Title: Nonlocal correlation in open and closed network scenarios and its application to the communication games. (VIRTUAL)

Speakers: Amit Kundu

Collection/Series: Quantum Foundations

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Abstract:

The study of quantum correlations in multipartite network scenarios represents a significant extension of nonlocality research beyond the standard Bell–CHSH framework. Quantum networks comprise multiple parties interconnected by several independent quantum sources that
distribute quantum states among them. A fundamental question in this context is whether the resulting input–output correlations generated by such networks admit a local classical explanation or not. Unlike standard Bell scenarios, the set of locally reproducible correlations in quantum
networks is generally non-convex. As a consequence, characterising nonlocality becomes highly nontrivial, and only limited classes of nonlinear inequalities are known, primarily for certain open network configurations where some parties have measurement inputs while others do not. For
many important scenarios—such as closed-loop networks and higher-dimensional open networks—no complete set of inequalities is known, making the identification of nonlocal correlations particularly challenging.
In this talk, I will briefly review selected open network scenarios and the corresponding nonlocality inequalities, as well as closed-loop network scenarios where inequality-based methods are unavailable. I will discuss alternative approaches used to determine the nature of correlations in
both cases. Finally, I will present some of our recent results along with an overview of current developments in the field.

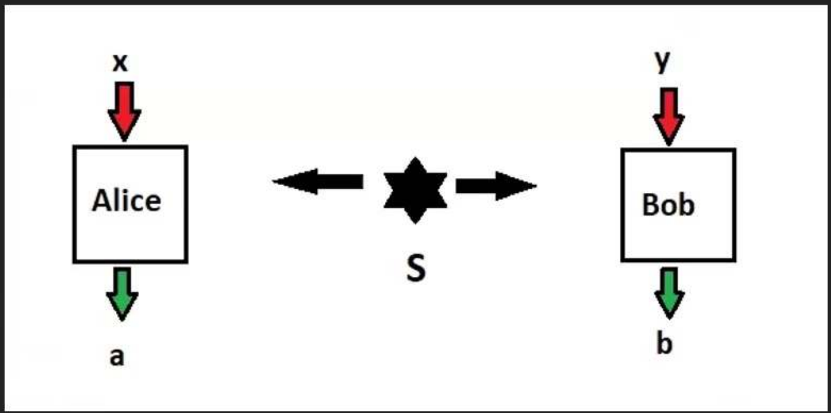
Nonlocal correlation in open and closed network scenarios and its application to the communication games.

Amit Kundu
SNBNCBS, Kolkata, India

Outline of the Talk…

- Define correlation
- What is Nonlocal correlation
- Network formation and its correlations
- Important results and a few recent updates
- Application
- Conclusion

Correlation: Input- Output Statistics



Input choices x for Alice.
Input choices y for Bob.
Output for Alice is a.
Output for Bob is b.

P(a, b | x, y) is the correlation.

Now this *S* can be anything, classical coin, quantum states or any physical resources

How to get correlation?

- The S(Source) can be anything, e.g., a box with a coin,
  colouring objects
  or a Quantum state
- The input choices also have a variety, e.g., Head/Tail, Two/One rupee coin
  Red/Blue, Ball/Cube
  or σ_x/σ_y

Nature of the Correlation

- A correlation is Classical(Local), when, $p_c(ab|xy) = \int p(a|x,\lambda)p(b|y,\lambda)d\lambda\rho(\lambda)$
- A correlation is Quantum, when, $p_Q(ab|xy) = \langle\psi| A_{a}^{x} \otimes B_{b}^{y} |\psi\rangle$
- Here a Important question is, Can the quantum correlation be the classical correlation?

$$p_c(ab|xy) = p_Q(ab|xy) \;?$$

The answer is, No! Not Always…

How to detect?

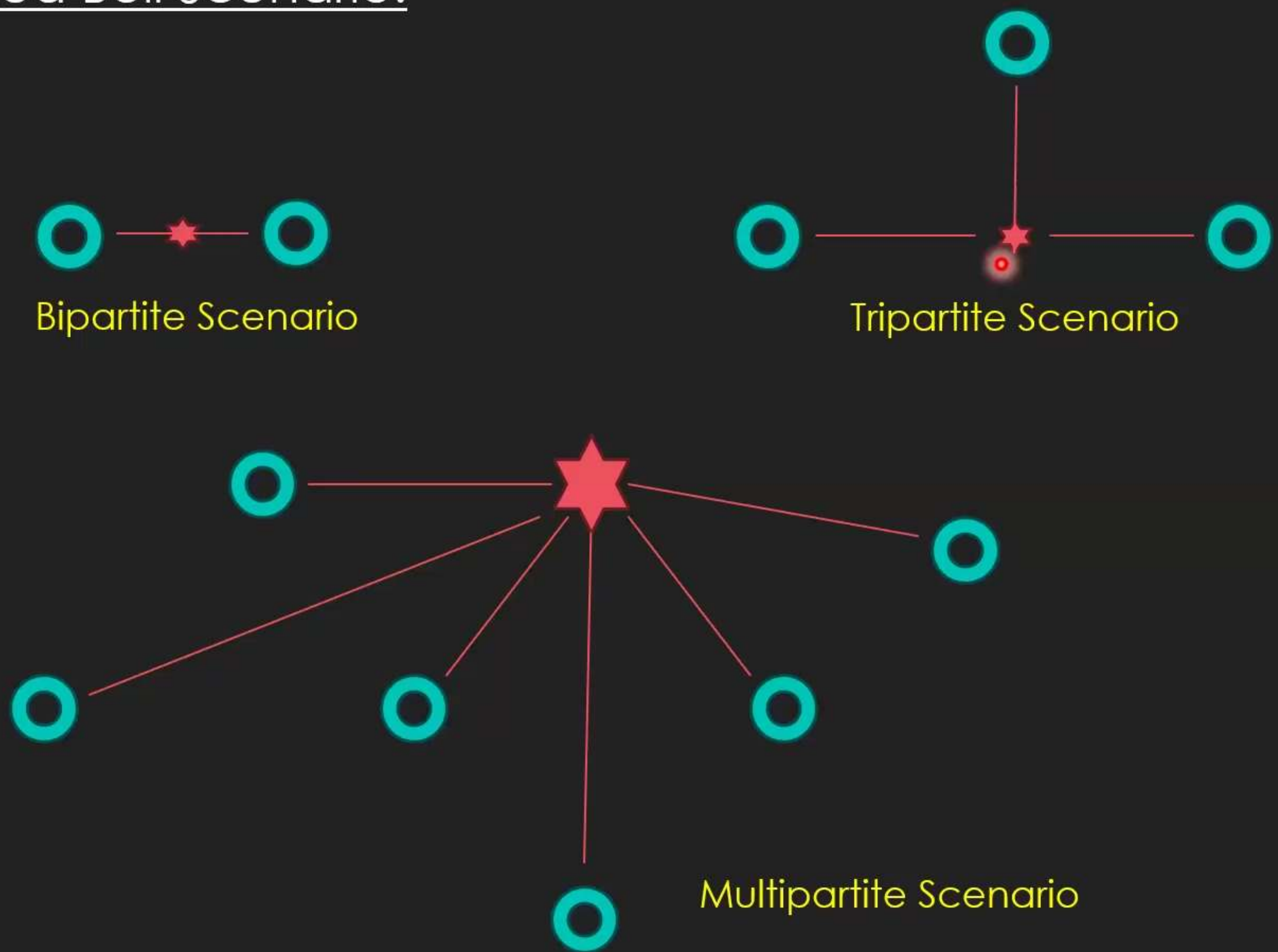
- We have Bell-CHSH Inequality: $|\langle A_0 B_0\rangle + \langle A_0 B_1\rangle + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle| \leq 2^*$

