Title: Lecture - Quantum Field Theory III, PHYS 777

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Subject: Quantum Fields and Strings

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Anomalies

- classical system with some global symmetry G
  - spacetime symmetries (e.g. conformal symmetry)
  - internal symmetries (flavor symmetry in QCD $SU(3) \times SU(3)$ flavor symmetry)
  - continuous
  - discrete (P, T, $\mathbb{Z}_N$, ...)

Anomalies

- classical system with some global symmetry G
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  - continuous
  - discrete (P, T, $\mathbb{Z}_N$, ...)

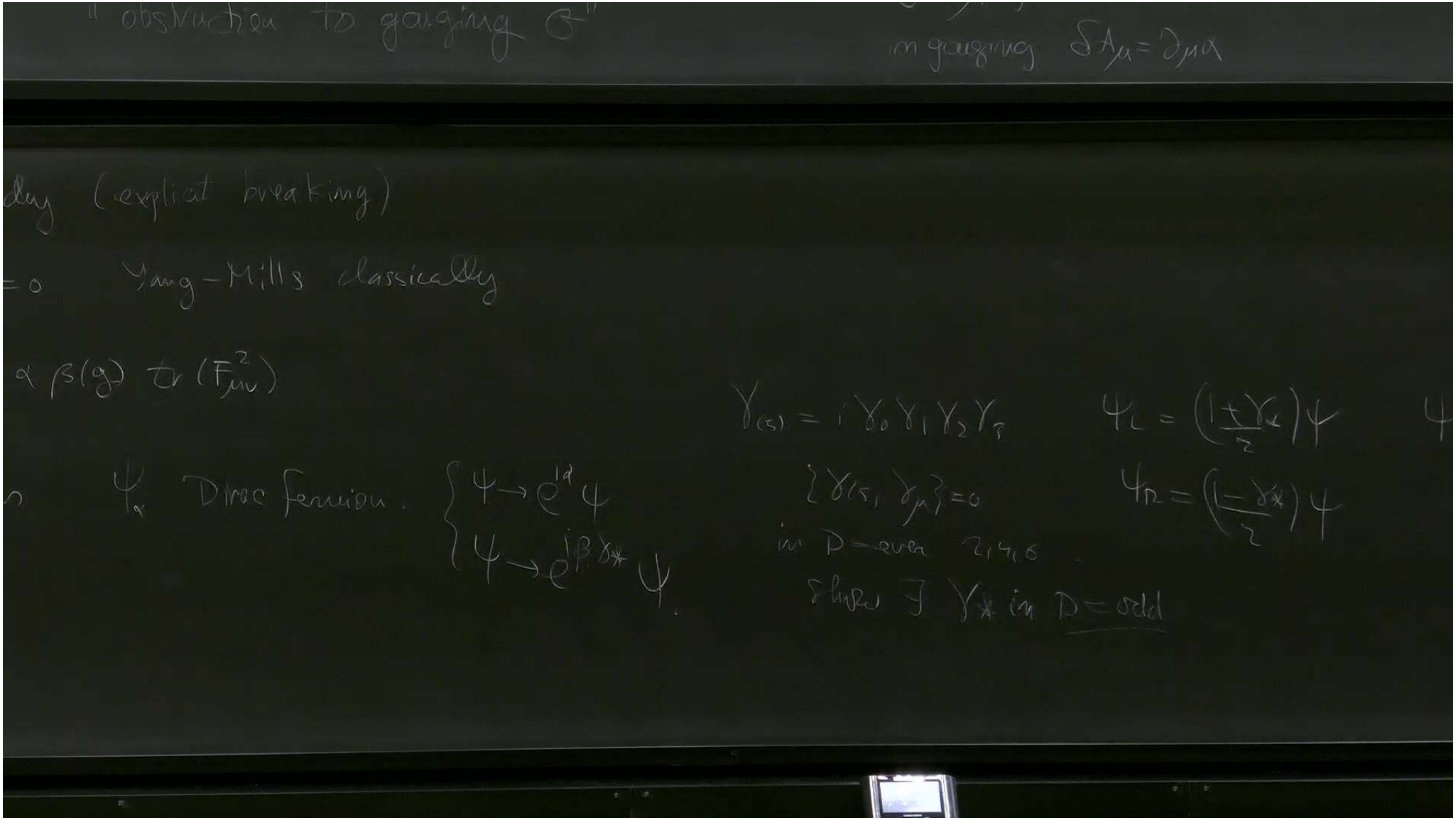
- Does quantization preserve the symmetries?

