Title: Lecture - Strong Gravity, PHYS 777

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Subject: Strong Gravity

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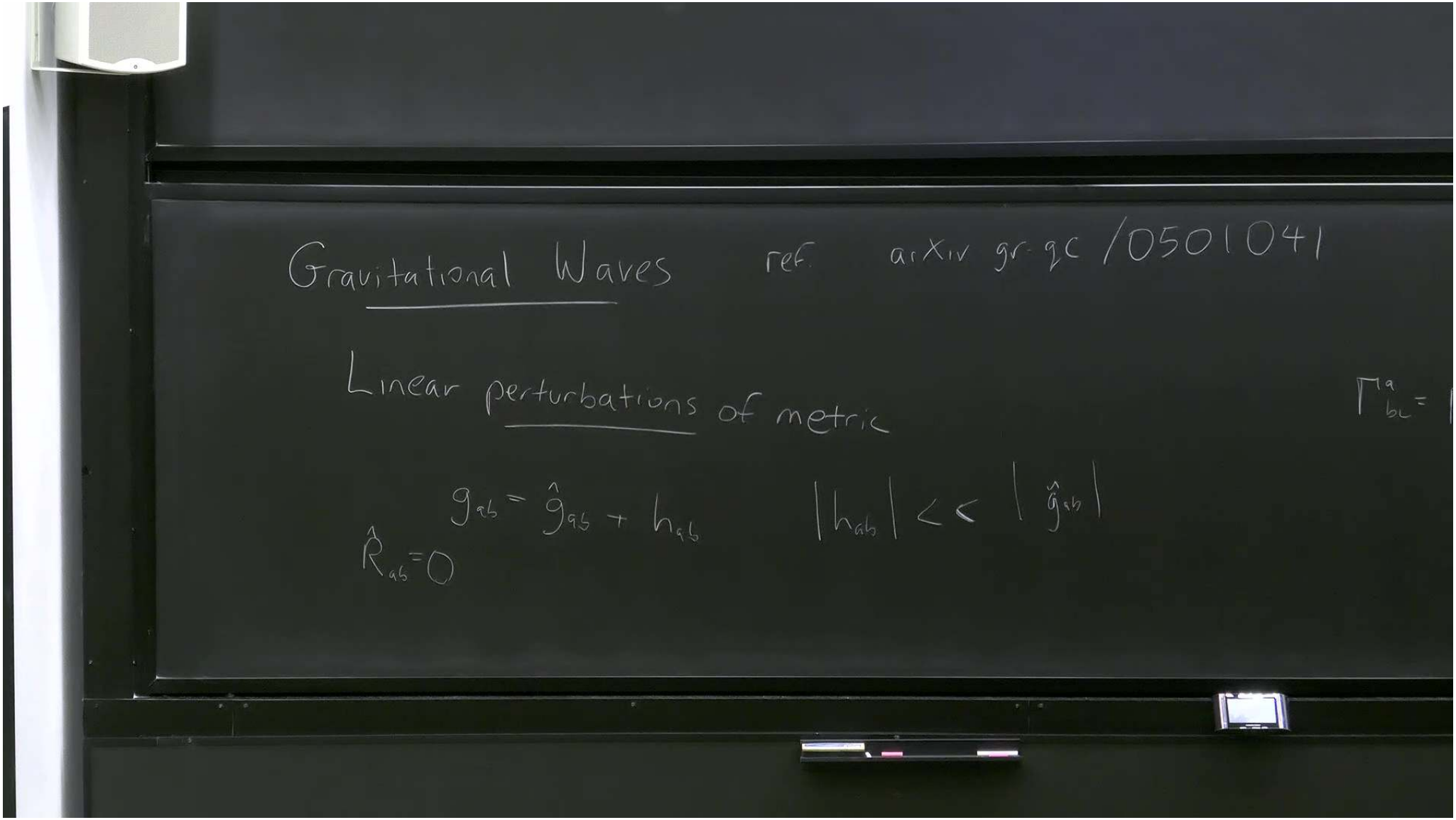
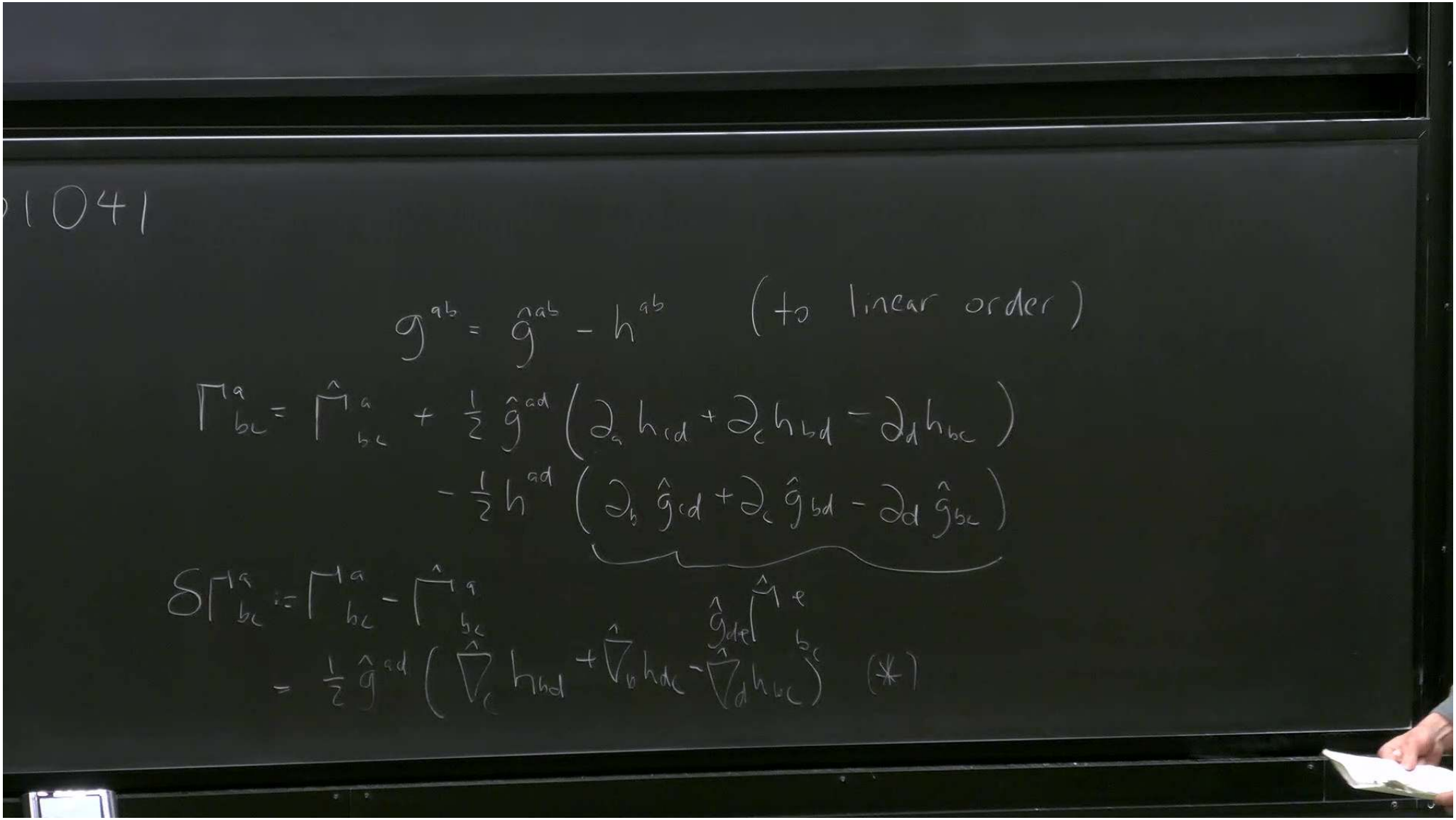

