Title: Lecture - Strong Gravity, PHYS 777

Speakers: William East

Collection/Series: Strong Gravity (Elective), PHYS 777, February 23 - March 27, 2026

Subject: Strong Gravity

Date: March 11, 2026 - 3:30 PM

URL: https://pirsa.org/26030016

CTT Eqns:

$$\tilde{D}_i \tilde{D}^i \psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\left[(\tilde{\mathcal{L}}X)_{ij} + \hat{A}_{ij}{}^{TT}\right]^2 \psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi \tilde{E}\psi^{-n+5}$$

$$(\tilde{\Delta}_{\mathcal{L}} X)^{ij} - \frac{2}{3}\psi^6 \tilde{D}^i K = 8\pi \tilde{p}^i$$

$$)^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}\psi^{-n+5} = 0$$

$$\tilde{E} = \psi^n E \qquad C = \ldots + 2\pi(n-5)\tilde{E}$$

$$n \geq 5$$

$$\psi = \bar{\psi} + \epsilon \qquad \tilde{D}_i\tilde{D}^i\epsilon = C\epsilon$$

$$C \geq 0$$

Local uniqueness by Max Principle

Dominant Energy condition: $-T^a{}_b n^b$ future pointing and causal

$$-E^2 + p_i p^i \le 0$$

$$\tilde{E}^2 \ge \tilde{\gamma}_{ij}\tilde{p}^i\tilde{p}^j$$

$n=3$ $$E^2 = \psi^{-16}\tilde{E}^2 \ge \psi^{-16}\tilde{\gamma}_{ij}\tilde{p}^i\tilde{p}^j = \gamma_{ij}p^i p^j$$

CTT Eqns: $\tilde{D}_i \tilde{D}^i \psi - \frac{1}{8} \tilde{R} \psi + \frac{1}{8} \left[(\tilde{\mathbb{L}} X)_{ij} + \hat{A}_{ij}^{TT} \right]^2 \psi$

$$(\tilde{\Delta}_{\mathbb{L}} X)^i - \frac{2}{3} \psi^6 \tilde{D}^i K = 8\pi \tilde{p}^i$$