ND: ∃ an anomaly! (irreducible property)

"obstruction to gauging G"

$\int A_\mu j^\mu + \dots$

in gauging $\delta A_\mu = \partial_\mu q$


"obstruction to gauging"
in gauging $\delta A_\mu = \partial_\mu \alpha$
dly (explicit breaking)
=0 Yang-Mills classically
$\alpha \beta(g) \mathrm{tr}(F_{\mu\nu}^2)$
ψ_α Dirac fermion. $\begin{cases} \psi \to e^{i\alpha}\psi \\ \psi \to e^{i\beta\gamma_*}\psi \end{cases}$
$\gamma_{(5)} = i\gamma_0\gamma_1\gamma_2\gamma_3$
$\{\gamma_{(5)}, \gamma_\mu\} = 0$
in D=even 2,4,6
$\psi_L = \left(\frac{1+\gamma_*}{2}\right)\psi$
$\psi_R = \left(\frac{1-\gamma_*}{2}\right)\psi$
ψ
$\exists$ γ_* in D=odd

in gauging $\delta A_\mu = \partial_\mu \alpha$

$\gamma_{(5)} = i\gamma_0\gamma_1\gamma_2\gamma_3$ $\quad \psi_L = \left(\frac{1+\gamma_*}{2}\right)\psi$ $\quad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$\{\gamma_{(5)}, \gamma_\mu\} = 0$ $\quad \psi_R = \left(\frac{1-\gamma_*}{2}\right)\psi$

in D = even $2, 4, 6$

$\gamma_*\psi$.

$\exists$ γ_* in D = odd

J anomaly (explicit breaking)

$T^{\mu}_{\mu} = 0$ Yang-Mills classically

$T^{\mu}_{\mu} \propto \beta(g)\, \mathrm{tr}(F^2_{\mu\nu})$

$\gamma_{(5)} = i\gamma_0\gamma_1\gamma_2\gamma_3$ $\quad \psi_L = \left(\frac{1+\gamma_*}{2}\right)\psi$

$\{\gamma_{(5)}, \gamma_\mu\} = 0$ $\quad \psi_R = \left(\frac{1-\gamma_*}{2}\right)\psi$

in D = even 2, 4, 6

$\exists\ \gamma_*$ in D = odd

theories ψ_α Dirac fermion. $\begin{cases} \psi \to e^{i\alpha}\psi \\ \psi \to e^{i\beta\gamma_*}\psi \end{cases}$

mixed anomaly between $U(1)_V \times U(1)_A$

if gauge $U(1)_V \Longrightarrow U(1)_A$ is broken

A_μ

$\partial_\mu j^A_\mu = K\, \epsilon^{\mu\nu} F_{\mu\nu}$

$= i\gamma_0\gamma_1\gamma_2\gamma_3$ $\quad \psi_L = \left(\frac{1+\gamma_*}{2}\right)\psi \quad \psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$

$\{\gamma_*, \gamma_\mu\} = 0$ $\quad \psi_R = \left(\frac{1-\gamma_*}{2}\right)\psi$

D = even 2, 4, 6 ...

