Title: Lecture - Quantum Information I (Elective), 635

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Collection/Series: Quantum Information I (Elective), PHYS 635, February 23 - March 27, 2026

Subject: Quantum Information

Date: March 17, 2026 - 2:15 PM

URL: https://pirsa.org/26030007

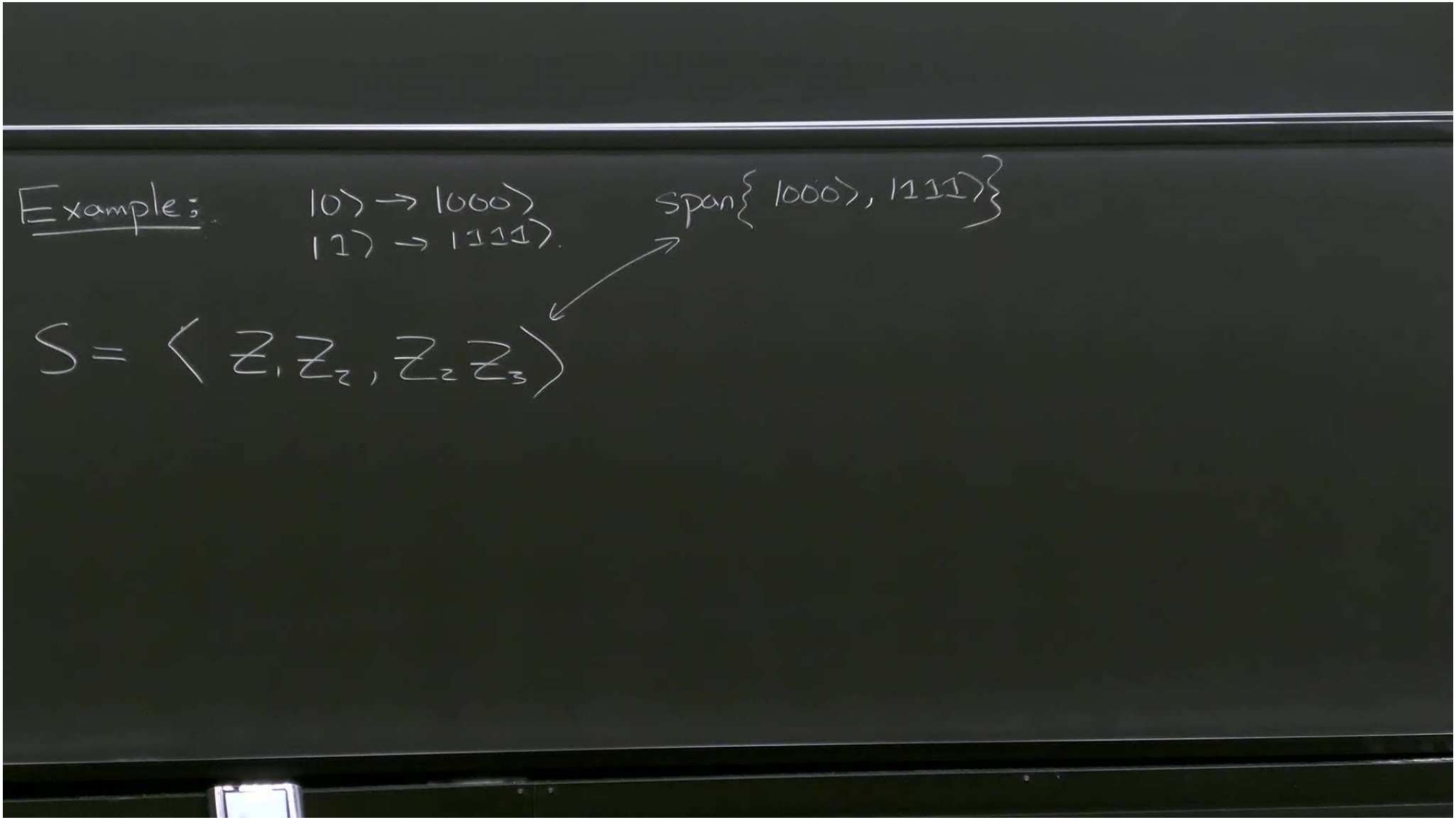
Lecture 9 : Stabilizer codes

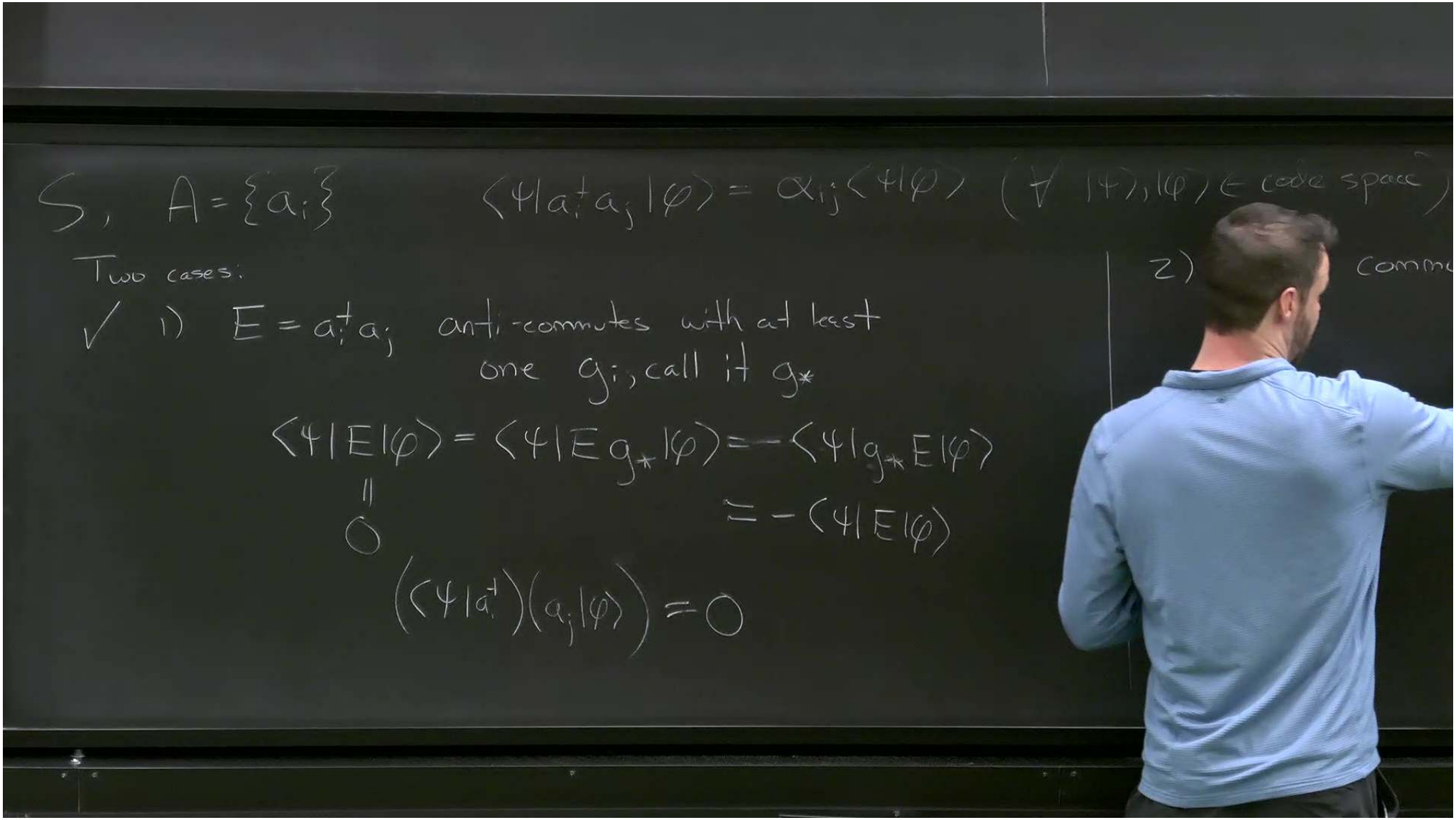
Recall: $S = \langle g_1, \ldots, g_{n-k} \rangle$ $\quad -\mathbb{I} \notin S$, $\quad g_i g_j = g_j g_i$

