
- $w < -1/3$ or

$a(t_1)$, $a(t_2)$

$$\Delta N = \ln a_2/a_1 = 2$$

$$a_2 = e^2 a_1$$

Scalar field : $\phi_{(t,x)}$ inflaton



$$S_{tot} = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R$$

inflaton

$$S_{tot} = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\left(\underbrace{\tfrac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi}_{KE} - \underbrace{V(\phi)}_{PE}\right)$$

$$T_{\mu\nu} = -\frac{2\,\delta S_m}{\delta g^{\mu\nu}}$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\left(\tfrac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi - V(\phi)\right)$$

$$d_\mu = \frac{\partial_\mu\phi}{(\partial\phi)^2}$$

$$T^0{}_0 = \rho_\phi = \tfrac{1}{2}\dot\phi^2 + V(\phi)$$

$$-\tfrac{1}{3}T^i{}_i = P_\phi = \tfrac{1}{2}\dot\phi^2 - V(\phi)$$

$\underbrace{-V(\phi))}_{p_E}$

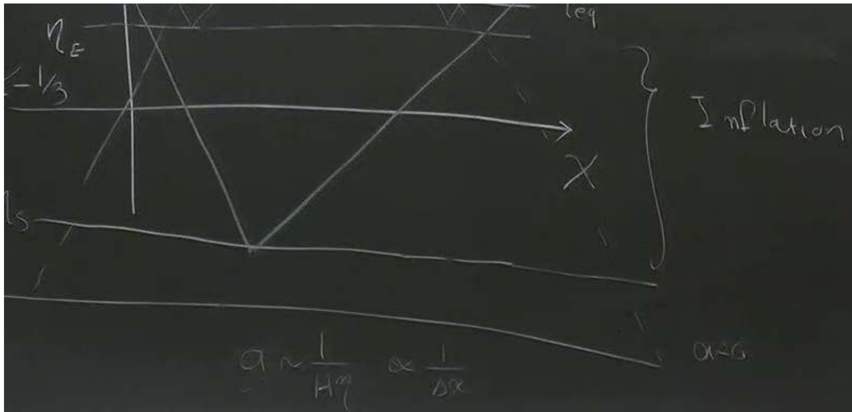
$$8\pi G \longrightarrow M_p^2 = \frac{\hbar c}{8\pi G} \qquad \hbar = c = 1$$

$$H^2 = \frac{1}{3M_p^2}\rho_\phi = \frac{1}{3M_p}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$-2\cancel{H}\dot{H} = \frac{1}{3M_p^2}\dot{\rho}_\phi = \frac{1}{\cancel{3}M_p^3}\left(-\cancel{3}\cancel{H}\,\dot{\phi}^2\right)$$

$$-\dot{H} = \dot{\phi}^2 / 2M_p^2$$

$$\epsilon = \frac{\frac{1}{2}\dot{\phi}^2}{M_p^2 H^2 \to \frac{1}{3}\rho_\phi} \ll 1 \qquad \text{Slow-roll}$$



$$R^c_H := \frac{1}{aH}$$

geometry $\leftarrow$ $\epsilon := -\frac{\dot H}{H^2} \overset{K=0}{=} +\frac{3}{2}(1+w)$

$$w = P/\rho$$

e-folding number $dN = H\,dt = \frac{\dot a}{a}dt = d\ln a$

$\circ \left| \eta = \frac{d\ln\epsilon}{dN} \right| < 1$, $|\ddot\phi| \ll |H\dot\phi|$ $\qquad \delta = \frac{\ddot\phi}{H\dot\phi} \ll 1$

$\Rightarrow \eta = 2(\epsilon + \delta)$ $\qquad \underbrace{\epsilon, \delta \ll 1 \iff \epsilon, \eta \ll 1}_{\text{Slow-roll conditions}}$

$\langle P_\phi \rangle \approx 0 \qquad \langle \rho_\phi \rangle \propto a^{-3}$

$$\begin{cases} \epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \approx \epsilon \\ \eta_V = M_p^2 V''/V \approx \epsilon - \delta \end{cases}$$