Since $\exists\ \gamma_*$ in D = odd

Exercise: it is impossible to simultaneously impose conservation of j_μ j_μ^A

$\partial_\mu j_\mu^A = K\,\epsilon^{\mu\nu} F_{\mu\nu} \Rightarrow$ $\langle j_\mu j_\nu \rangle$, $\langle j_\mu j_\nu^A \rangle$ $\langle j_\mu^A j_\nu^A \rangle$

if gauge $U(1)_V \Longrightarrow U(1)_A$ is broken

A_μ

$\partial_\mu j^A_\mu = K$

- "'t Hooft anomalies" global symmetry

1. Symmetry cannot be gauged

2. $\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle$ this obeys the Ward identities of G

$x_i \leftrightarrow x_j$

$\partial_\mu j^A_\mu = K\, \epsilon^{\mu\nu} F_{\mu\nu} \Longrightarrow$ $\langle j_\mu j_\nu \rangle$, $\langle j_\mu j^A_\nu \rangle$ $\langle j^A_\mu j^A_\nu \rangle$

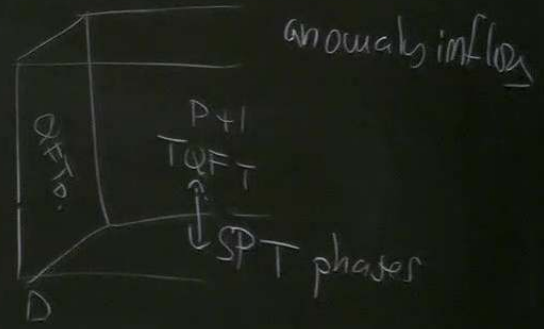
Topological quantities that do not change under RG flow

UV G, ? 't Hooft anomalies α_{UV}

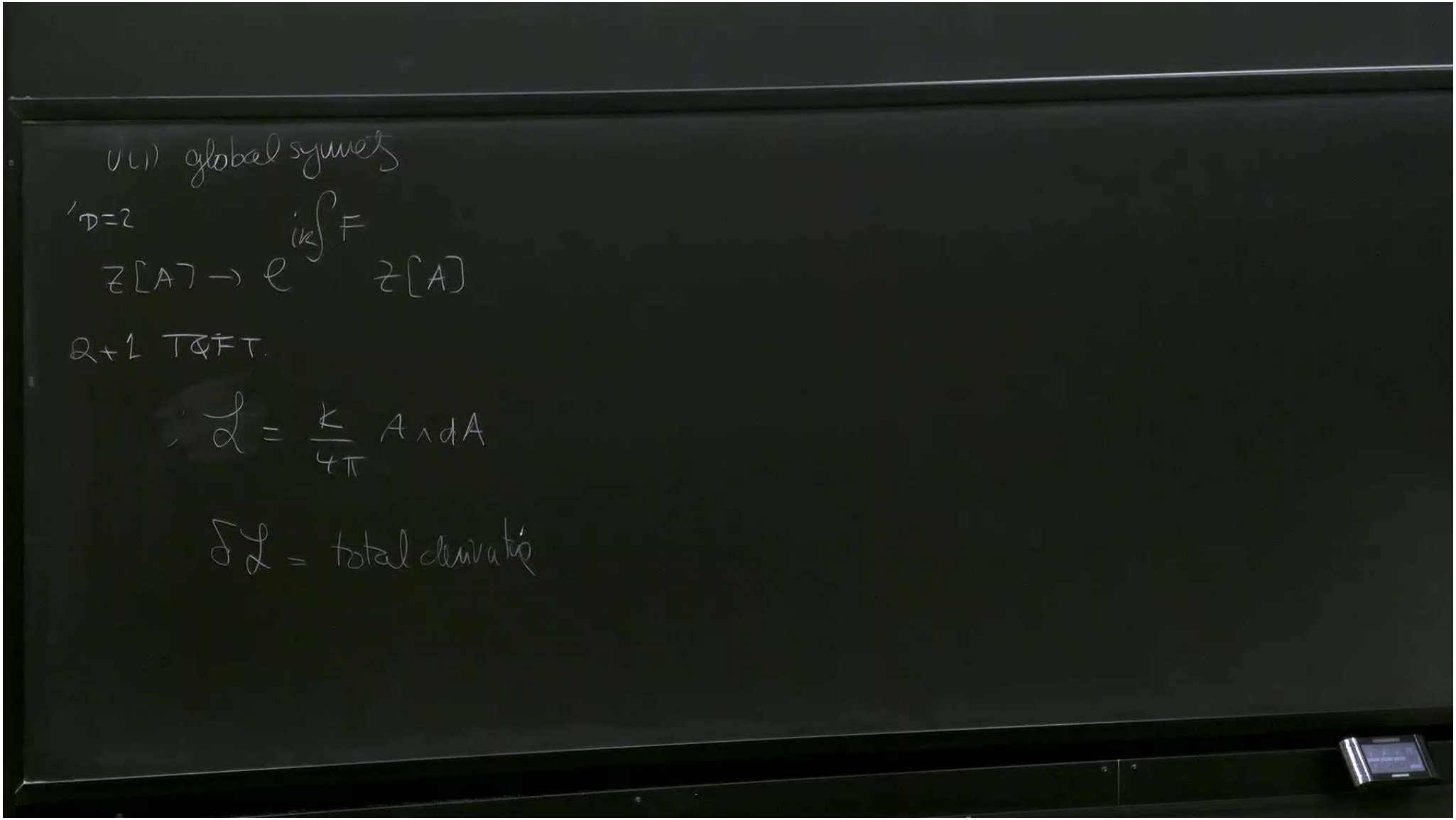
$Z_D[A] \xrightarrow[A]{\text{gauge transf of}} e^{i\int \alpha} Z[A] \Longleftrightarrow \int \alpha \longleftrightarrow$ Topological quantum field theories in D+1