Gravitational Waves ref. arXiv gr-qc/0501041
Linear perturbations of metric
$g_{ab} = \hat{g}_{ab} + h_{ab}$ $|h_{ab}| << |\hat{g}_{ab}|$
$\hat{R}_{ab} = 0$


1041
$g^{ab} = \hat{g}^{ab} - h^{ab}$ (to linear order)
$\Gamma^a_{bc} = \hat{\Gamma}^a_{bc} + \frac{1}{2}\hat{g}^{ad}(\partial_b h_{cd} + \partial_c h_{bd} - \partial_d h_{bc})$
$- \frac{1}{2} h^{ad}(\partial_b \hat{g}_{cd} + \partial_c \hat{g}_{bd} - \partial_d \hat{g}_{bc})$
$\delta\Gamma^a_{bc} := \Gamma^a_{bc} - \hat{\Gamma}^a_{bc}$
$= \frac{1}{2}\hat{g}^{ad}(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{dc} - \hat{\nabla}_d h_{bc})$ (*)
$\hat{g}_{de}\hat{\Gamma}^e_{bc}$

$$g_{ab} = \hat{g}_{ab} + h_{ab} \qquad |h_{ab}| << |\hat{g}_{ab}|$$

$$\hat{R}_{ab} = 0$$

$$\delta\Gamma^a_{bc} := \Gamma^a_{bc} - \hat{\Gamma}^a_{bc} = \frac{1}{2} \hat{g}^{ad} \left(\hat{\nabla}_b h_{dc} + \hat{\nabla}_c h_{bd} - \hat{\nabla}_d h_{bc} \right)$$

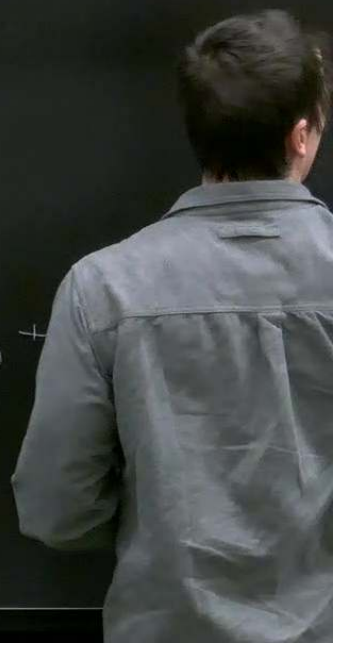
$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce} \Gamma^e_{bd} - \Gamma^a_{de} \Gamma^e_{bc}$$

$$= \partial_c \hat{\Gamma}^a_{bd} - \partial_d \hat{\Gamma}^a_{bc} + \partial_c \delta\Gamma^a_{bd} - \partial_d \delta\Gamma^a_{bc}$$

$$= \hat{R}^a{}_{bcd} + \hat{\nabla}_c \delta\Gamma^a_{bd} - \hat{\nabla}_d \delta\Gamma^a_{bc}$$

$$\delta R^a{}_{bcd} = \frac{1}{2} \left[\hat{\nabla}_c \hat{\nabla}_b h^a{}_d + \hat{\nabla}_c \hat{\nabla}_d h^a{}_b - \hat{\nabla}_c \hat{\nabla}^a h_{bd} - \hat{\nabla}_d \hat{\nabla}_b h^a{}_c - \hat{\nabla}_d \hat{\nabla}_c h^a{}_b + \right.$$

