Title: Towards the Quantization of Cylindrically Symmetric Spacetimes

Speakers: Javier Peraza

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Abstract:

In this talk, we will provide an overview of recent developments in the quantization of Cylindrically symmetric spacetimes, which contain the Einstein-Rosen gravitational waves. This class of solutions leads to an integrable field theory, and its phase space can be expressed directly in terms of initial data living on a null generator. The corresponding Poisson algebra appears in the double null sheet formulation of full 3+1 gravity (Reisenberger, 2007), as essentially a Poisson subalgebra on each null generator. This suggests that the algebraic quantization introduced by Korotkin-Samtleben in the context of cylindrically symmetric gravity is relevant for the search for the full quantum theory. We will present some results concerning this quantum algebra, where two Wick-type algebraic structures emerge. Finally, we will discus the known quantization of the Einstein-Rosen waves with one polarization, due to Ashtekar-Pierri, as a subsector of the general two polarizations case.

Towards the Quantization of Cylindrically Symmetric Spacetimes

Javier Peraza

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QG Seminar

Work in progress: D. Korotkin (Concordia University, Montreal, Canada)
M. Reisenberger (Universidad de la República, Uruguay)

Outline

* Motivation
* Classical Theory
* Properties of the Algebraic Quantization
* Outlook

1. Motivation

* Poisson structure in phase space of Double Null Sheets as initial data hypersurfaces (Reisenberger, arxiv: 0712.2541)

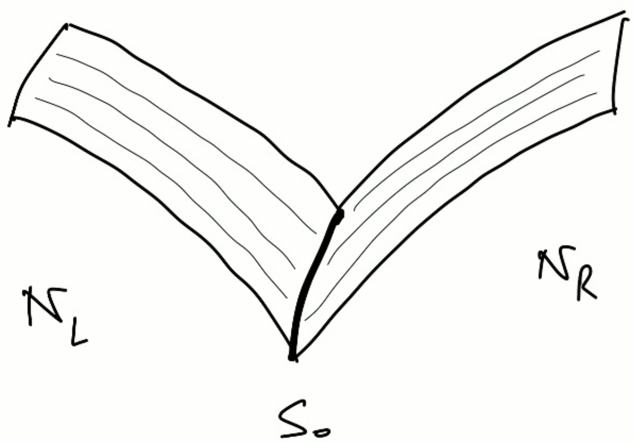
* Integrability properties of σ-models from dimensionally reduced gravity (Belinskii, Zakharov, Neugebauer, Geroch, ...)

* Algebraic Quantization in Cylindrically Symmetric Spaces (Korotkin, Samtleben, arxiv: 9710210)

* Quantization in exactly soluble midisuperspaces (Ashtekar, Pierri, arxiv: 9606085 ; Kuchař, PRD, 4: 955, 1971)

2. Classical Theory.

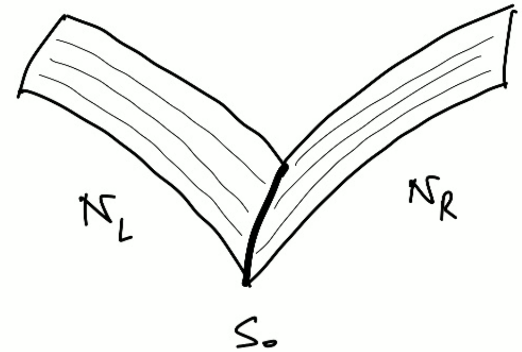
- Poisson structure in phase space of Double Null Sheets as initial data hypersurfaces (Reisenberger, arxiv: 0712.2541, 070314, 1211.3880, 1304.10284)



N_R and N_L are <u>null</u> 3-surfaces swept out by null geodesics emerging normally (n_L, n_R) from a 2-disk S_0, with area A_0.

- θ^1, θ^2: coordinates on S_0 (constant along generators)
- v : parameter along generators such that the cross sectional area of local neighboring generators is $A(v) = A_0\, v^2$.