A: background

anomaly inflow



IR (??) Candidate IR theory is $\alpha_{IR} = \alpha_{UV}$


U(1) global symmetry
D=2
$Z[A] \to e^{ik\int F} Z[A]$
2+1 TQFT.
$\mathcal{L} = \frac{k}{4\pi} A \wedge dA$
$\delta\mathcal{L}$ = total derivative

Conformal anomalies

$$\langle T_{\mu\nu}(p) \quad T_{\rho\sigma}(-p) \rangle =$$

- translational inv. $\partial^\mu T_{\mu\nu} = 0$ $\quad p^\mu T_{\mu\nu} = 0$
- conformal inv $\quad T^\mu{}_\mu = 0$

use $(p_\mu, \eta_{\mu\nu})$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})F_1(p^2) + \eta_{\mu\nu}\eta_{\rho\sigma}F_2(p^2) + (P_\mu P_\rho$$

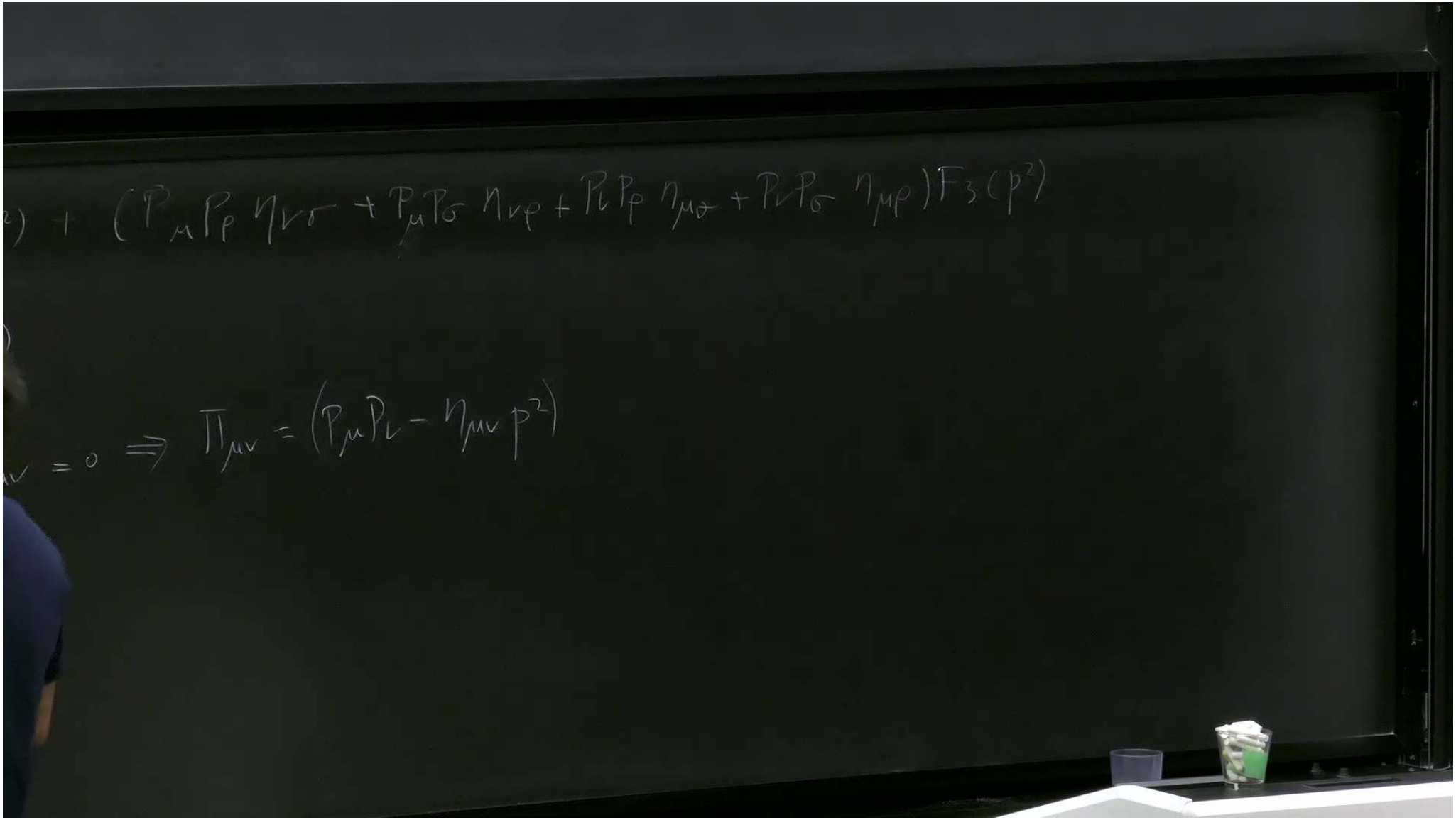
$$(P_\mu P_\nu \eta_{\rho\sigma} + P_\rho P_\sigma \eta_{\mu\nu})F_4(p^2) + P_\mu P_\nu P_\rho P_\sigma F_5(p^2)$$

want to impose conservation

$$\langle p^\mu T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = 0$$

$$\begin{cases} 2F_3 + F_4 + p^2 F_5 = 0 \\ F_1 + p^2 F_3 = 0 \\ F_2 + p^2 F_4 = 0 \end{cases}$$

$$p^\mu T_{\mu\nu} = 0$$


$+ (P_\mu P_\rho \eta_{\nu\sigma} + P_\mu P_\sigma \eta_{\nu\rho} + P_\nu P_\rho \eta_{\mu\sigma} + P_\nu P_\sigma \eta_{\mu\rho}) F_3(p^2)$
$= 0 \Rightarrow \Pi_{\mu\nu} = (P_\mu P_\nu - \eta_{\mu\nu} p^2)$

$$\Pi_{\mu\rho} = \tilde{P}_\mu \tilde{P}_\nu \qquad \tilde{P}_\mu = \epsilon_{\mu\nu}$$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{1}{p^2}\Big[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\Big]\, G(p^2)$$