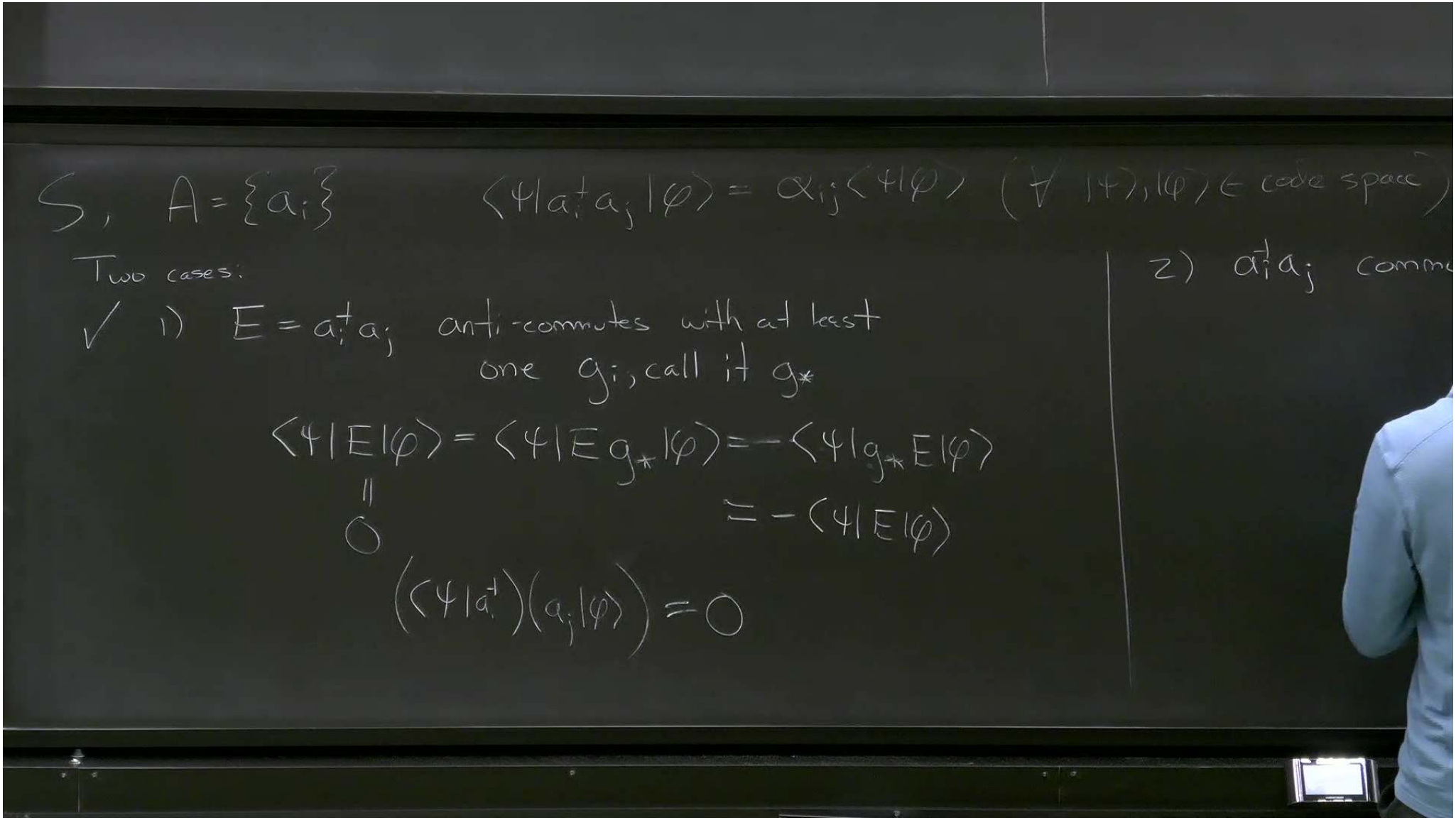
"Stabilizer subgroup" of G_n

Describes 2^k dimensional subspace of 2^n dim Hilbert space.

Idea: "Stabilizer code" = code subspace specified this way.


Example:
$|0\rangle \to |000\rangle$
$|1\rangle \to |111\rangle$.
span$\{|000\rangle, |111\rangle\}$
$S = \langle Z_1 Z_2, Z_2 Z_3 \rangle$


S, $A=\{a_i\}$ $\langle\psi|a_i^\dagger a_j|\varphi\rangle = \alpha_{ij}\langle\psi|\varphi\rangle$ ($\forall\ |\psi\rangle, |\varphi\rangle \in$ code space)
Two cases:
✓ 1) $E = a_i^\dagger a_j$ anti-commutes with at least one g_i; call it g_*
$\langle\psi|E|\varphi\rangle = \langle\psi|E g_*|\varphi\rangle = -\langle\psi|g_* E|\varphi\rangle$
$= 0$
$= -\langle\psi|E|\varphi\rangle$
$(\langle\psi|a_i^\dagger)(a_j|\varphi\rangle) = 0$
2) Commu


S, A = {a_i}
⟨ψ|a_i† a_j|φ⟩ = α_ij ⟨ψ|φ⟩ (∀ |ψ⟩, |φ⟩ ∈ code space)
Two cases:
✓ 1) E = a_i† a_j anti-commutes with at least one g_j; call it g*
⟨ψ|E|φ⟩ = ⟨ψ|E g* |φ⟩ = −⟨ψ|g* E|φ⟩
= −⟨ψ|E|φ⟩
0
(⟨ψ|a_i†)(a_j|φ⟩) = 0
2) a_i† a_j comm

$|\psi\rangle, |\phi\rangle \in$ code space)

2) $a_i^\dagger a_j$ commutes with all g_i

✓ 2a) $a_i^\dagger a_j \in S \implies \langle\psi|a_i^\dagger a_j|\phi\rangle = \langle\psi|\phi\rangle$.