Free Data on Double Null Sheet



* "Bulk" data on N_L and N_R:

$e_{ab}(\theta^1, \theta^2, v) \rightarrow$ conformal 2-metric

$$ds^2 = h_{ab}\, d\theta^a d\theta^b = \frac{\rho}{1-\mu\bar{\mu}} (dz + \mu d\bar{z})(d\bar{z} + \bar{\mu} dz)$$

$$e_{ab} = h_{ab} / \sqrt{\det h}$$

Beltrami differential (μ)

* "Surface" data on S_0:

$\rho_0(\theta^1, \theta^2) \rightarrow$ area density on S_0

$\lambda(\theta^1, \theta^2) \rightarrow$ conformal factor on S_0: $-\ln |n_L \cdot n_R|$

$\tau_a(\theta^1, \theta^2) \rightarrow$ twist constants on S_0: $\dfrac{n_L \cdot \nabla_a n_R - n_R \cdot \nabla_a n_L}{n_L \cdot n_R}$

Poisson brackets for free data:

Brackets not shown vanish.

$$\begin{aligned}\{\mu(\mathbf{1}), \bar{\mu}(\mathbf{2})\} &= 4\pi G\,\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)\,H(\mathbf{1},\mathbf{2})\Big[\frac{1-\mu\bar{\mu}}{v_A}\Big]_{\mathbf{1}} \\ &\quad \times\Big[\frac{1-\mu\bar{\mu}}{v_A}\Big]_{\mathbf{2}}\, e^{\int_{\mathbf{1}}^{\mathbf{2}}(\bar{\mu}d\mu-\mu d\bar{\mu})/(1-\mu\bar{\mu})}\end{aligned}$$

for $\mathbf{1}$, $\mathbf{2}$ in the same branch, $\mathcal{N}_A$.

$$\begin{aligned}\{\rho_0(\theta_1), \lambda(\theta_2)\} &= 8\pi G\delta^2(\theta_2-\theta_1) \\ \{\rho_0(\theta), \tau[f]\} &= -8\pi G \pounds_f \rho_0(\theta) \\ \{\lambda(\theta), \tau[f]\} &= -8\pi G\Big[\pounds_f\lambda + \frac{\pounds_f\mu}{(1-\mu\bar{\mu})^2}(\partial_{v_R}\bar{\mu}-\partial_{v_L}\bar{\mu})\Big]_\theta\end{aligned}$$

$$\begin{aligned}\{\tau[f_1], \tau[f_2]\} = -16\pi G\Big[\tau[[f_1,f_2]] - \frac{1}{2}\int_{S_0}\pounds_{[f_1,f_2]}\epsilon \\ + \int_{S_0}\big[\frac{\pounds_{f_1}\mu}{(1-\mu\bar{\mu})^2}\{\epsilon\pounds_{f_2}\bar{\mu} - \frac{1}{2}\pounds_{f_2}\epsilon(\partial_{v_R}\bar{\mu}+\partial_{v_L}\bar{\mu})\} - (1\leftrightarrow 2)\big]\Big].\end{aligned}$$

For $\mathbf{1}$ in $\mathcal{N}_R - S_0$

$$\{\mu(\mathbf{1}), \lambda(\theta_2)\} = 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)[v_R\partial_{v_R}\mu]_{\mathbf{1}}$$

$$\{\mu(\mathbf{1}), \tau[f]\} = -16\pi G\Big[\pounds_f\mu - \frac{1}{4}\frac{\pounds_f\rho_0}{\rho_0}v_R\partial_{v_R}\mu\Big]_{\mathbf{1}}.$$

For $\mathbf{1}$ in S_0

$$\begin{aligned}\{\mu(\mathbf{1}), \lambda(\mathbf{2})\} &= 0 \\ \{\mu(\mathbf{1}), \tau[f]\} &= -8\pi G[\pounds_f\mu]_{\mathbf{1}}.\end{aligned}$$