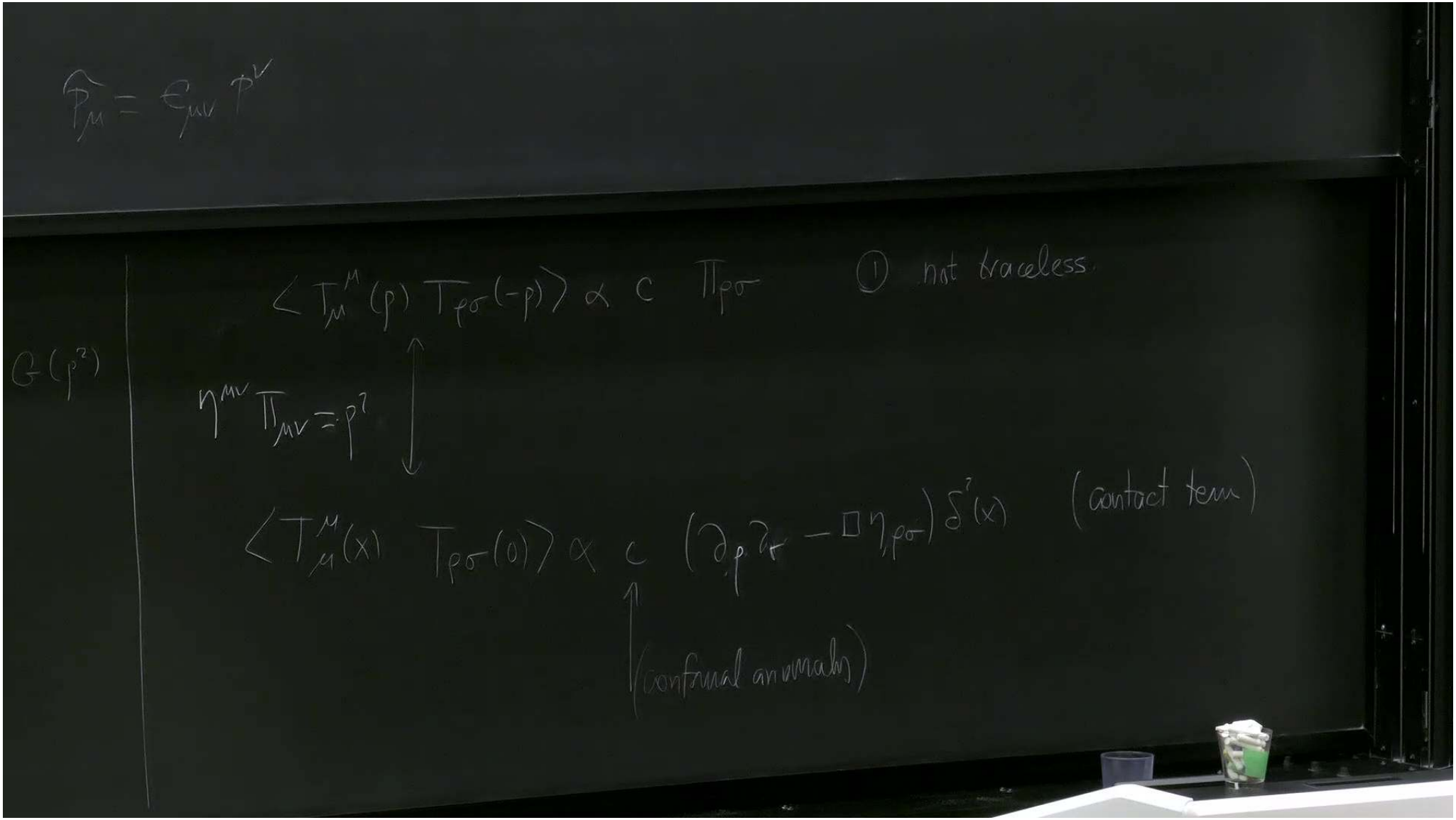
$$+ \Pi_{\mu\nu}\Pi_{\rho\sigma}\Big[H(p^2) + \frac{2D-4}{D-1}\,\frac{k(p^2)}{p^2}\Big]$$

$\eta_{\mu\nu}$

Let's go to $D=?$

impose scale invariance

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = C\,\frac{\Pi_{\mu\nu}\Pi_{\rho\sigma}}{p^2}$$


$\tilde{P}_\mu = \epsilon_{\mu\nu} P^\nu$
$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle \propto c\ \Pi_{\rho\sigma}$
① not traceless
$G(p^2)$
$\eta^{\mu\nu}\Pi_{\mu\nu} = p^2$
$\langle T^\mu_\mu(x)\, T_{\rho\sigma}(0)\rangle \propto c\,(\partial_\rho\partial_\sigma - \Box\eta_{\rho\sigma})\,\delta^2(x)$
(contact term)
(conformal anomaly)