  This inequality defines the local bound for the Two-party, Two-Input, Two-Output Quantum scenario: The violation of this inequality gives the correlation

  Non-Local…

*(Phys. Rev. Lett. **23**, 880, 1969)

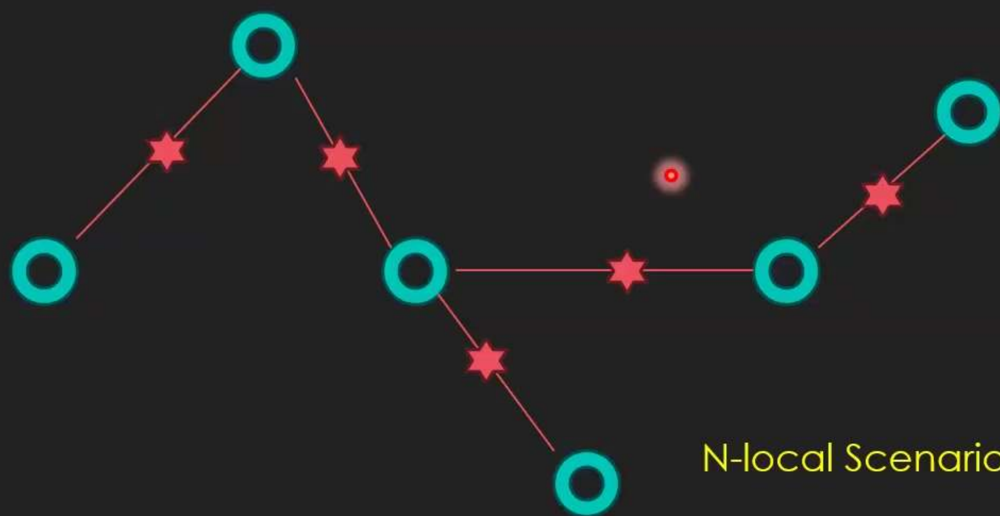
Extended Bell Scenario:



Beyond Bell Scenario:

Bipartite Scenario

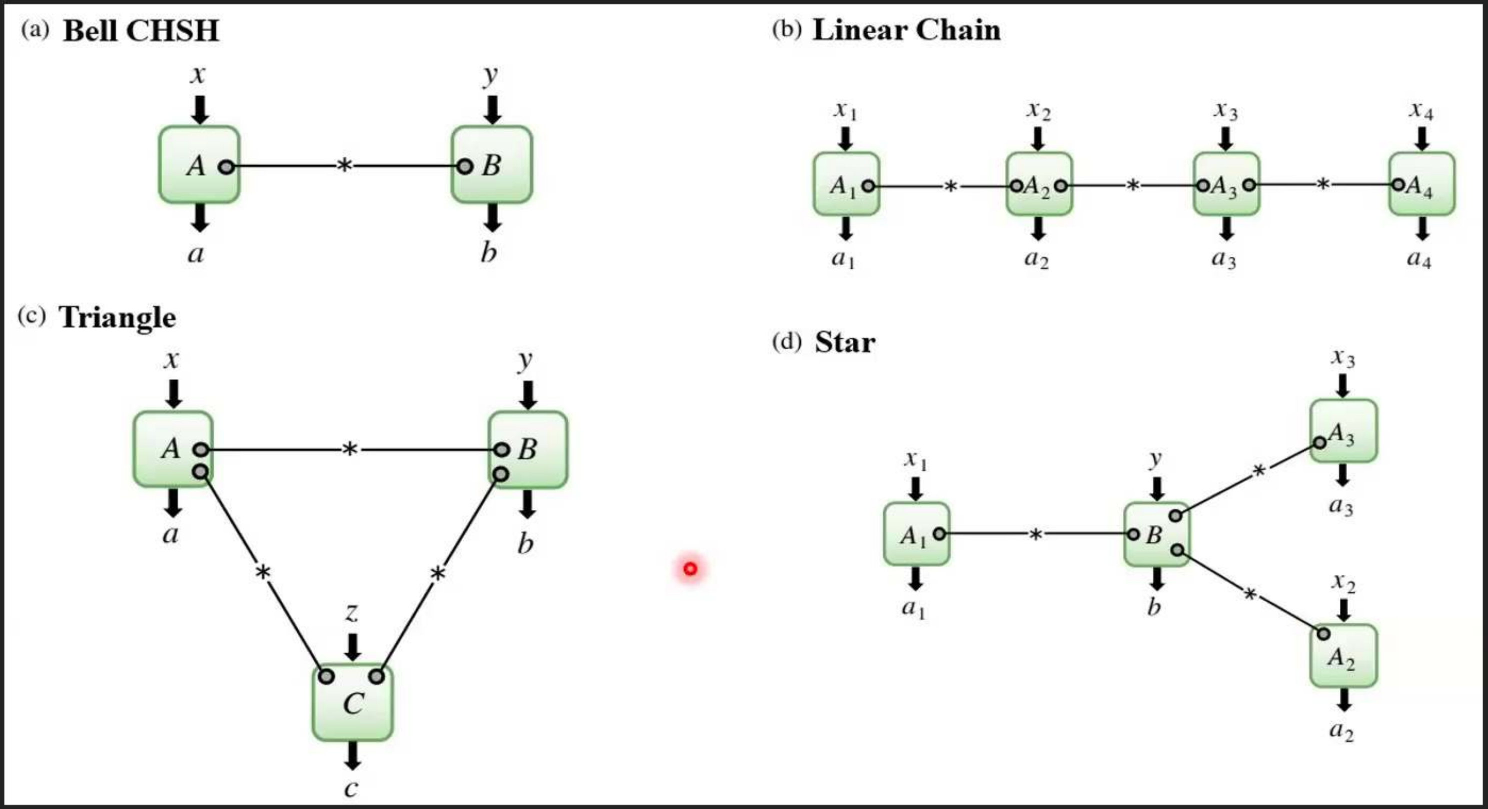
Bilocal Scenario



N-local Scenario

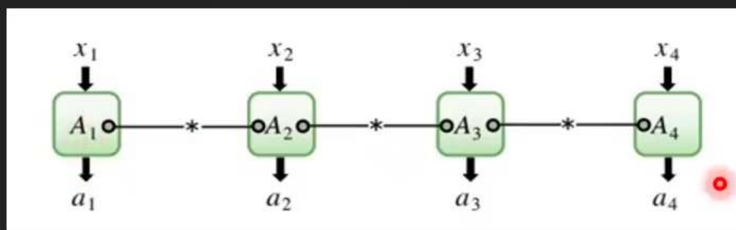
This is what we call the Network Scenario.

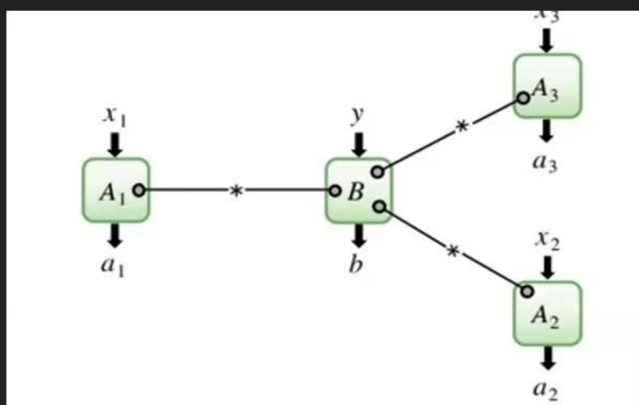
Different networks based on the distribution



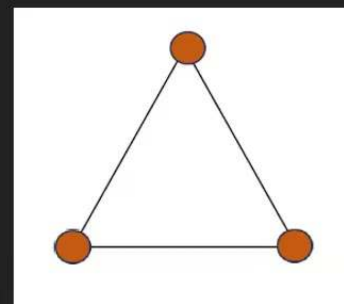
Open and Closed Network

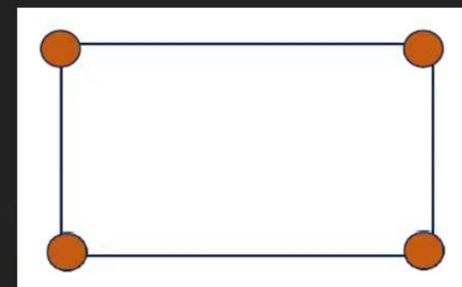
Open Network

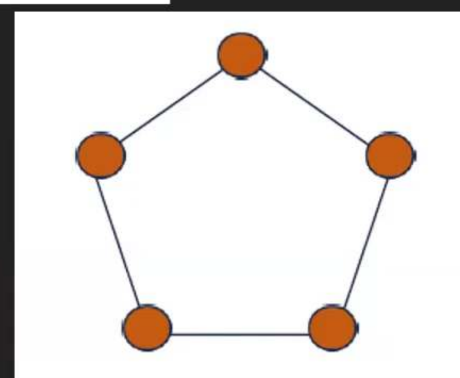




Closed Network

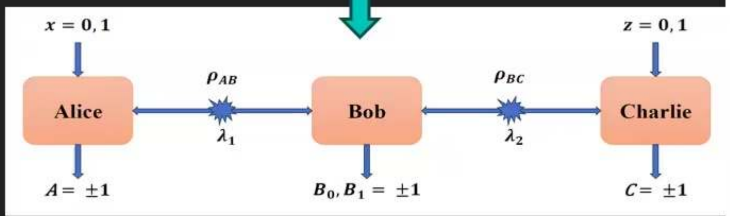






Bilocal Scenario

Bob has two particles,
Uses joint measurement



Source Independence:
$\rho(\lambda_1, \lambda_2) = \rho(\lambda_1)\rho(\lambda_2)$

$$p(a, b, c|x, y, z) = \int d\lambda_1 d\lambda_2 p(\lambda_1)p(\lambda_2)p(a|x, \lambda_1)p(b|y, \lambda_1, \lambda_2)p(c|z, \lambda_2) \quad (3)$$

Bilocal Inequality

$$S_{biloc} = \sqrt{|I|} + \sqrt{|J|} \leq 2$$

Where

$$I = \langle (A_0 + A_1)B_0(C_0 + C_1) \rangle$$

$$J = \langle (A_0 - A_1)B_1(C_0 - C_1) \rangle$$

Correlation that violates Bilocal Inequality

- All Pure entangled states from both sources violate the Bilocal Inequality.

  In analogy, all pure entangled states violate the CHSH inequality.

- What about mixed states?

  If both sources share mixed states in the form,

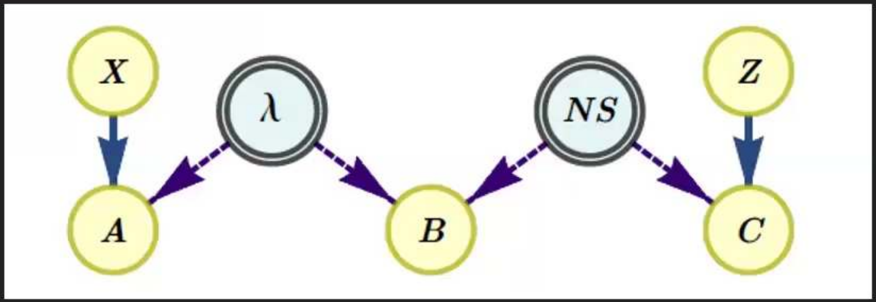
$$\rho_{AB} = \frac{1}{4}\left(\mathbb{1} + \vec{m}_A \cdot \vec{\sigma} \otimes \mathbb{1} + \mathbb{1} \otimes \vec{m}_B \cdot \vec{\sigma} + \sum_{ij} t_{ij}^{AB} \sigma_i \otimes \sigma_j\right)$$

Violation of Bilocal Inequality $S_{\text{biloc}}^{\max} = 2\sqrt{\xi_1 \zeta_1 + \xi_2 \zeta_2}.$

⟶ ρ_{AB} or ρ_{BC} or both violates Bell CHSH inequality.