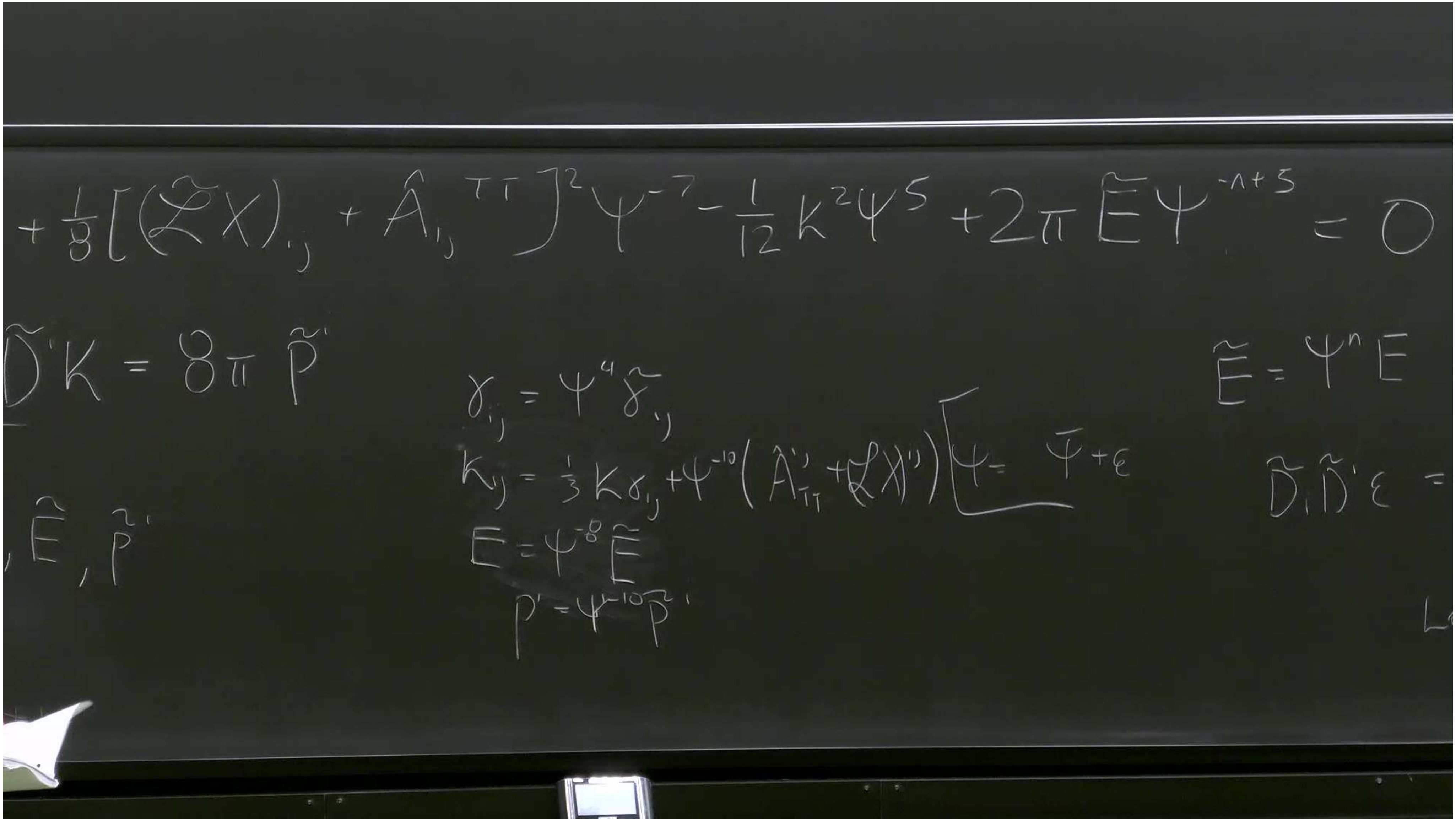
$\gamma_{ij} = \psi^4$

$K_{ij} = \frac{1}{3} K$

Free data: $\tilde{\gamma}_{ij}, K, \hat{A}^{TT}_{ij}, \hat{E}, \tilde{p}^i$

5 1 2

Constrained data: ψ, X^i 3+1


$+\frac{1}{8}[(\tilde{L}X)_{,j} + \hat{A}_{ij}^{TT}]^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}\psi^{-n+5} = 0$
$\tilde{D}^i K = 8\pi\tilde{p}^i$
$\gamma_{ij} = \psi^4\tilde{\gamma}_{ij}$
$\tilde{E} = \psi^n E$
$K_{ij} = \frac{1}{3}K\gamma_{ij} + \psi^{-10}(\hat{A}^{ij}_{TT} + (\tilde{L}X)^{ij})$
$\psi = \bar{\psi} + \epsilon$
$\tilde{D}_i\tilde{D}^i\epsilon =$
$, \hat{E}, \tilde{p}^i$
$E = \psi^{-8}\tilde{E}$
$p^i = \psi^{-10}\tilde{p}^i$

Simple example

Vacuum $E = p^i = 0$

Maximal + "Waveless" $K = \hat{A}^{ij}_{TT} = 0$

Conformal flat $\tilde{\gamma}_{ij} = f_{ij}$

BCs $\psi \to 1$, $X^i \to 0$ as $r \to \infty$

$\partial_i \partial^i \psi + \frac{1}{8} (\mathcal{L} X)^{ij} (\mathcal{L} X)_{ij} \psi^{-7} = 0 \qquad (\Delta_L X)^i = 0$