For $\mathbf{1}$ in $\mathcal{N}_L - S_0$

$$\begin{aligned}\{\mu(\mathbf{1}), \lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)[v_L\partial_{v_L}\mu]_{\mathbf{1}} \\ \{\mu(\mathbf{1}), \tau[f]\} &= -4\pi G\Big[\frac{\pounds_f\rho_0}{\rho_0}v_L\partial_{v_L}\mu\Big]_{\mathbf{1}}.\end{aligned}$$

For $\mathbf{1}\in\mathcal{N}_R$ (including $\mathbf{1}\in S_0$)

$$\begin{aligned}\{\bar{\mu}(\mathbf{1}), \lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)\Big[(v_R\partial_{v_R}\bar{\mu})_{\mathbf{1}} \\ &\quad + \Big(\frac{1}{v_R}\Big)_{\mathbf{1}} e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar{\mu})/(1-\mu\bar{\mu})}(\partial_{v_L}\bar{\mu})_{\mathbf{1}_0}\Big] \\ \{\bar{\mu}(\mathbf{1}), \tau[f]\} &= -8\pi G\Big[\Big(2\pounds_f\bar{\mu} - \frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}v_R\partial_{v_R}\bar{\mu}\Big)_{\mathbf{1}} \\ &\quad - \Big(\pounds_f\bar{\mu} - \frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}\partial_{v_L}\bar{\mu}\Big)_{\mathbf{1}_0}\Big(\frac{1}{v_R}\Big)_{\mathbf{1}} e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar{\mu})/(1-\mu\bar{\mu})}\Big]\end{aligned}$$

where $\mathbf{1}_0\in S_0$ is the origin of the generator through $\mathbf{1}$.
For $\mathbf{1}\in\mathcal{N}_L$

$$\begin{aligned}\{\bar{\mu}(\mathbf{1}), \lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)\Big[(v_L\partial_{v_L}\bar{\mu})_{\mathbf{1}} \\ &\quad + \Big(\frac{1}{v_L}\Big)_{\mathbf{1}} e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar{\mu})/(1-\mu\bar{\mu})}(\partial_{v_R}\bar{\mu})_{\mathbf{1}_0}\Big] \\ \{\bar{\mu}(\mathbf{1}), \tau[f]\} &= -8\pi G\Big[\Big(\frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}v_L\partial_{v_L}\bar{\mu}\Big)_{\mathbf{1}} \\ &\quad + \Big(\pounds_f\bar{\mu} - \frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}\partial_{v_R}\bar{\mu}\Big)_{\mathbf{1}_0}\Big(\frac{1}{v_L}\Big)_{\mathbf{1}} e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar{\mu})/(1-\mu\bar{\mu})}\Big].\end{aligned}$$

(Reisenberger, PRL 101:211101, 2008)

* Only data on same generator have non-zero bracket !!

* "One generator algebra": $\delta(\theta_2-\theta_1)$ removed, $\mu, \bar{\mu}$ functions on single line

$$\{\mu(1), \mu(2)\} = \{\bar{\mu}(1), \bar{\mu}(2)\} = 0$$

$$\{\mu(1), \bar{\mu}(2)\} = \frac{4\pi G}{\sqrt{\rho_1 \rho_2}} \delta(\theta_2-\theta_1) H(1,2) [1-\mu\bar{\mu}]_1 [1-\mu\bar{\mu}]_2 e^{\int_1^2 \frac{\bar{\mu}d\mu - \mu d\bar{\mu}}{1-\mu\bar{\mu}}}$$

where $H(1,2)=1$ if 2 follows 1 along the generator, 0 otherwise.

* Poisson algebra of $\mu, \bar{\mu}$ in cylindrically symmetric gravity.
  Quantization exists (Korotkin, Samtleben '98).