2b) $a_i^\dagger a_j \notin S$ $\quad \langle\psi|a_i^\dagger a_j|\phi\rangle$

For stab. subgroups $\longrightarrow$ $=$

$Z(S) = \{g \in G_n : gh = hg \ \forall h \in S\}$

$N(S) = \{g \in G_n : ghg^{-1} \in S\}$

Bad / uncorrectable errors: $a_i^\dagger a_j \in Z(S) \setminus S$
$= N(S) \setminus S$

$$(\langle\psi|a_i^\dagger)(a_j|\phi\rangle) = 0$$

Bad / uncorrectable erro

Thm Let S be a stab subgroup describing $\mathcal{C}(S)$. Then $A = \{a_i\}$ are correctable whenever

$$\forall a_i, a_j \in A \quad a_i^\dagger a_j \notin N(S) \setminus S$$

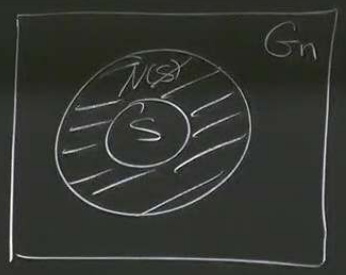
Subgroups

Bad / uncorrectable errors: $a_i^\dagger a_j \in Z(S) \setminus S$
" $N(S) \setminus S$

$N(S) = \{ g \in G_n : ghg^{-1} \in S \}$

Proof: construct α_{ij} Hermitian s.t.

$$\langle \psi | a_i^\dagger a_j | \varphi \rangle = \alpha_{ij} \langle \psi | \varphi \rangle$$



$a_i^\dagger a_j \notin N(S) \setminus S$

→ ① $a_i^\dagger a_j \in S$ ⟶ choose $\alpha_{ij} = 1 = \alpha_{ji}$

→ ② $a_i^\dagger a_j \in G_n \setminus N(S)$ ⟶ choose $\alpha_{ij} = \alpha_{ji} = 0$

am s.t.

$\alpha_{ij} < 4\pi$.

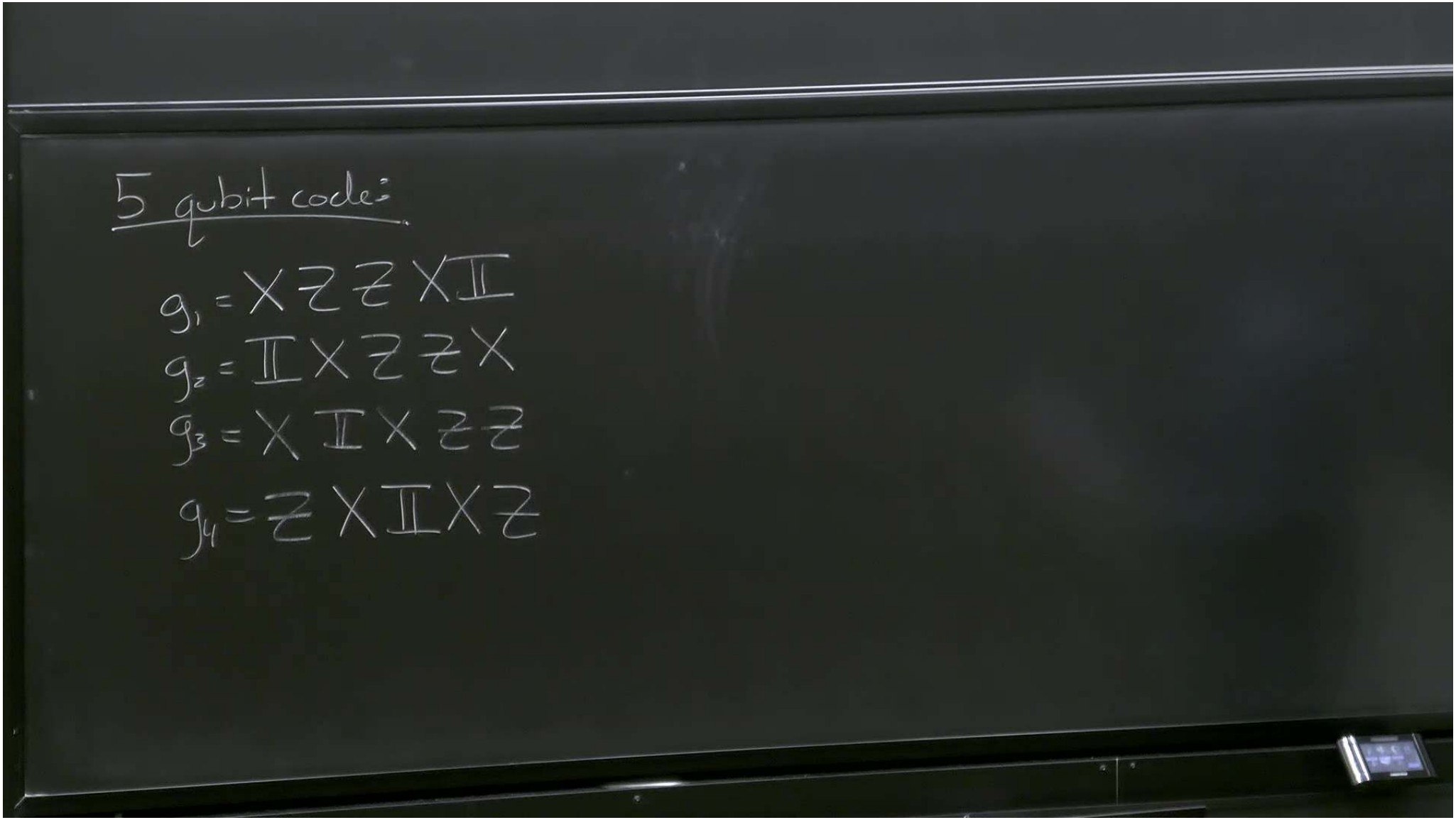
Example: $S = \langle Z_1 Z_2, Z_2 Z_3 \rangle$