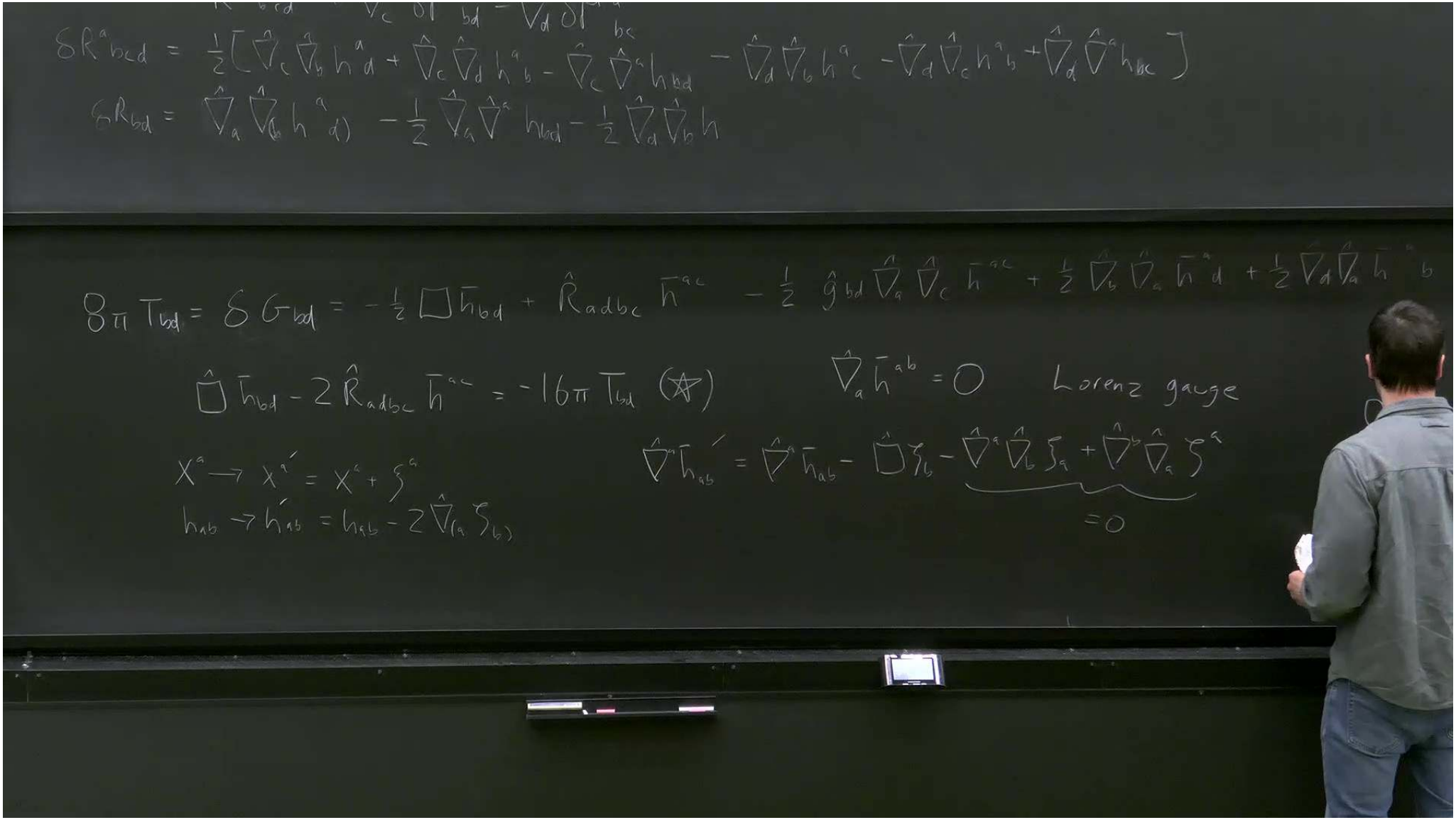
$$\delta R_{bd} = \hat{\nabla}_a \hat{\nabla}_{(b} h^a{}_{d)} - \frac{1}{2} \hat{\nabla}_a \hat{\nabla}^a h_{bd} - \frac{1}{2} \hat{\nabla}_d \hat{\nabla}_b h$$



$$= \frac{1}{2}\hat{g}^{cd}\left(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{dc} - \hat{\nabla}_d h_{bc}\right) \quad (*)$$

$$h = \hat{g}^{ab} h_{ab}$$

$$- \hat{\nabla}_d \hat{\nabla}_c h^a{}_b + \hat{\nabla}_d \hat{\nabla}^a h_{bc} \Big]$$