$\Sigma_0 = \mathbb{R}^3 \Rightarrow \gamma_{ij} = f_{ij}, \; K_{ij} = 0$

$\Sigma_0 = \mathbb{R}_3 / B_R$

at $r = R$ $\quad D_i s^i = 0 \iff \frac{1}{r^2}\partial_r(\psi^4 r^2) = 0$

$$\partial_r \psi + \frac{\psi}{2r} = 0$$

$$\frac{M}{2} = R$$

$$\partial_i \partial^i \psi = 0$$

Unique Soln : $\psi = 1 + \frac{R}{r}$

$$\gamma_{ij} dx^i x^j = \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

Schwarzschild in isotropic coördinates

Constrained data: ψ, X^i 3+1

Other formulations of Constraint Eqn

Conformal thin sandwich (CTS)

Extended CTS

Remarks on choosing free data

$$\tilde{\gamma}_{ij} \simeq \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij} - f_{ij}$$

Constrained data: ψ, X^i 3+1

Other formulations of Constraint Eqn:

Conformal thin sandwich (CTS)

Extended CTS

Remarks on choosing free data

$$\tilde{\gamma}_{ij} \simeq \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij} - f_{ij}$$

$$\xi_a = \hat{t}^a + \Omega_{ab}\hat{\phi}^a$$

$$\mathcal{L}_\xi g_{ab} \simeq 0$$

Local Uniqueness by Max Principle

Free data

$\tilde{\gamma}_{ij}$, K, $\tilde{\alpha}$, $\partial_t \tilde{\gamma}^{ij}$

$\tilde{\gamma}_{ij}$, K, $\partial_t K$, $\partial_t \tilde{\gamma}_{ij}$

Constrained data

ψ, β^i

ψ, α, β^i

Prelude: Example of Maxwell Eqns

$$0 = \partial^a F_{ab} = \partial^a \partial_a A_b - \partial^a \partial_b A_a \quad , \quad A_a \;\; \partial_t A_a$$

$b = t$

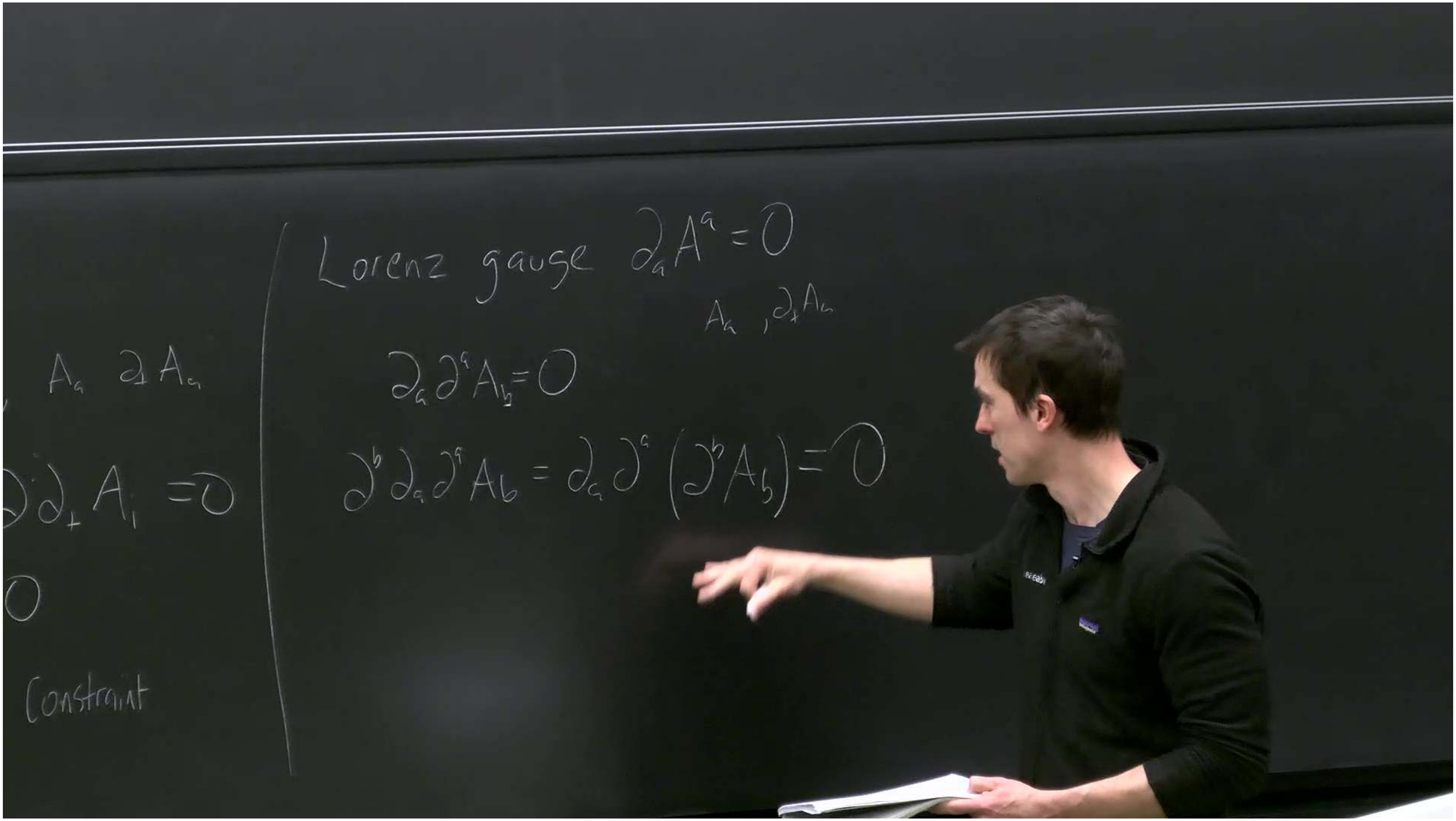
$$-\partial_t^2 A_t + \partial_i \partial^i A_t + \partial_t^2 A_t - \partial^i \partial_t A_i = 0$$