$A = \{X_1, X_2, X_3\}$.

$X_1 X_2, X_2 X_3, X_1 X_3$.

oose $\alpha_{ij} = 1 = \alpha_{ji}$

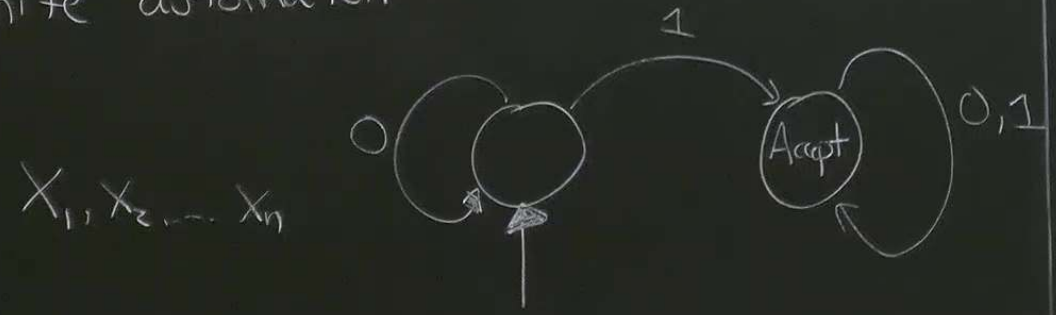
$\Rightarrow$ choose $\alpha_{ij} = \alpha_{ji} = 0$