PHYSICAL REVIEW A **96**, 020304(R) (2017)

What about Genuineness?

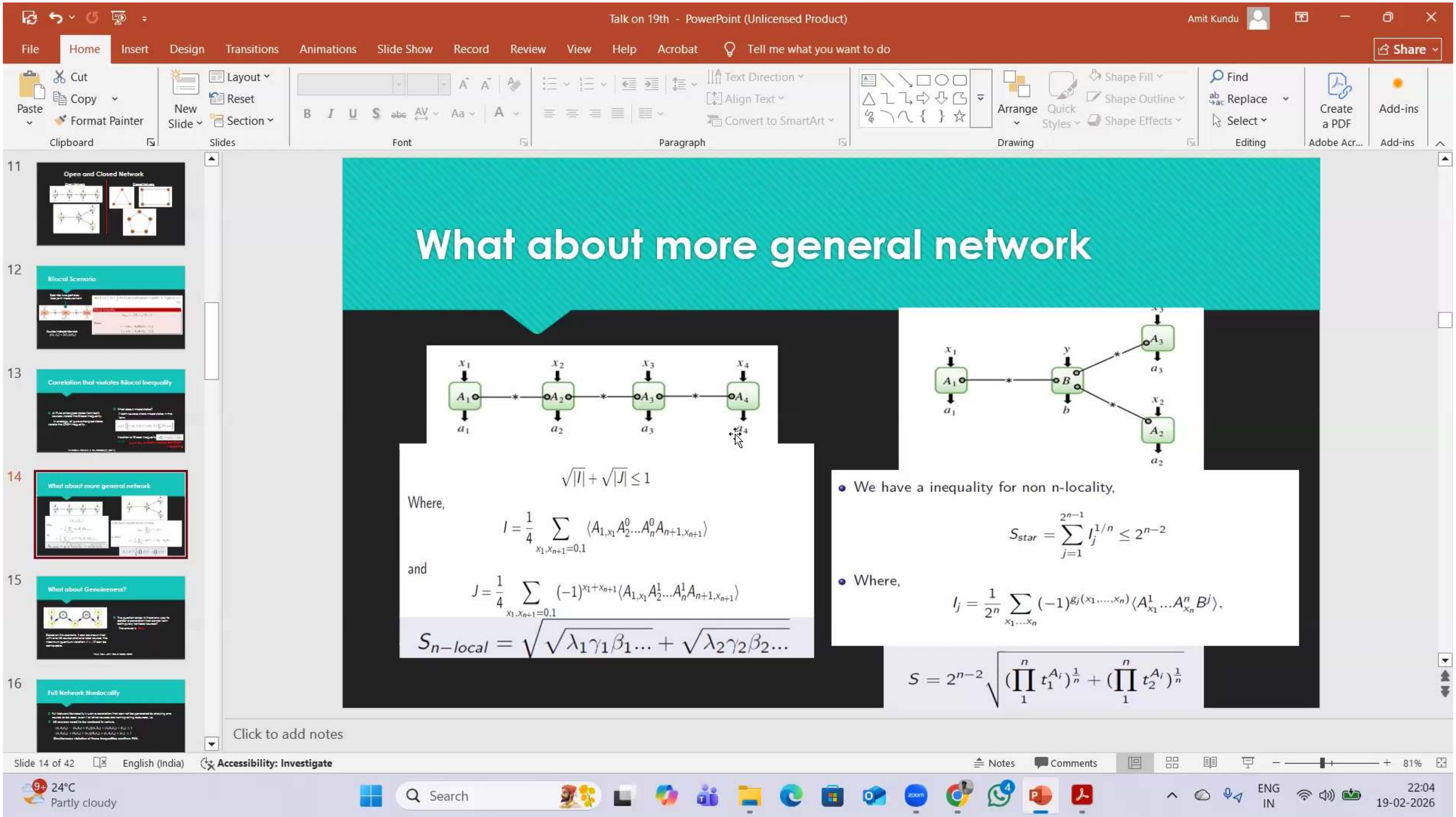


Based on this scenario, it can be shown that with one NS source and one local source, the maximum quantum violation, $S = \sqrt{2}$, can be achievable.

- The question arises: Is there any way to detect a correlation that comes from both purely nonlocal sources?

  The answer is Yes…

Phys. Rev. Lett. **128**, 010403, 2022


Talk on 19th - PowerPoint (Unlicensed Product)
What about more general network
$\sqrt{|I|} + \sqrt{|J|} \leq 1$
Where,
$I = \frac{1}{4} \sum_{x_1, x_{n+1}=0,1} \langle A_{1,x_1} A_2^0 \ldots A_n^0 A_{n+1,x_{n+1}} \rangle$
and
$J = \frac{1}{4} \sum_{x_1, x_{n+1}=0,1} (-1)^{x_1+x_{n+1}} \langle A_{1,x_1} A_2^1 \ldots A_n^1 A_{n+1,x_{n+1}} \rangle$
$S_{n-local} = \sqrt{\sqrt{\lambda_1 \gamma_1 \beta_1 \ldots} + \sqrt{\lambda_2 \gamma_2 \beta_2 \ldots}}$
We have a inequality for non n-locality,
$S_{star} = \sum_{j=1}^{2^{n-1}} I_j^{1/n} \leq 2^{n-2}$
Where,
$I_j = \frac{1}{2^n} \sum_{x_1 \ldots x_n} (-1)^{g_j(x_1, \ldots, x_n)} \langle A_{x_1}^1 \ldots A_{x_n}^n B^j \rangle,$
$S = 2^{n-2} \sqrt{(\prod_1^n t_1^{A_i})^{\frac{1}{n}} + (\prod_1^n t_2^{A_i})^{\frac{1}{n}}}$
Click to add notes
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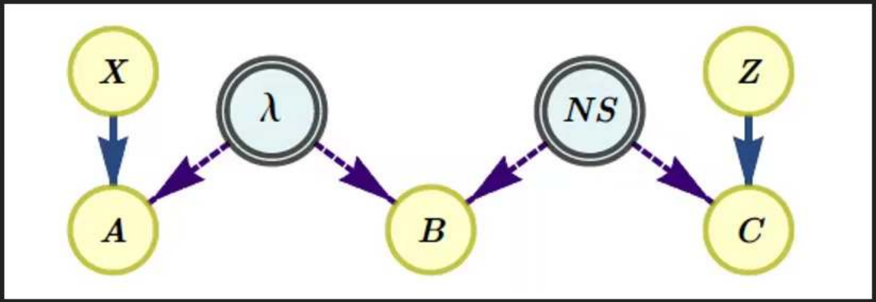
What about Genuineness?