$\delta R^a{}_{bcd} = \frac{1}{2}[\hat\nabla_c\hat\nabla_b h^a{}_d + \hat\nabla_c\hat\nabla_d h^a{}_b - \hat\nabla_c\hat\nabla^a h_{bd} - \hat\nabla_d\hat\nabla_b h^a{}_c - \hat\nabla_d\hat\nabla_c h^a{}_b + \hat\nabla_d\hat\nabla^a h_{bc}]$
$\delta R_{bd} = \hat\nabla_a\hat\nabla_b h^a{}_d) - \frac{1}{2}\hat\nabla_a\hat\nabla^a h_{bd} - \frac{1}{2}\hat\nabla_d\hat\nabla_b h$
$8\pi T_{bd} = \delta G_{bd} = -\frac{1}{2}\Box\bar h_{bd} + \hat R_{adbc}\bar h^{ac} - \frac{1}{2}\hat g_{bd}\hat\nabla_a\hat\nabla_c\bar h^{ac} + \frac{1}{2}\hat\nabla_b\hat\nabla_a\bar h^a{}_d + \frac{1}{2}\hat\nabla_d\hat\nabla_a\bar h^a{}_b$
$\Box\bar h_{bd} - 2\hat R_{adbc}\bar h^{ac} = -16\pi T_{bd}$ (★)
$\hat\nabla_a\bar h^{ab} = 0$ Lorenz gauge
$x^a \to x'^a = x^a + \xi^a$
$h_{ab} \to h'_{ab} = h_{ab} - 2\hat\nabla_{(a}\xi_{b)}$
$\hat\nabla^a\bar h'_{ab} = \hat\nabla^a\bar h_{ab} - \Box\xi_b - \hat\nabla^a\hat\nabla_b\xi_a + \hat\nabla^b\hat\nabla_a\xi^a$
$=0$

$$\hat{\nabla}_a \hat{\nabla}_c \bar{h}^{ac} + \frac{1}{2} \hat{\nabla}_b \hat{\nabla}_a \bar{h}^{ad} + \frac{1}{2} \hat{\nabla}_d \hat{\nabla}_a \bar{h}^{ab}$$

$$\bar{h}_{ab} = h_{ab} - \frac{1}{2} h \hat{g}_{ab}$$

$$\bar{h} = -h$$

$$\bar{h}^{ab} = 0 \quad \text{Lorenz gauge}$$

Original gauge $\xi^a(t, x^i)$

Lorenz gauge: $\xi^a(0, x^i)$, $\partial_t \xi^a(0, x^i)$

$$\hat{\Box} \xi_b - \underbrace{\hat{\nabla}^a \hat{\nabla}_b \xi_a + \hat{\nabla}^b \hat{\nabla}_a \xi^a}_{=0}$$

Trace of (☆)

$\Box \bar{h} = -16\pi T$

Particular cases

(i) Vacuum ($T_{ab} = 0$)

$h(t=0)$, $\partial_t h(t=0) \Rightarrow h = 0$ for all time

Hence $h_{ab} = \bar{h}_{ab}$

(ia) Vacuum, flat background ($\hat{g}_{ab} = \eta_{ab}$) [wavezone]