5 qubit code:
$g_1 = XZZXI$
$g_2 = IXZZX$
$g_3 = XIXZZ$
$g_4 = ZXIXZ$

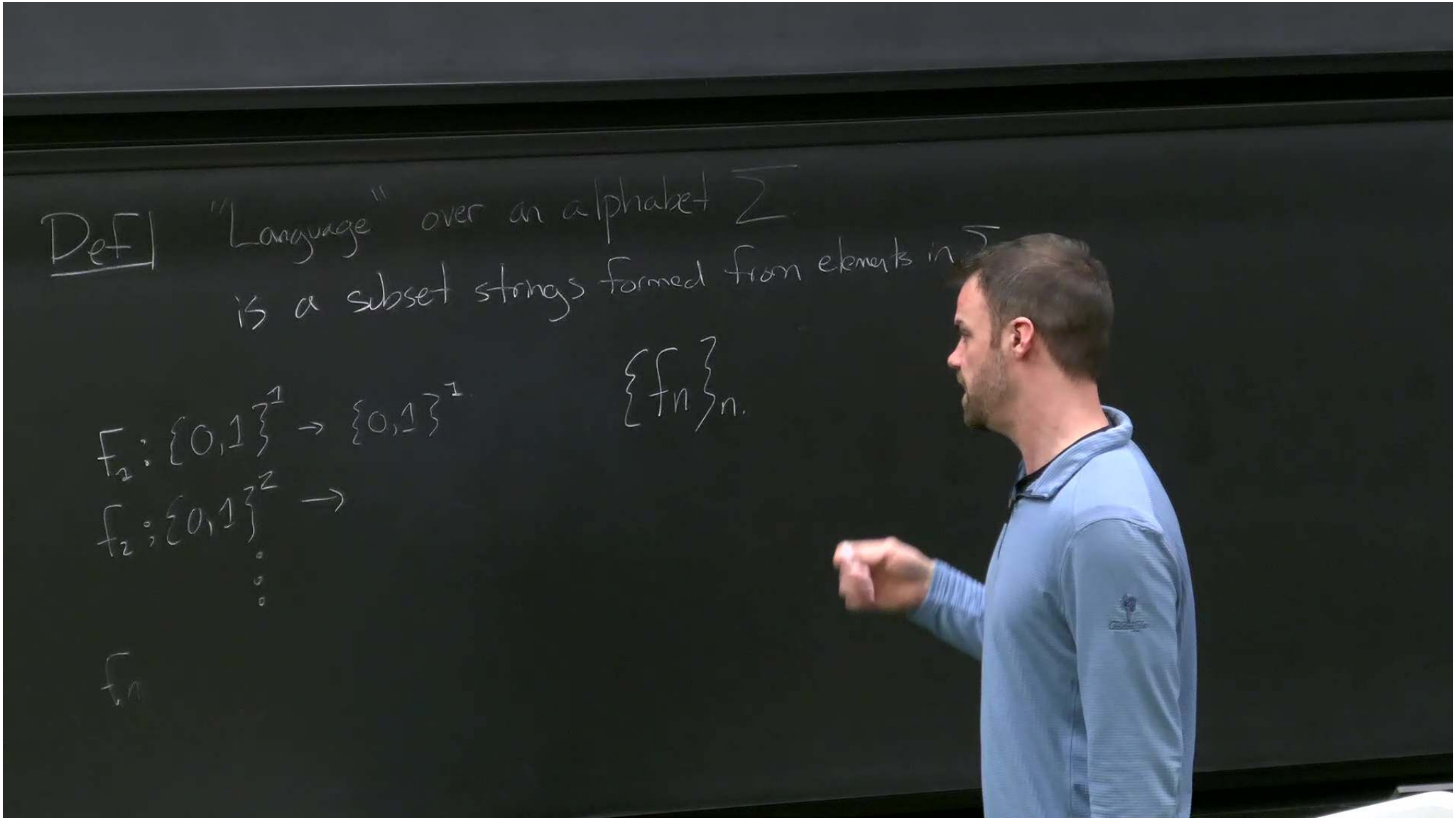
Lecture 10 - Theory of computation

"Model of computation"