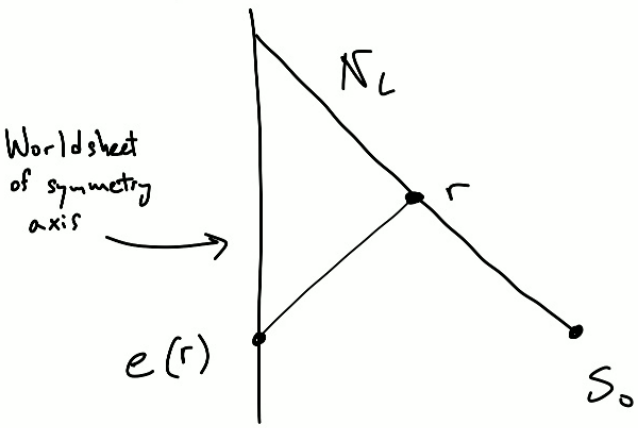
* Cylindrically Symmetric General Relativity is an integrable system.

* Korotkin & Samtleben (KS, arXiv: 9710210) used this machinery to obtain "good variables" with which to quantize the system.

* Going from Double Null Sheet Data to KS variables:

  Reisenberger & Fuchs, arXiv: 1704.06992

* Good variables: Monodromy matrix $M(\omega)$



* Let ρ be the area density of cylinders ($\Box\rho = 0$).

* $\rho = \rho^+ + \rho^-$, null decomposition of ρ.

* Let ω be the spectral parameter of the theory, which is related to ρ by $\omega = 2\rho(r) - \rho^+$

Then, at the axis,

$$M(\omega) = e(r(\omega)) = \underbrace{\hat{V}(r(\omega); \omega + i0)}_{\text{deformed zweibein}} \hat{V}^t(r(\omega); \omega - i0)$$

* A (non-trivial) calculation shows

$$\left\{ \overset{1}{M}(\sigma), \overset{2}{M}(\omega) \right\} = p.v. \left(\frac{32\pi G'}{\sigma - \omega} \right) \overset{1}{Sym}\, \overset{2}{Sym} \left(\Omega\, \overset{1}{M}(\sigma)\, \overset{2}{M}(\omega) \right),$$

where $\overset{1}{A} := A \otimes Id$, $\overset{2}{A} := Id \otimes A$ and $\Omega := \frac{1}{2} \left(\overset{1}{\sigma}_1 \overset{2}{\sigma}_1 + \overset{1}{\sigma}_2 \overset{2}{\sigma}_2 + \overset{1}{\sigma}_3 \overset{2}{\sigma}_3 \right)$ is the Casimir of $sl(2)$.

* Classically, constraints are:
  * $\det M(\omega) = 1$
  * $M(\omega) - M^t(\omega) = 0$

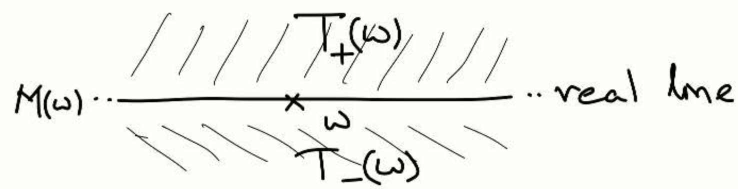
* Also: $\{ p_0, M(\omega) \} = 0$, $\{ \ln(\lambda), M(\omega) \} = \frac{1}{8\pi G'} \partial_\omega M(\omega)$,

$$\{ p_0, \ln(\lambda) \} = \frac{1}{8\pi G'}$$

3. Properties of the Algebraic Quantization

* As it was shown in KS arxiv: 9710210, the classical monodromy matrix $M(v)$ can be holomorphically factorized as

$$M(v) = T_+(v)\, T_-^{\,t}(v)$$



* In terms of variables $T_\pm$, the quantization is given by a twisted Yangian double at critical level
(i.e., $Y_+ \oplus Y_-$ with infinite dimensional center and twist)

* In terms of $M(\omega)$,

$$R(v-\omega)\overset{1}{M}(v)\,R^{\eta}(v-\omega+2i\hbar)\,\overset{2}{M}(\omega) = \overset{2}{M}(\omega)\,R^{\eta}(v-\omega+2i\hbar)\,\overset{1}{M}(v)\,R(\omega-v)\,\frac{v-\omega-2i\hbar}{v-\omega+2i\hbar}$$

with $R(x) = (x - i\hbar/2)\,Id - i\hbar\,\Omega$; $R^{\eta}(x) = (x - i\hbar/2)\,Id - i\hbar\,\Omega^{\eta}$; $\Omega^{\eta} = -\Omega^{t}$

* Conformal factor, $\ln(\lambda)$, naturally acts as an infinitesimal shift.