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Phys. Rev. Lett. **128**, 010403, 2022

Full Network Nonlocality

- Full Network Nonlocality is such a correlation that can not be generated by allowing one source to be local, even if all other sources are non-signalling resources. i.e.
- **All sources need to be nonlocal in nature.**

$$-\langle A_1B_2C_3\rangle - \langle A_2B_2\rangle + \langle C_3\rangle[\langle A_1B_2\rangle + \langle A_2B_2C_3\rangle + \langle C_3\rangle \leq 1$$

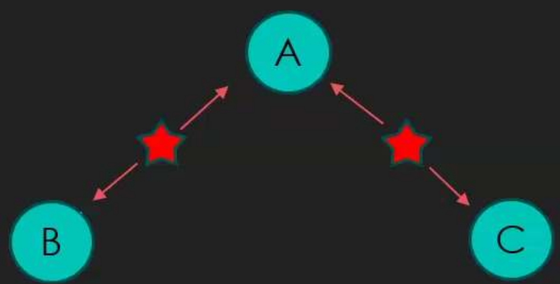
$$-\langle A_1B_2C_3\rangle + \langle B_2C_2\rangle + \langle A_1\rangle[\langle B_2C_3\rangle + \langle A_1B_2C_2\rangle + \langle A_1\rangle \leq 1$$

Simultaneous violation of these inequalities confirms FNN.

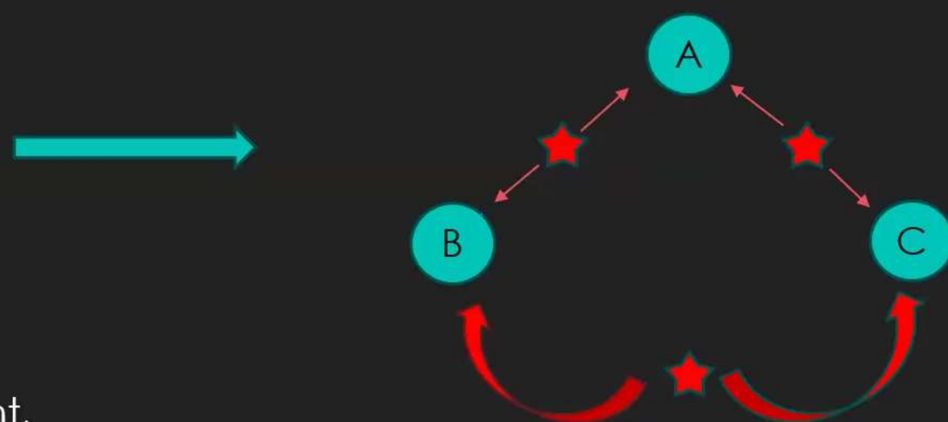
Joint Measurement Bases

- Another important aspect of a network is the joint measurement bases; the bases on which the middle party measures their multiple particle on a single shot.
- We have **BSM**. (Till the discussion)
- Is BSM always advantageous to reveal nonlocality in a network scenario?
- Here we need to go to another network structure scenario, which has structural differences and fundamental differences: **Closed Network**

Closed network(No-Input scenario)



The same old Bilocal Scenario,
Where only Bob has a joint measurement,
i.e., BSM



Three sources distribute states
To three parties, and each party now
Has joint measurement.

Here, the correlation is $P(a, b, c)$

Nonlocality in closed network

- $P(a,b,c)$, is the correlation here in the simplest closed network, triangle network, without any input choices. How do we define nonlocality here?

$$P_C(a, b, c) = \iiint d\alpha d\beta d\gamma \mu(\alpha)\mu(\beta)\mu(\gamma)p(a \mid \beta\gamma)p(b \mid \gamma\alpha)p(c \mid \alpha\beta)$$

- The quantum correlation is: $P_Q(a,b,c) = |<\varphi_a|<\varphi_b|<\varphi_c||\psi_\alpha>|\psi_\beta>|\psi_\gamma>|^2$

The nonlocality in triangle network is $P_C(a, b, c) \neq P_Q(a,b,c)$

Detection is Hard…. As no inequality is there…

Correlation from BSM

- Consider here BSM for each party, Alice, Bob and Charlie, with their two particle from two sources which distribute three maximally entangled bases.

Bases as BSM →

$$\phi^+ = \frac{1}{2}(|00\rangle + |11\rangle)$$

$$\phi^- = \frac{1}{2}(|00\rangle - |11\rangle)$$

$$\psi^+ = \frac{1}{2}(|01\rangle + |10\rangle)$$

$$\psi^+ = \frac{1}{2}(|01\rangle - |10\rangle)$$

+

State as maximally entangled state → The resulting correlation is local*

Entropy **2019**, 21, 325

Elegant Joint Measurement(EJM)

$$\vec{m_1} = (+1,+1,+1)$$
$$\vec{m_2} = (+1,-1,-1)$$
$$\vec{m_3} = (-1,+1,-1)$$
$$\vec{m_4} = (-1,-1,+1)$$

Four vertices of the tetrahedron inscribed in the Poincaré sphere.

$$|\Phi_j\rangle = \sqrt{\frac{3}{2}}|\vec{m}_j, -\vec{m}_j\rangle + i\frac{\sqrt{3}-1}{2}|\psi^-\rangle$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}|\vec{m}_j, -\vec{m}_j\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}|-\vec{m}_j, \vec{m}_j\rangle,$$

The elegant properties come from,

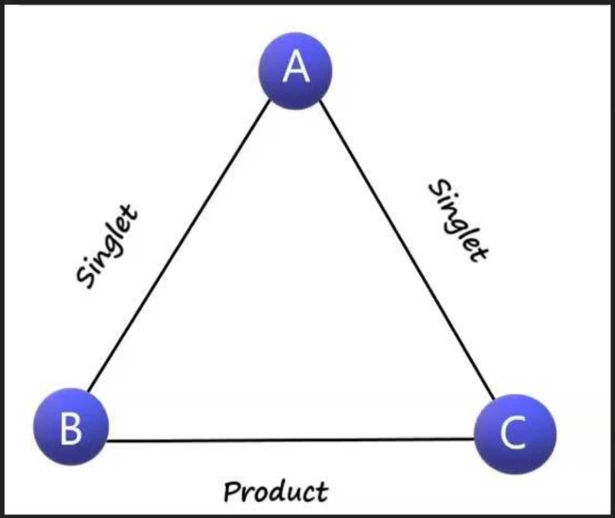
$$\langle\Phi_j|\vec{\sigma}\otimes\mathbb{1}|\Phi_j\rangle = \frac{1}{2}\vec{m}_j$$
$$\langle\Phi_j|\mathbb{1}\otimes\vec{\sigma}|\Phi_j\rangle = -\frac{1}{2}\vec{m}_j.$$

EJM as a basis and maximally entangled states from each source to the parties Creates Non-Trilocal correlation(Gisin's conjecture).