$T^\mu_\mu \propto \beta(g)\ \mathrm{tr}(F^2_{\mu\nu})$

gauge theories ψ_α Dirac fermion. $\begin{cases} \psi \to e^{i\alpha}\psi \\ \psi \to e^{i\beta\gamma_*}\psi \end{cases}$

mixed anomaly between $U(1)_V \times U(1)_A$

if gauge $U(1)_V \Longrightarrow U(1)_A$ is broken

$\gamma_{(5)} = i\gamma_0\gamma_1\gamma_2\gamma_3$ $\quad \psi_L = \left(\frac{1+\gamma_*}{2}\right)\psi$ $\quad \psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$

$\{\gamma_{(5)}, \gamma_\mu\} = 0$ $\quad \psi_R = \left(\frac{1-\gamma_*}{2}\right)\psi$

in D = even $2, 4, 6$

since $\exists\ \gamma_*$ in D = odd

Exercise: it is impor

$\partial_\mu j^A_\mu = \kappa\, \epsilon^{\mu\nu} F_{\mu\nu} \Longrightarrow$ $D=2$ $\quad \langle j_\mu\ j_\nu \rangle$

couple CFT to a background metric $\sim \int T^{\mu\nu} h_{\mu\nu} + \dots$

D $\boxed{T^\mu_\mu = cR}$

D=3? $\boxed{T^\mu_\mu =}$

$$\langle T^\mu_\mu(0) \rangle_h = \int dx \langle T^\mu_\mu(0)\ T_{\rho\sigma}(x) \rangle_{h=0} h^{\rho\sigma}(x)$$

$$= c\int dx\ (\partial_\rho\partial_\sigma - \Box\eta_{\rho\sigma})\delta^2(x)\, h^{\rho\sigma}(x) \propto c\,(\partial_\rho\partial_\sigma - \Box\eta_{\rho\sigma})\, h^{\rho\sigma}(x) = c\, R_{\text{linearized}}$$

$T^{\mu\nu} h_{\mu\nu} + \cdots$

D $\boxed{T^\mu_\mu = cR}$ $D=2$

$D=3$? $\boxed{T^\mu_\mu = 0}$ no conformal anomalies in odd D.

$D=4$ $\boxed{T^\mu_\mu = \underline{\underline{a}}\,(E_4) + \underline{\underline{c}}\,\mathrm{Weyl}^2}$ a, c can be computed from 2pt and 3pt of $T_{\mu\nu}$

