Title: TBA - Mathematical Physics

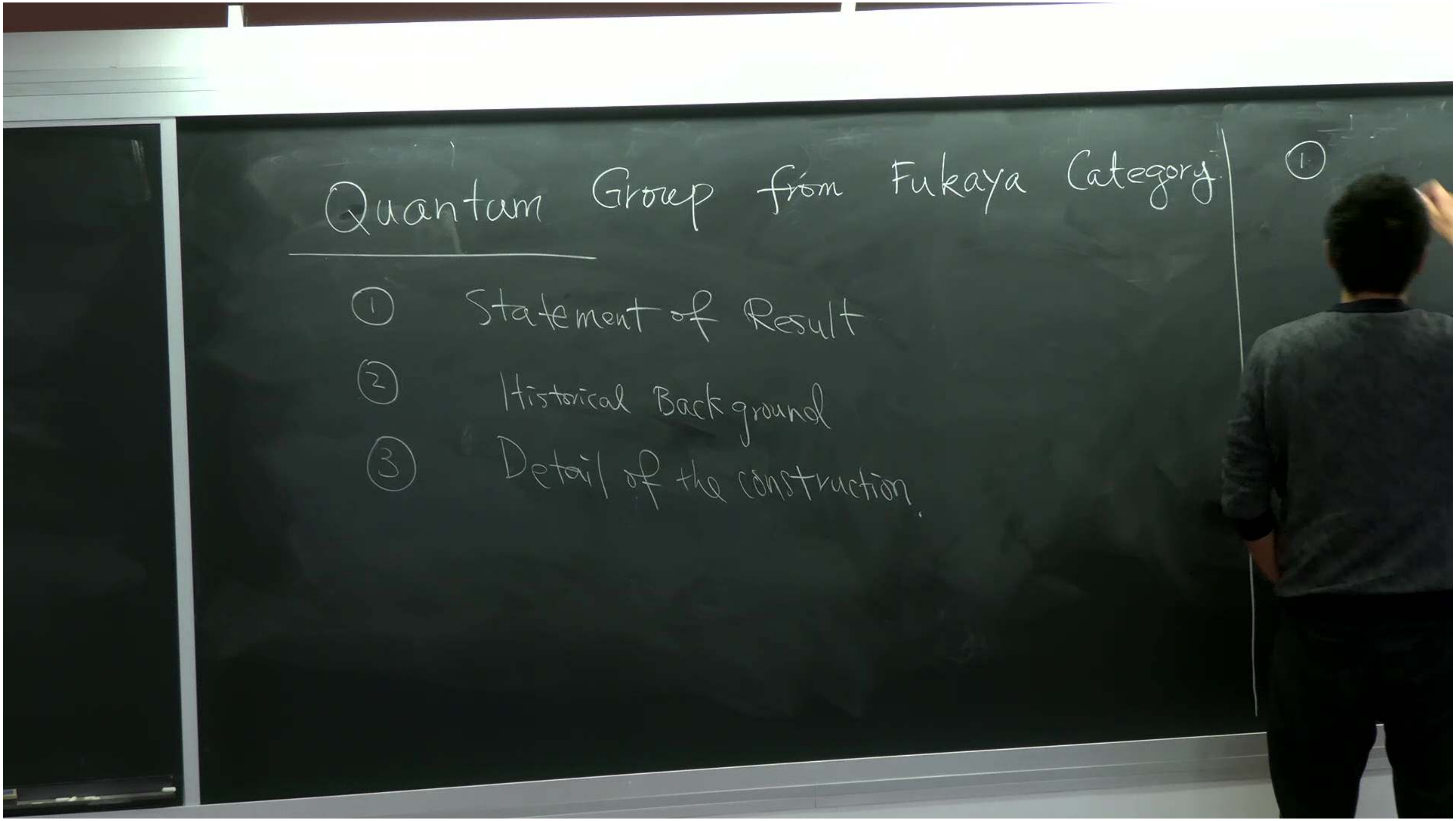
Speakers: Peng Zhou

Collection/Series: Mathematical Physics

Subject: Mathematical physics

Date: February 20, 2026 - 7:30 PM

URL: https://pirsa.org/26020030


Quantum Group from Fukaya Category
① Statement of Result
② Historical Background
③ Detail of the construction
①

ategory:

① $\mathfrak{g}$: ADE Lie alg with Dynkin Quiver Q

$\mathfrak{g} = sl_2$. $Q = \bullet$

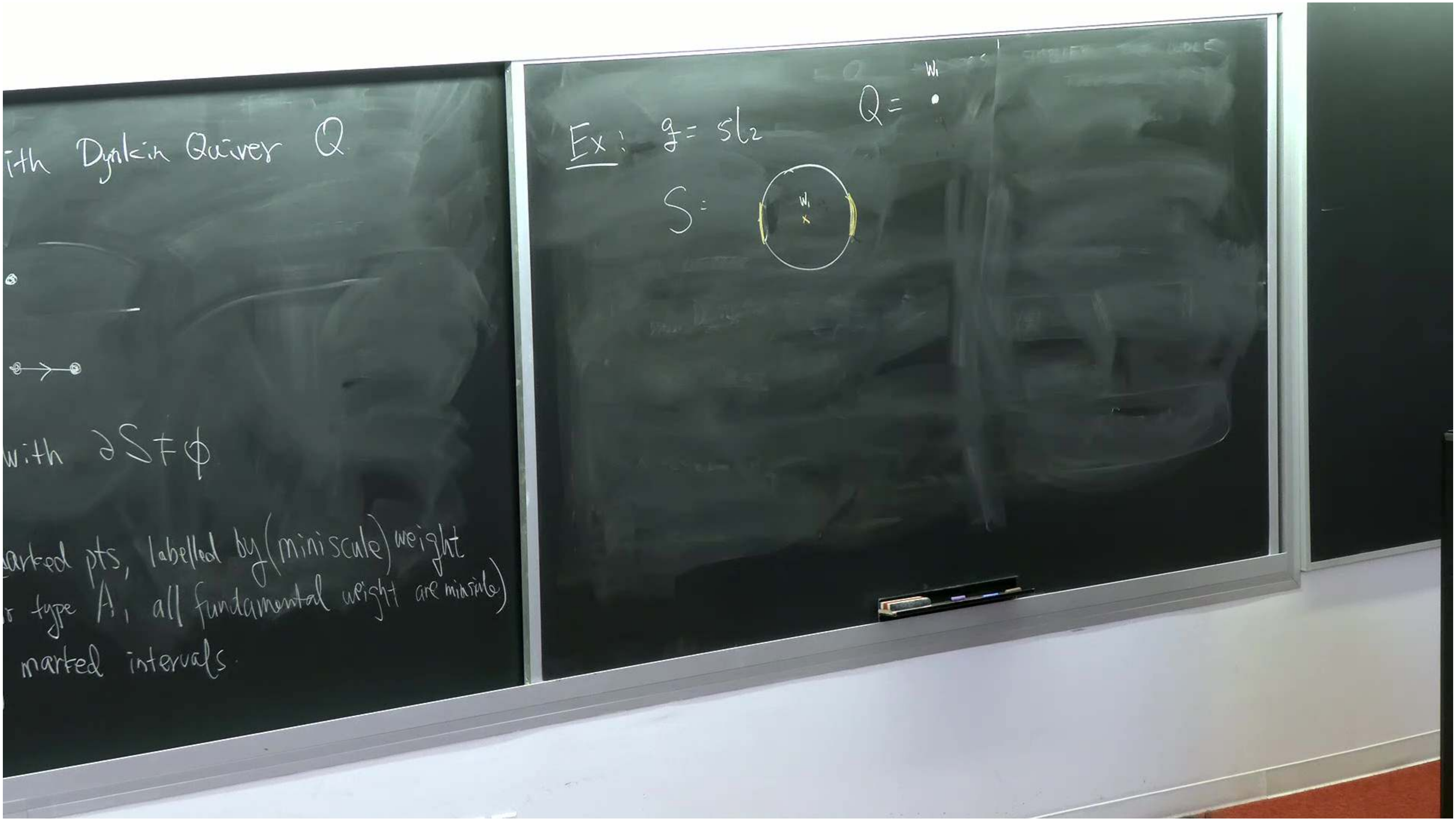
$\mathfrak{g} = sl_3$ $Q = \bullet \to \bullet$

S: smooth surface with $\partial S \neq \phi$

with decorations:

- interior marked pts, labelled by (miniscule) weight
  (for type A, all fundamental weight are miniscule)
- boundary marked intervals.