Later, the neural network method supports Gisin's Conjecture numerically* that $P_{QEJM}(a, b, c)$ is **nontrilocal**.

*npj Quantum Information (2020) 6:70

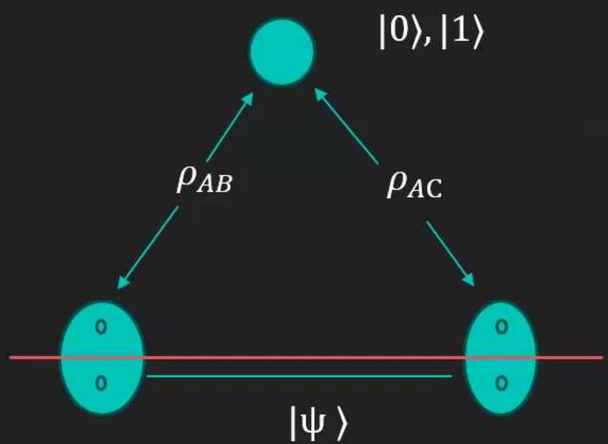
What if not all states from the sources are Entangled?



- Three maximally Entangled states → Nonlocal
- Two maximally entangled, one product state → Nonlocal
- Two products, one maximally entangled → Local

Fritz Distribution*

New J. Phys. **14** 103001,2012



$$\rho\_\{AB\} = \rho\_\{AC\} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)/\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

$|\psi\rangle$ to be a bipartite two-qubit state which violates the CHSH inequality with respect to measurements in the two bases $\{|\phi_0\rangle, |\phi_1\rangle\}$, $\{|\omega_0\rangle, |\omega_1\rangle\}$, which are the same for both parties.

This can be mapped in a Bell scenario, and the correlation is nonlocal. But it is not a general prescription.

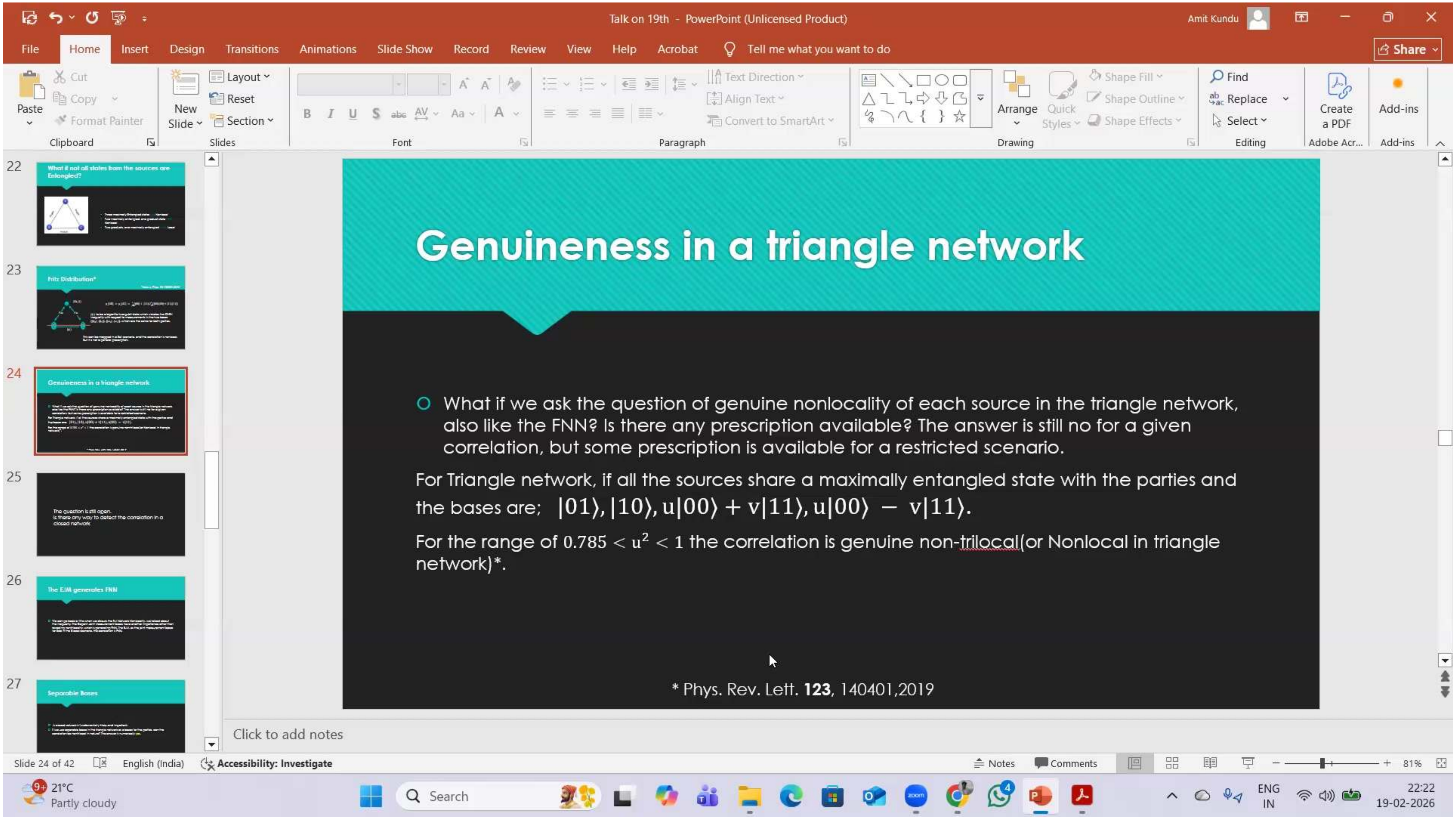
Genuineness in a triangle network

- What if we ask the question of genuine nonlocality of each source in the triangle network, also like the FNN? Is there any prescription available? The answer is still no for a given correlation, but some prescription is available for a restricted scenario.

For Triangle network, if all the sources share a maximally entangled state with the parties and the bases are; $|01\rangle, |10\rangle, \mathrm{u}|00\rangle + \mathrm{v}|11\rangle, \mathrm{u}|00\rangle - \mathrm{v}|11\rangle$.

For the range of $0.785 < \mathrm{u}^2 < 1$ the correlation is genuine non-trilocal(or Nonlocal in triangle network)*.