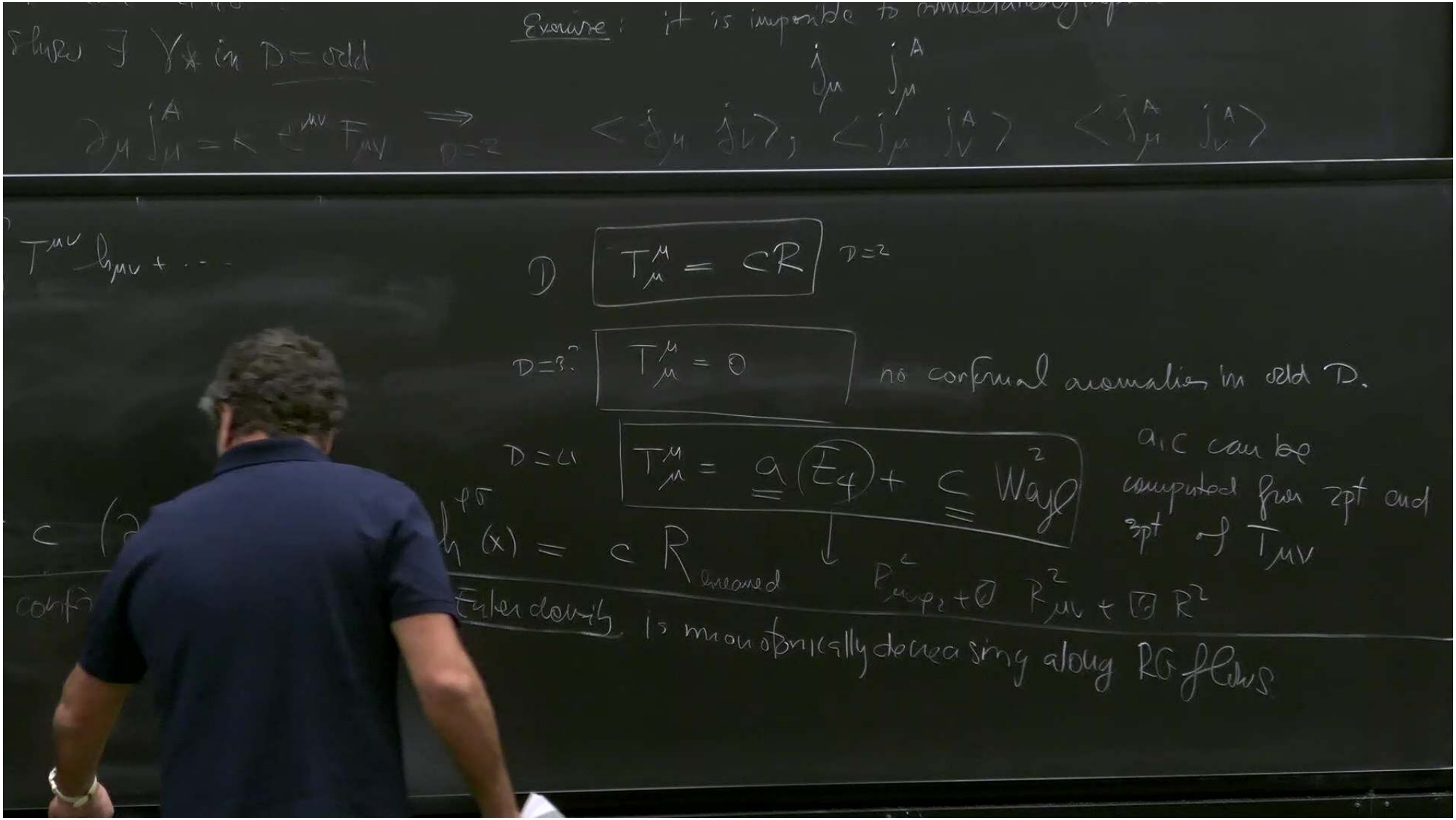
$c\,(\partial_\rho\partial_\sigma - \Box\eta_{\rho\sigma})\,h^{\rho\sigma}(x) = c\,R_{\text{linearized}}$ $\quad R^2_{\mu\nu\rho\sigma} + \emptyset\, R^2_{\mu\nu} + \boxed{?}\, R^2$

conformal anomaly associated to Euler density is monotonically decreasing along RG flows.

$C_{UV} > C_{IR}$

topological quantities that do not change under RG flow

UV ... 't Hooft anomalies α_{UV}


Exercise: it is impossible to
$\partial_\mu j^A_\mu = \kappa\, \epsilon^{\mu\nu} F_{\mu\nu}$
$j_\mu \quad j^A_\mu$
$\langle j_\mu j_\nu \rangle, \langle j_\mu j^A_\nu \rangle \quad \langle j^A_\mu j^A_\nu \rangle$
$T^{\mu\nu} h_{\mu\nu} + \cdots$
$T^\mu_\mu = cR$
$D=2$
$D=3$?
$T^\mu_\mu = 0$
no conformal anomalies in odd D.
$D=4$
$T^\mu_\mu = a\,(E_4) + c\,W^2$
a, c can be computed from 2pt and 3pt of $T_{\mu\nu}$
$h_\mu(x)$
$c\,R^2_{\text{squared}}$
$R^2_{\mu\nu} + \boxed{?}\, R^2$
Euler density
is monotonically decreasing along RG flows