Title: Lecture - Quantum Information I (Elective), 635

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Lecture 5 (extra comments)

$$I(A:B)_\rho \equiv \min_{\sigma_A,\sigma_B} D(\rho_{AB} \| \sigma_A \otimes \sigma_B)$$

$$= D(\rho_{AB} \| \rho_A \otimes \rho_B)$$

$$= S(A)_\rho + S(B)_\rho - S(AB)_\rho$$

$$I(A:B)_{\Psi^+} = 2$$

$$I(A:B)_{\rho_{cl}} = 1$$

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ρ_{A}

Def

Def

$$|\psi\rangle_{AB} \neq |\psi_1\rangle_A \otimes |\psi_2\rangle_B$$

$$\rho_{AB} = \frac{1}{2}\left(|00\rangle\langle 00|_{AB} + |11\rangle\langle 11|_{AB}\right)$$

$\boxed{\text{Def.}}$ State σ_{AB} <u>Separable</u>.

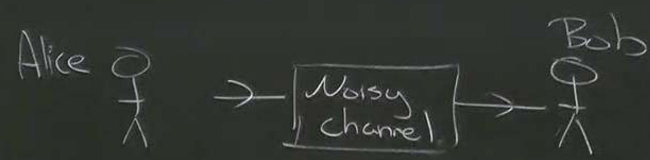
iff $\sigma_{AB} = \sum_i p^i \sigma_A^i \otimes \sigma_B^i$

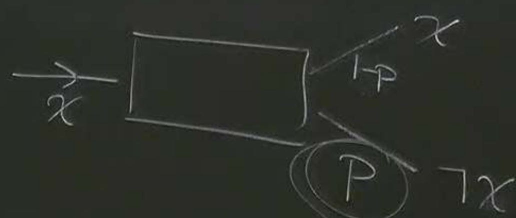
$\boxed{\text{Def}}$ ρ_{AB} entangled if not separable

$$E_R(A:B)_\rho = \min_{\sigma \in Sep} D(\rho_{AB} \| \sigma_{AB})$$

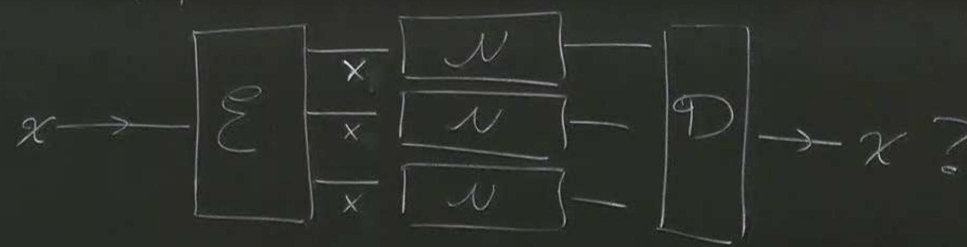
ρ_{AB} entangled if not separable

Lecture 6 - Classical error correction.





repetition



$0 \xrightarrow{\mathcal{E}} 000 \to 101 \to 1$

majority vote

$000 \to 0$

$001 \to 0$

$\vdots$

$110 \to 1$

$p_{fail} \sim 3p^2 < p$

x ?

$\boxed{\text{Def}}$ A classical ECC is defined by:

1) L = "logical space",
2) P = "physical space",
3) $E: L \to P$,
4) Set of allowed errors $\{e_i\}_i$
5) $D: P \to L$