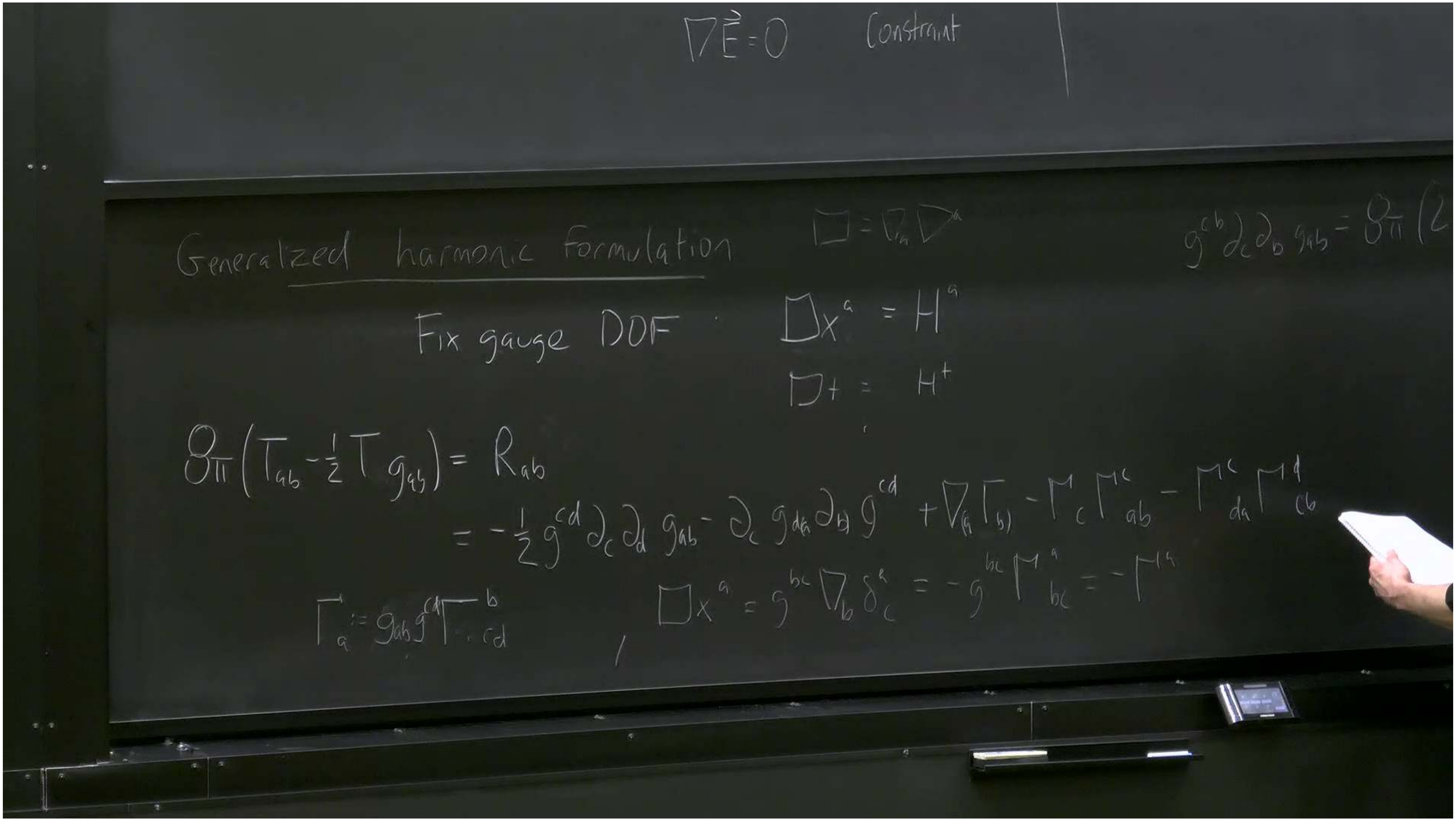
$$\partial_i \partial^i A_t - \partial^i \partial_t A_i = 0$$