* Bethe ansatz - like approach: Niedermaier, Samtleben, arXiv: 0008016
  Niedermaier, arXiv: 0007227

* Quantization of $M(\omega)$ preserved by "Quantum" Geroch group

$$M(\omega) \longmapsto M^{\sigma}(\omega) = s(\omega + i\hbar/2)\, M(\omega)\, s^{t}(\omega - i\hbar/2)$$

(J.P., Paternain, Reisenberger, arXiv: 1906.04856)

Digging in the Quantum Algebra.

* Write $M(\omega) = \begin{pmatrix} a(\omega) & d(\omega) \\ e(\omega) & b(\omega) \end{pmatrix}$, each entry a hermitian operator.

* It can be proven that $d(\omega) - e(\omega)$ is in the center, and it generates an ideal $\Rightarrow$ $d \equiv e$ w.l.g.

* $q\det(M) = q\det(T_+) \cdot q\det(T_-) = 1$ only under certain analiticity assumptions (Runov, arXiv: 2010.09967)

* The algebra can be nicely written as: $M(\omega) = \begin{pmatrix} a(\omega) & d(\omega) \\ d(\omega) & b(\omega) \end{pmatrix}$

$$a(v)\, a(\omega) = e^{i\theta(v,\omega)}\, a(\omega)\, a(v)$$

$$b(v)\, b(\omega) = e^{i\theta(v,\omega)}\, b(\omega)\, b(v)$$

$$a(v)\, b(\omega) - e^{i(\theta+\theta_1)(v,\omega)}\, b(\omega)\, a(v) = \frac{2i\hbar}{v-\omega}\left(d(v)\, d(\omega) + e^{i(\theta+\theta_1)(v,\omega)}\, d(\omega)\, d(v) \right)$$

$$a(v)\, b(\omega) - e^{i(\theta+\theta_1)(v,\omega)}\, b(\omega)\, a(v) = b(v)\, a(\omega) - e^{i(\theta+\theta_1)(v,\omega)}\, a(\omega)\, b(v)$$

$$a(v)\, d(\omega) = e^{i\theta(v,\omega)}\, \frac{v-\omega-2i\hbar}{v-\omega}\, d(\omega)\, a(v) - \frac{2i\hbar}{v-\omega}\, d(v)\, a(\omega)$$

$$b(v)\, d(\omega) = e^{i\theta(v,\omega)}\, \frac{v-\omega-2i\hbar}{v-\omega}\, d(\omega)\, b(v) - \frac{2i\hbar}{v-\omega}\, d(v)\, b(\omega)$$

$$e^{i\theta(v,\omega)} = \frac{v-\omega+i\hbar}{v-\omega-i\hbar}$$

$$e^{i\theta_1(v,\omega)} = \frac{v-\omega-2i\hbar}{v-\omega+2i\hbar}$$

* We want to capitalize on the two Wick algebras for the elements on the diagonal:

$$\begin{cases} a(v)\, a(\omega) = e^{i\theta(v,\omega)}\, a(\omega)\, a(v) \\ b(v)\, b(\omega) = e^{i\theta(v,\omega)}\, b(\omega)\, b(v) \end{cases}$$

* Intuition from known quantization of Einstein Rosen waves with one polarization (Ashtekar, Pierri, arXiv: 9606085) (AP)

* Review of AP results:

* Classically, $a = b^{-1} \equiv e^{\varphi}$, E.o.M.: $-\partial_t^2 \varphi + \frac{\partial_\rho \varphi}{\rho} + \partial_\rho^2 \varphi = 0$

with solution: $$\varphi(\rho, t) = \int_0^{+\infty} \left[A_+(\lambda)\, J_0(\lambda \rho)\, e^{i\lambda t} + A_-(\lambda)\, J_0(\lambda \rho)\, e^{-i\lambda t} \right] d\lambda$$

J_0 : Bessel function of the first kind

* Since $\{A_+(\lambda), A_-(\lambda')\} = \delta(\lambda - \lambda')$, quantization is carried in terms of creation and annihilation operators,

$$A_- |0\rangle = 0 \quad , \quad A_+ = A_-^\dagger$$

* In particular, $\langle 0 | A_-^\dagger(\lambda) A_-(\lambda') | 0 \rangle = \delta(\lambda - \lambda')$

* We consider the abelian sector of the two polarization case,

meaning: <u>assume</u> there exists a physically acceptable (unitary) representation of the full algebra (a, b, d) on a Hilbert space $\mathcal{H}$.

$\Longrightarrow$ take the null eigenspace of $d(\omega)$, $\mathcal{H}_0 \subset \mathcal{H}$

$$d(\omega)\, \mathcal{H}_0 = 0 \qquad \forall\, \omega \in \mathbb{R}.$$

* We get two Wick algebras with a (closed) mixed relation

$$\begin{cases} a(\sigma)\, a(\omega) = e^{i\theta(\sigma,\omega)}\, a(\omega)\, a(\sigma) \\ b(\sigma)\, b(\omega) = e^{i\theta(\sigma,\omega)}\, b(\omega)\, b(\sigma) \\ a(\sigma)\, b(\omega) = e^{i(\theta+\theta_1)(\sigma,\omega)}\, b(\omega)\, a(\sigma) \end{cases}$$

<u>Obs:</u> reminiscent of ZF algebras !!

* Idea of Quantization

* $a(\omega) = e^{\varphi(\omega)}$; $b(\omega) = e^{\psi(\omega)}$

* $$\varphi(\sigma) = \int_0^{+\infty} \left[A_-(\lambda)\, e^{-i\lambda\sigma} + A_+(\lambda)\, e^{i\lambda\sigma}\right] d\lambda$$

$$\psi(\sigma) = \int_0^{+\infty} \left[B_-(\lambda)\, e^{-i\lambda\sigma} + B_+(\lambda)\, e^{i\lambda\sigma}\right] d\lambda$$

* $A_-|0\rangle = B_-|0\rangle = 0$, and we can show that

$$\langle 0| A_-^\dagger(\lambda)\, A_-(\lambda')\,|0\rangle = \frac{1 - e^{-\hbar\lambda}}{\lambda}\, \delta(\lambda - \lambda')$$

* Coincides with AP quantization mod $\hbar^2$!!!

$$a(\sigma)\, a(\omega) = e^{i\theta(\sigma,\omega)}\, a(\omega)\, a(\sigma)$$

$$b(\sigma)\, b(\omega) = e^{i\theta(\sigma,\omega)}\, b(\omega)\, b(\sigma)$$

$$a(\sigma)\, b(\omega) = e^{i(\theta+\theta_1)(\sigma,\omega)}\, b(\omega)\, a(\sigma)$$

4. Outlook

* Simplification of the algebra, but still work to do.

* Taking $a(\omega)$ and $b(\omega)$ as d.o.f.

* Understanding Abelian sector as an example of the problem

  Solve Constraint First, then Quantize
  vs.
  Quantize First, then solve constraint

* In particular, finding suitable candidate for qdet in Abelian case might shed light to qdet in general case

... (Continuation)

* Integrability implies infinite hierarchy of (non-local) charges in the classical theory. What information can we get from them?

* Looking for a characterization of the full Hilbert space, $\mathcal{H}$, in terms of suitable hermitian operators.

* Adding ρ_0 and λ in the picture.

Thank you!