e_i = map acting on P

$\forall x \in L, \forall e_i \quad D(e_i(E(x))) = x$

$110 \to$

$L = \{0,1\}$

$001 \leftrightarrow 100$

$P = \{000, \ldots\}$

$e_1 = \{001, 010, 100, 000, \ldots$ non-linear $\}$

$(1,1,1)+(0,0,1) = (1,1,0)$

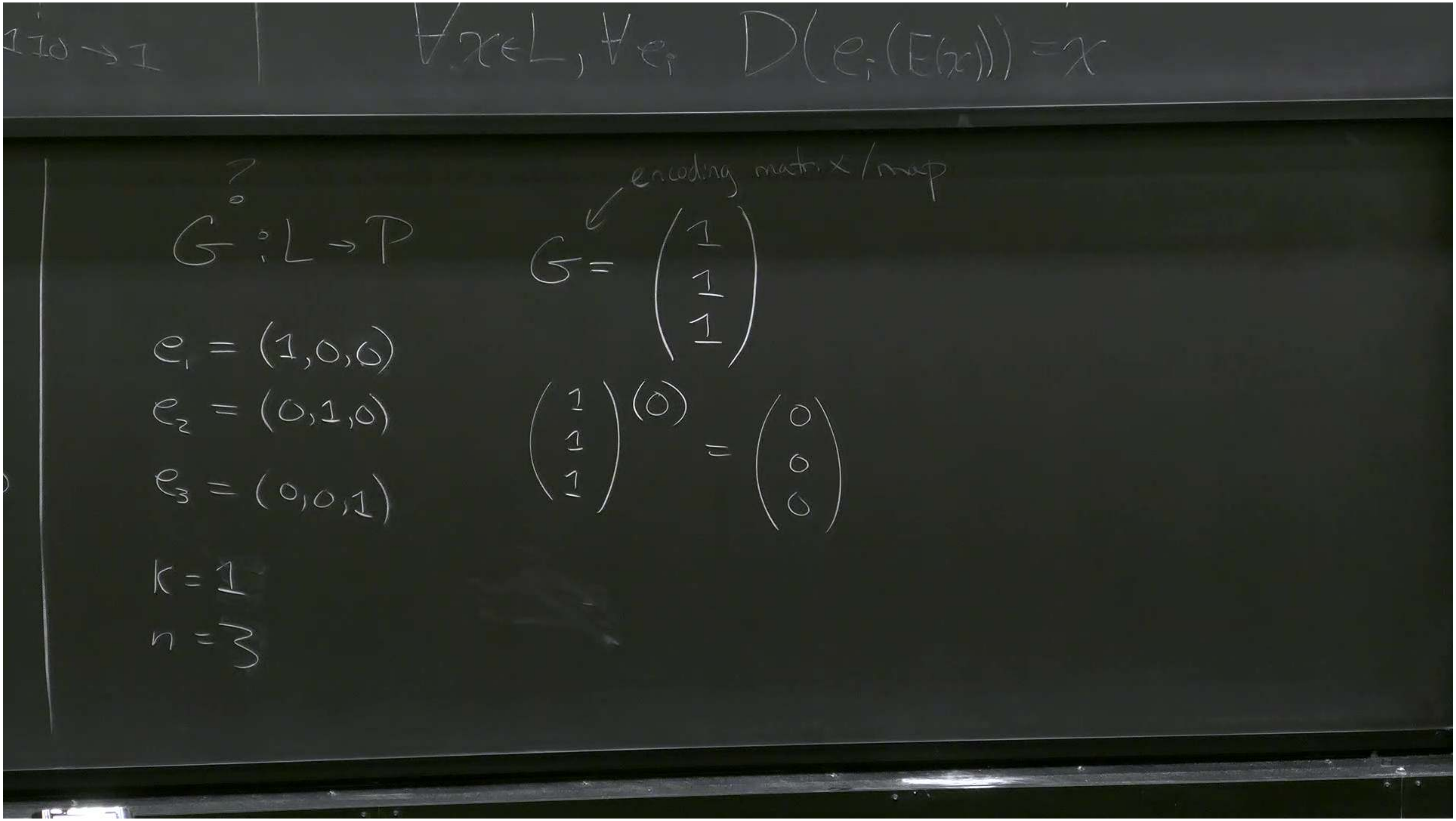
Linear codes

$L = \mathbb{F}_2^k$

$P = \mathbb{F}_2^n$

$E =$ linear map

$e_i(y) = y + e_i$



$110 \to 1$
$\forall x \in L, \forall e_i \; D(e_i(E(x))) = x$
$G \overset{?}{:} L \to P$
encoding matrix / map
$G = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$e_1 = (1,0,0)$
$e_2 = (0,1,0)$
$e_3 = (0,0,1)$
$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
$k = 1$
$n = 3$

"parity check matrix"

$$H \text{ s.t. } \ker H = \operatorname{Im} G \,, \quad HG = 0$$

$$\operatorname{Im} G = \operatorname{span}\{v_1, \dots v_k\}$$

$$\mathbb{F}_2^n = \operatorname{span}\{v_1, \dots v_k, w_{k+1}, \dots w_n\}$$

$$H = \begin{pmatrix} w_{k+1} \\ w_{k+2} \\ \vdots \\ w_n \end{pmatrix}$$

$$H(y+e_i) = H(e_i)$$

Lemma: In Linear code H, errors $\{e_i\}$ can be corrected iff $\{He_i\}$ are all distinct.

Proof 1) distinct $\Rightarrow$ correctable

look at He_i, determine e_i, $(y' + e_i) = (y + e_i) + e_i = y$

2) correctable $\Rightarrow$ distinct.

suppose $\exists\ i, j$ s.t. $He_i = He_j$

$\Rightarrow H(e_i + e_j) = 0$

$$\Rightarrow H(e_i+e_j)=0$$

$$e_i+e_j+y=y'$$

$$\hookrightarrow \quad y+e_i=y'+e_j$$

$$\Rightarrow\Leftarrow$$

$$H=\begin{pmatrix}1&1&0\\1&0&1\end{pmatrix}$$

$$He_1=\begin{pmatrix}1&1\\1&0\end{pmatrix}$$

$$HG=\begin{pmatrix}1&1&0\\1&0&1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$n, k.$

$$He_1 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$HG = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$