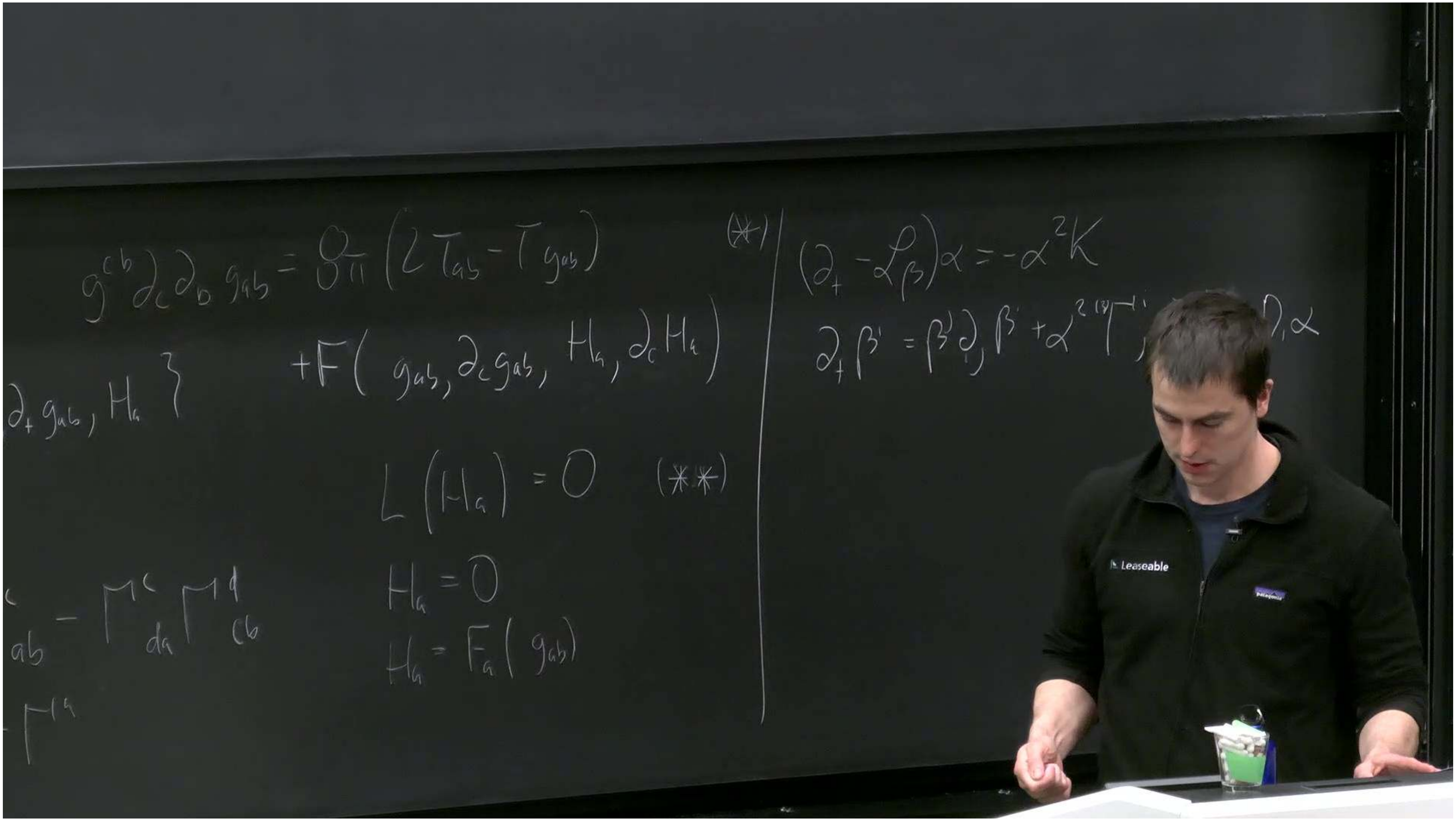
$$\nabla \vec{E} = 0 \quad \text{Constraint}$$