* Phys. Rev. Lett. **123**, 140401,2019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The EJM generates FNN

- We can go back a little when we discuss the Full Network Nonlocality, we talked about the inequality. The Elegant Joint Measurement bases have another importance other than revealing nontrilocality, which is generating FNN. The EJM, as the joint measurement bases for Bob in the Bilocal scenario, the correlation is FNN.

Separable Bases

- A closed network is fundamentally tricky and important.
- If we use separable bases in the triangle network as a bases for the parties, can the correlation be nontrilocal in nature? The answer is numerically yes.

Nonlocality possible with separable bases in Closed Network?

- Considering the No-input minimal triangle network with cardinality 3-3-3.
- Each party have two particles from two different independent sources.
- Each party measure both the particle with a fixed measurement basis.
- This time, the basis is separable, and all the sources distribute maximally entangled states
- $P_Q(a, b, c)$ is Local or Not?

$$\pi_1 = |\psi_1\rangle\langle\psi_1|$$

$$\pi_2 = |\psi_2\rangle\langle\psi_2| + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$$

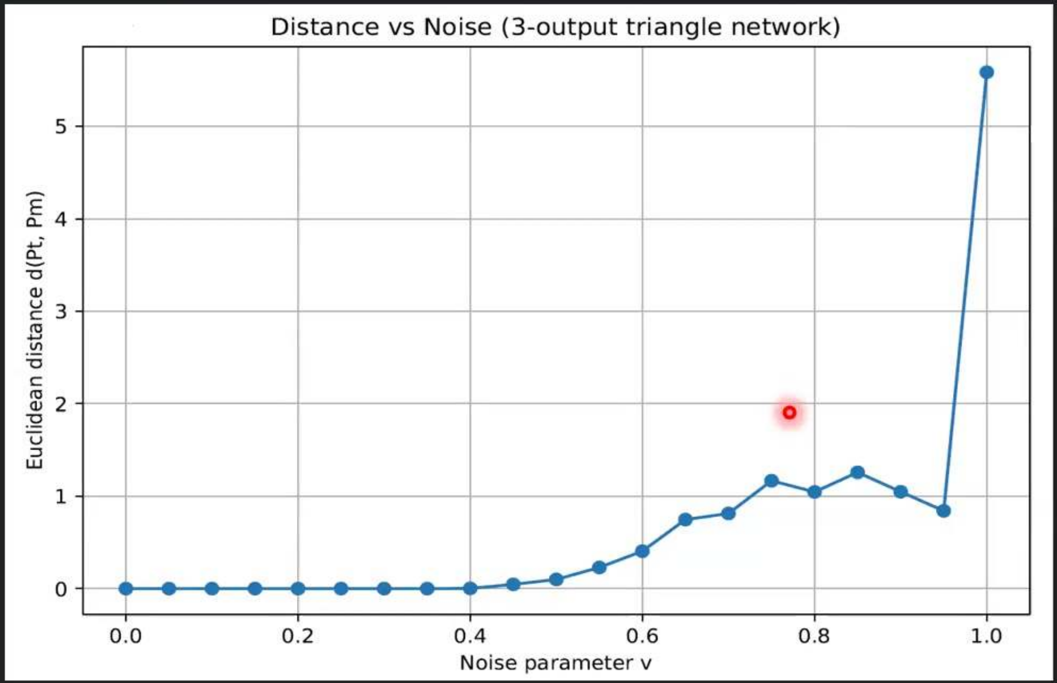
$$\pi_2 = |\psi_2\rangle\langle\psi_2| + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$$

Where*,

- $\psi_1 = |01>$
- $\psi_2 = (|\phi^+\rangle - |10\rangle)/\sqrt{2}$
- $\psi_3 = (|\phi^+\rangle + |10\rangle)/\sqrt{2}$

*arXiv:2501.15807

Neural Network Method as evidence



So…

- Network is important in both manners, first, its fundamental importance, i.e., correlation for a complex graph for multiple parties, more close to a real-world scenario.
- Second, it is the basic building blocks of the Quantum internet and Quantum repeater, i.e., important for long-distance communication.
- There are a lot of open areas to explore, and very importantly, the correlation structure of a closed network.

Closed network has a real-world application…

Mozart Café problem:

This is a rendezvous search task!

What is a Rendezvous* task?

- A task of meeting at a point without communication. Two or more persons, robots, cars, drones, etc., are targeted to meet at a dedicated point.
- There are some restrictions, based on which they have to meet with a high success probability or within a finite time.
- The party or agents don't know another person's location and movement.
- They cannot communicate with others(most important).
- The scenario can be anything.

* "The Theory of Search Games and Rendezvous" by Steve Alpern, Shmuel Gal

The scenarios:

- The Graph:
  - The graph can be discrete or continuous.
  - It can be Line(1D) or Ring(2D) or infinite grid or complicated graph.
  - Vertex has no unique ID, either.

- Symmetric or Asymmetric
  - Symmetric: No unique ID to the parties + they will follow the same strategy.
  - Asymmetric: Unique ID may or may not be given; they will follow different strategy.
  - Initial position can be known or unknown.
  - Randomised walk strategy or any fixed strategy.

1. R. Klasing, E. Markou, and A. Pelc, Theoretical Computer Science 390, 27 (2008), ISSN 0304-3975.
2. S. Bhagat, A. Chakraborty, B. Das, and K. Mukhopadhyaya, CoRR abs/2202.03350 (2022).

Our scenario:

- The Graph:
  - The graph can be discrete or continuous.
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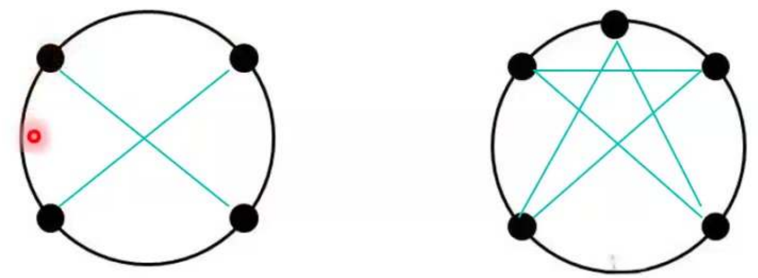
NB: We consider here N>2(gathering), and they will meet at a single shot.