$h_{Ti} = \partial_t h_{ti} = 0 \Rightarrow h_{ta} = 0$ for all time

(ib) $\hat{g}_{ab} = g^{Kerr}_{ab}$

(ii) Non-vacuum case, flat background — Can't assume $h = 0$

Propagation of GWs

$$\Box h_{ij}^{TT} = 0$$

$$h_{ij}^{TT} = A_{ij} \exp(i$$

In wavezone, use gauge DOFs

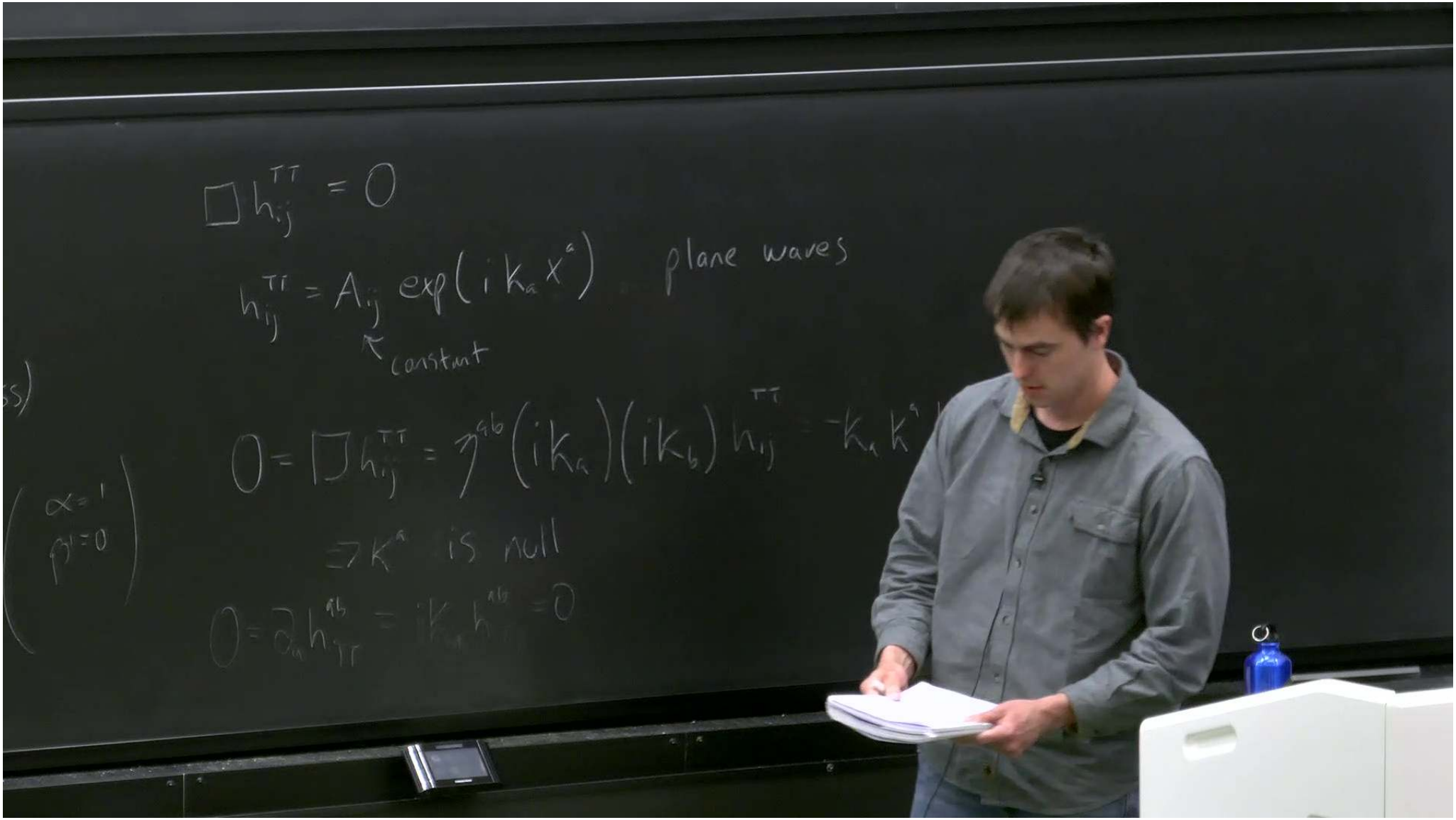
$$h_{ti}^{TT} = 0 \qquad \eta^{ab} h_{ab}^{TT} = 0 \quad \text{(traceless)}$$