Lorenz gauge

$$\partial_a \partial^a A_b = 0$$


Lorenz gauge $\partial_a A^a = 0$
$A_a \; , \partial_t A_a$
$\partial_a \partial^a A_b = 0$
$\partial^b \partial_a \partial^a A_b = \partial_a \partial^a (\partial^b A_b) = 0$
$A_a \quad \partial_t A_a$
$\partial_t A_1 = 0$
0
Constraint


$\nabla \vec{E} = 0$ Constraint
Generalized harmonic formulation
$\Box = \nabla_a \nabla^a$
$g^{cb}\partial_c \partial_b g_{ab} = 8\pi ($
Fix gauge DOF
$\Box x^a = H^a$
$\Box t = H^t$
$8\pi \left(T_{ab} - \frac{1}{2} T g_{ab}\right) = R_{ab}$
$= -\frac{1}{2} g^{cd} \partial_c \partial_d g_{ab} - \partial_c g_{d(a} \partial_{b)} g^{cd} + \nabla_{(a} \Gamma_{b)} - \Gamma_c \Gamma^c_{ab} - \Gamma^c_{da} \Gamma^d_{cb}$
$\Gamma_a := g_{ab} g^{cd} \Gamma^b_{cd}$
$\Box x^a = g^{bc} \nabla_b \delta^a_c = -g^{bc} \Gamma^a_{bc} = -\Gamma^a$



$g^{cb}\partial_c\partial_b g_{ab} = 8\pi(2T_{ab} - T g_{ab})$
$+F(g_{ab}, \partial_c g_{ab}, H_a, \partial_c H_a)$
$\partial_t g_{ab}, H_a \}$
$L(H_a) = 0$ (**)
$H_a = 0$
$H_a = F_a(g_{ab})$
(*)
$(\partial_t - \mathcal{L}_\beta)\alpha = -\alpha^2 K$