2D Ring Graph:

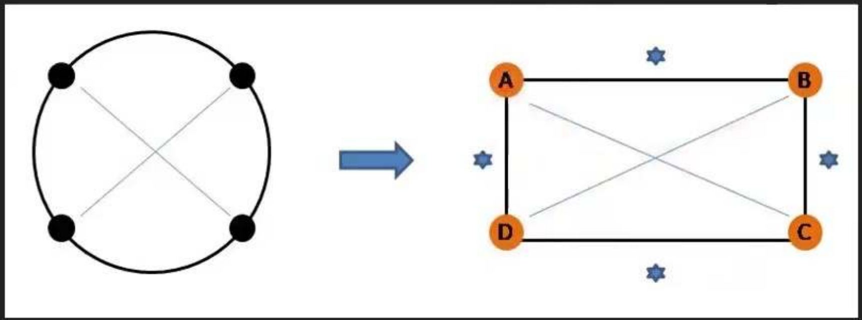


a. Ring Graph with even number of agents.

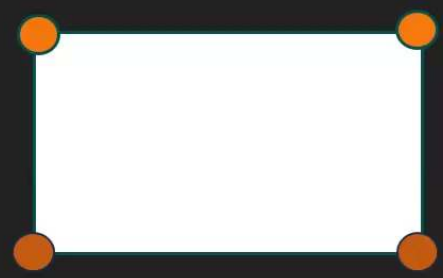
b. Ring Graph with odd number of agents.

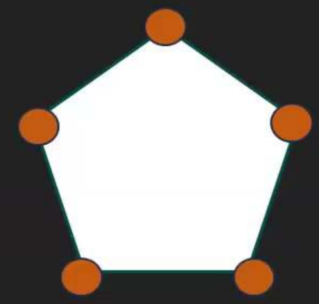
- A ring graph with 4/5 agents(black dots).
- Each of their position is a valid meeting point.
- Each point is connected to the others.
- The goal: They all have to move to meet in a single shot or single step.
- (a) They have four options: stay or move to one of the other 3 paths. For(b), they have five options.
- On randomised walk strategy, on a single point the meeting probability is $\frac{1}{4\times4\times4\times4} = \frac{1}{256}$

Quantum closed loop network:



- The ring graph is cast as a quantum square network with 4 parties.
- Every two parties(AB, BC, CD, DA) share quantum states from four independent sources.
- Every party measure a single measurement on a joint measurement basis on their particles from two different states.
- The four outcomes will now decide which path they will choose.
- And we got an advantage.



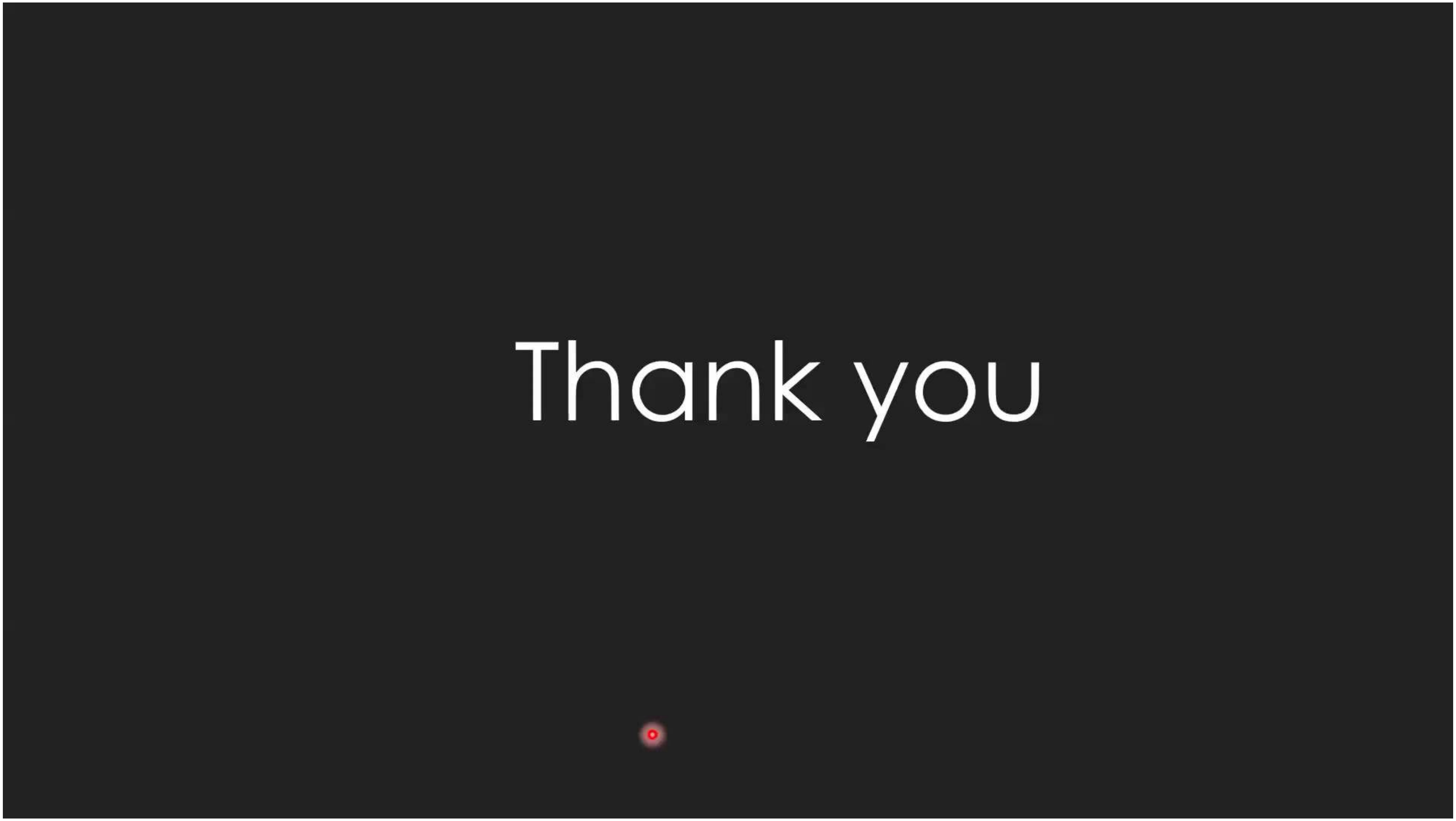


	4 agents in Ring:	5 agents in a Ring:
	Q Advantage: 24.9%	Q Advantage: 20%

- *This applies to higher polygon structures also.

Conclusion:

- We go beyond the Bell scenario.
- Multiple parties, multiple sources and joint measurement are important elements.
- Finding the correlation and its nonlocality is the main task here.
- More applications of these structures need to be studied here.
- Finding detection tools in the closed network is another open problem here.
- Joint measurement bases and their generalisation are important aspects of network nonlocality.
- Full network nonlocality for more general, dimension independent networks should also be an important aspect.


Thank you