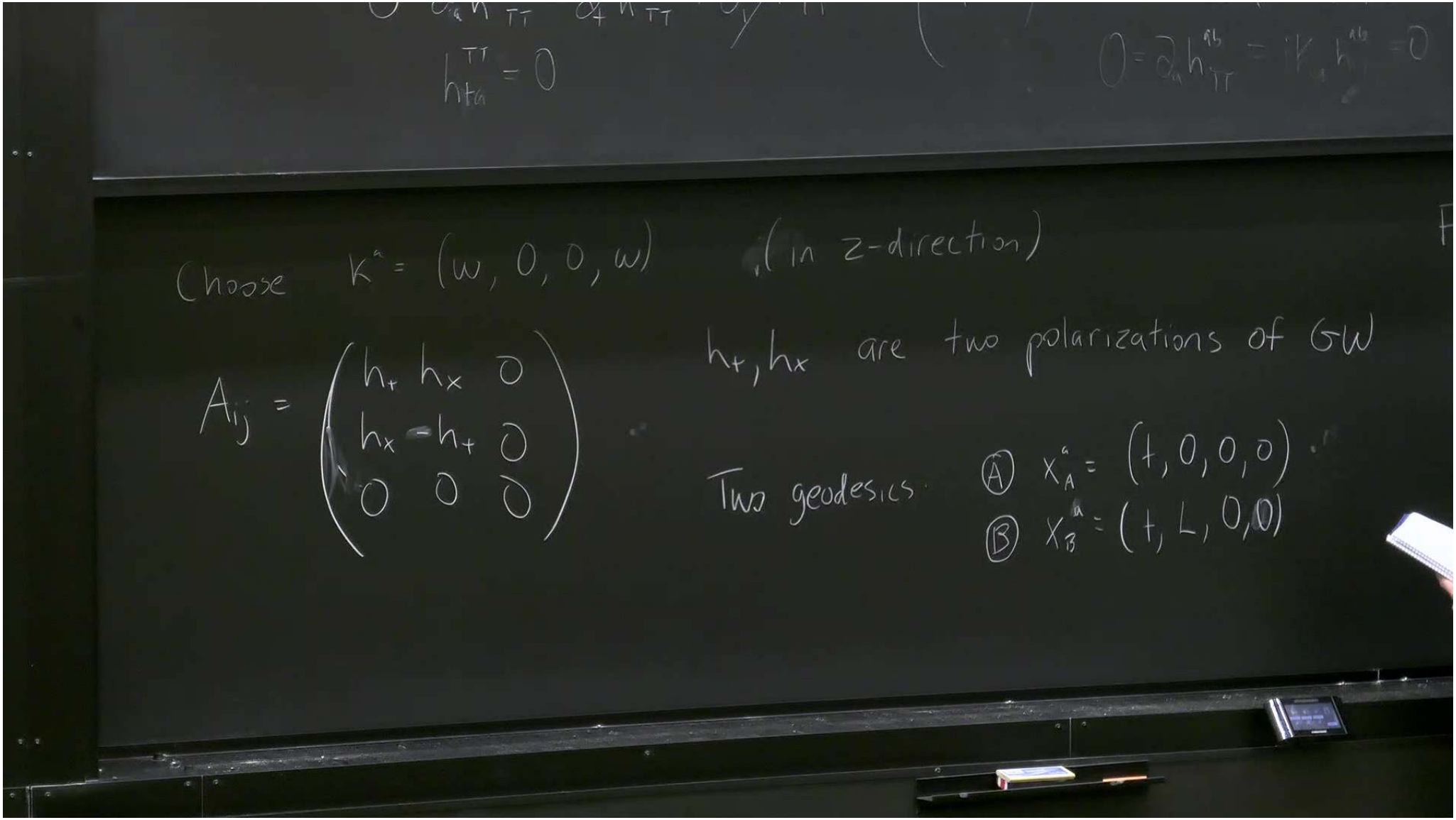
$$h_{ab}^{TT} = \bar{h}_{ab}$$

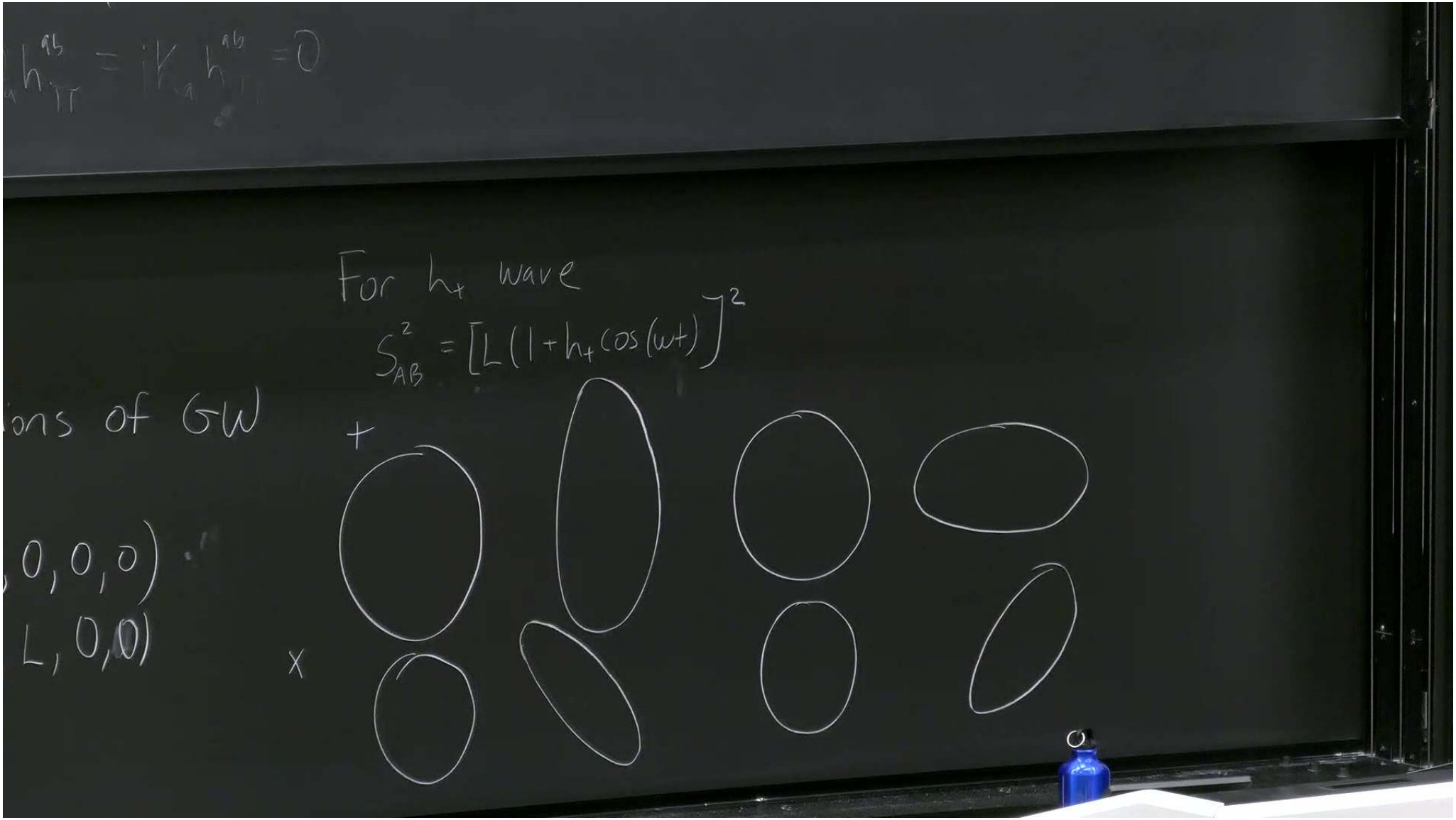
$$0 = \partial_a h^{at}_{TT} = \partial_t h^{tt}_{TT} + \partial_i h^{it}_{TT}$$

$$\left(\begin{array}{l} \alpha = 1 \\ \beta^i = 0 \end{array} \right)$$

$$h^{TT}_{ta} = 0$$


$\Box h_{ij}^{TT} = 0$
$h_{ij}^{TT} = A_{ij} \exp(i k_a x^a)$
plane waves
constant
$0 = \Box h_{ij}^{TT} = \eta^{ab}(ik_a)(ik_b) h_{ij}^{TT}$
$\Rightarrow k^a$ is null
$0 = \partial_a h_{TT}^{ab} = i k_a h^{ab}$
$\alpha = 1$
$\beta^i = 0$


$h^{TT}_{t\alpha} = 0$
$0 = \partial_\alpha h^{\alpha\beta}_{TT} = i k_\alpha h^{\alpha\beta} = 0$
Choose $k^a = (\omega, 0, 0, \omega)$ (in z-direction)
$A_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$h_+, h_\times$ are two polarizations of GW
Two geodesics
(A) $x^a_A = (t, 0, 0, 0)$
(B) $x^a_B = (t, L, 0, 0)$


$h^{ab}_{TT} = i k_a h^{ab}_T = 0$
For h_+ wave
$S^2_{AB} = [L(1 + h_+ \cos(\omega t)]^2$
ions of GW
0, 0, 0)
L, 0, 0)
+
x

Rotate by ϕ about z-axis

$$h_x \pm i h_+ \rightarrow \left(h_x \pm i h_+\right) e^{\mp 2i\phi}$$