Finite automaton

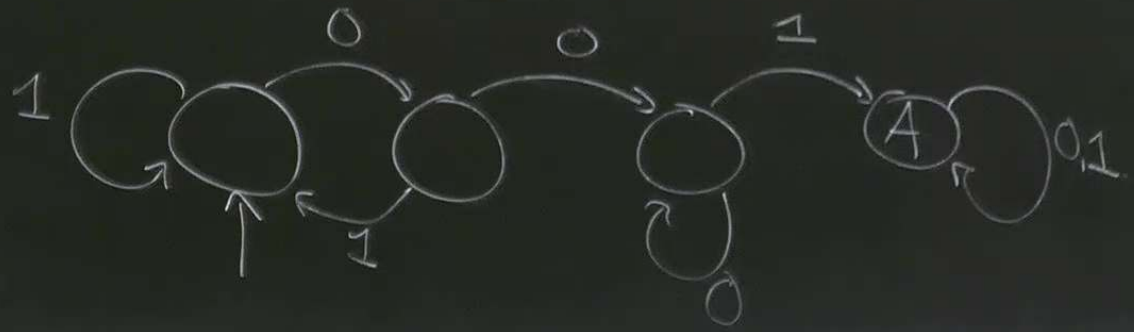


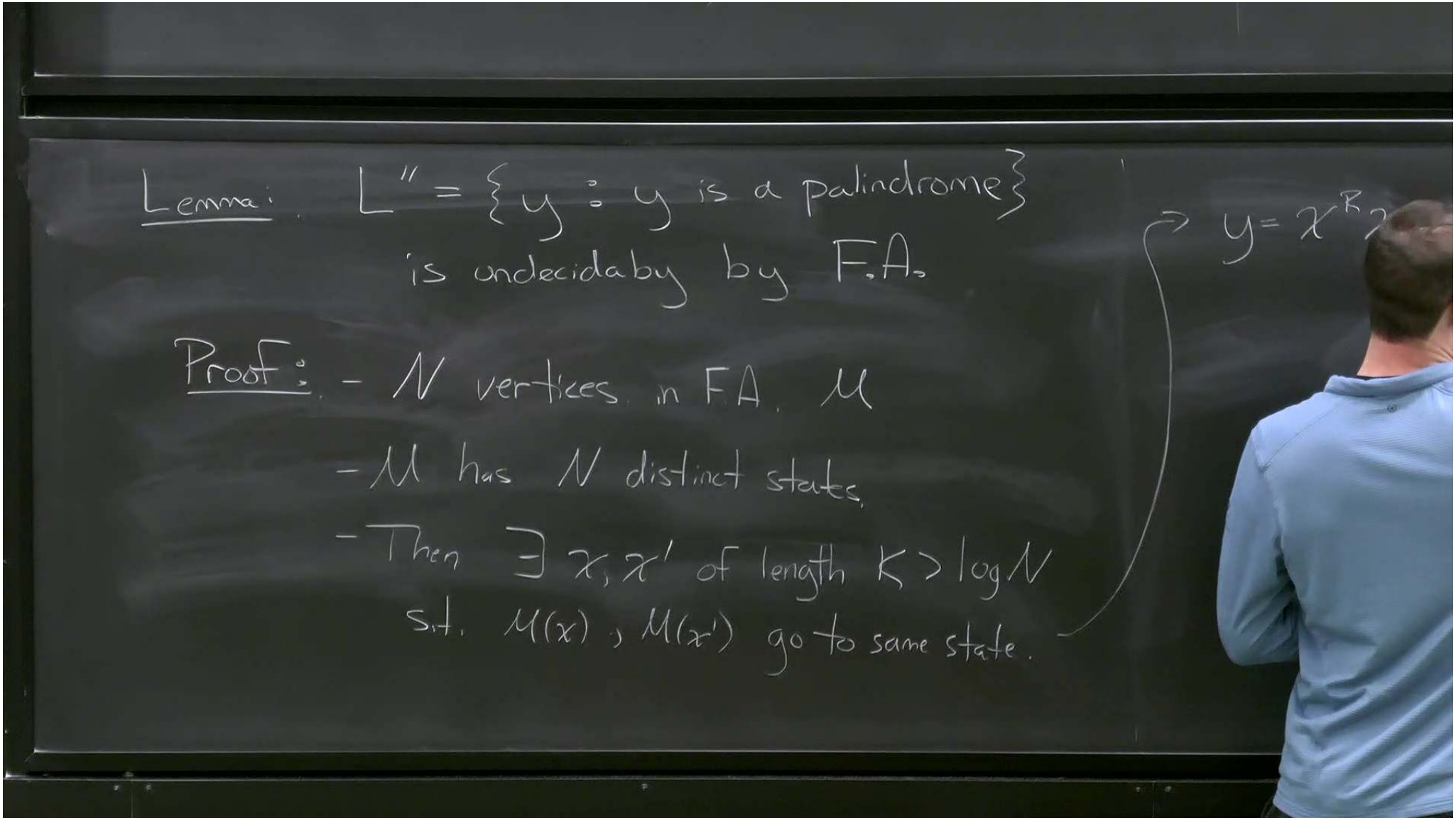
$X_1, X_2 \ldots X_n$

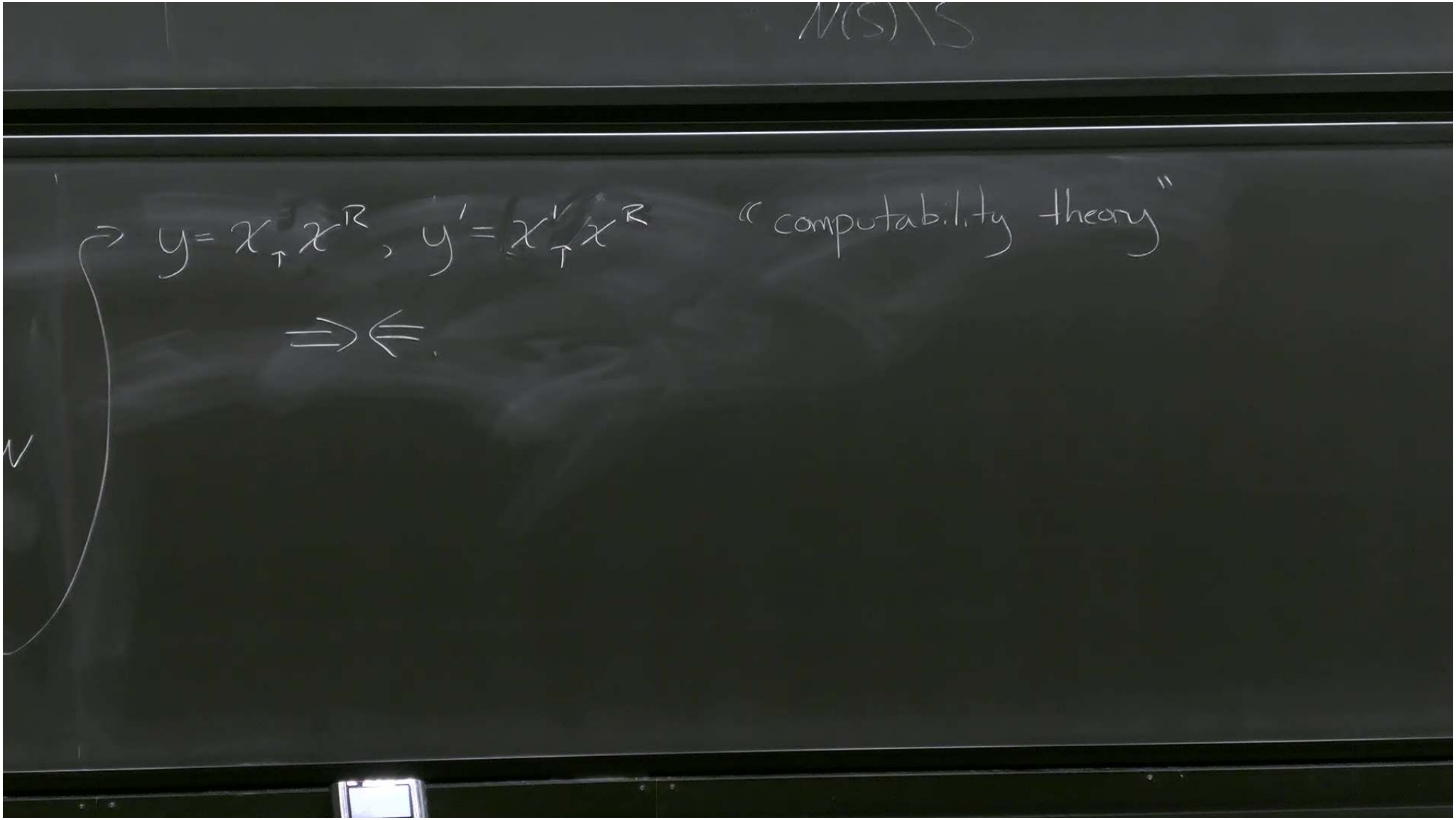

Def| "Language" over an alphabet Σ
is a subset strings formed from elements in Σ
$\{f_n\}_n$.
$f_1 : \{0,1\}^1 \to \{0,1\}^1$
$f_2 : \{0,1\}^2 \to$
f_n

Complexity(L) $\overset{\text{e.g.}}{=}$ minimal # of vertices in any F.A. recognizing L

Exercise] $L' = \{x : x \text{ contains } 001 \text{ as substring}\}$

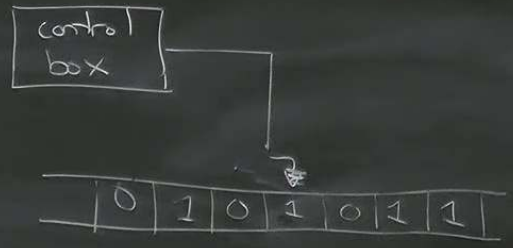



Lemma: $L'' = \{y : y \text{ is a palindrome}\}$
is undecidaby by F.A.
Proof: - N vertices in F.A. M
- M has N distinct states.
- Then $\exists\, x, x'$ of length $K > \log N$
s.t. $M(x)$, $M(x')$ go to same state.
$y = x^R$?


$y = x_{\uparrow} x^R$, $y' = x'_{\uparrow} x^R$
"computability theory"
$\Rightarrow \Leftarrow$

- M has N distinct states. $2^K > N$
- Then $\exists\ x, x'$ of length $K > \log N$ s.t. $M(x)$, $M(x')$ go to same state.

Turing Machine



(current state of box , current value in cell of tape)

↓

(new state of box , new value in current cell , direction move R/W head)