Ex: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with ∂S ≠ ∅
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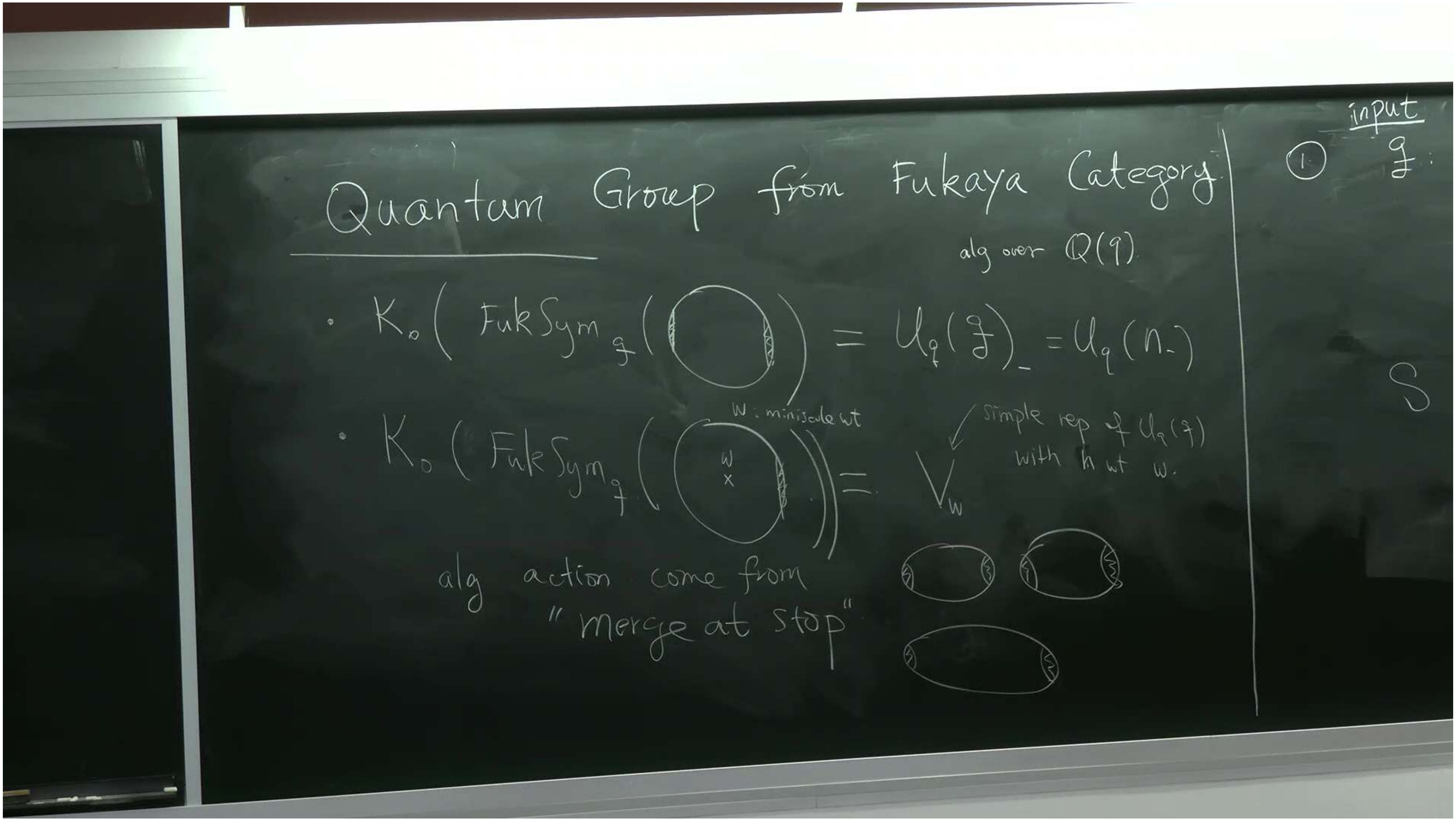
marked intervals.
Ex: g = sl2
Q = •
w1
S:
w1


Quantum Group from Fukaya Category
alg over Q(q)
• K0(FukSym g ( )) = Uq(g)_ = Uq(n_)
w: minuscule wt
• K0(FukSym g (w x)) = Vw
simple rep of Uq(g) with h wt w.
alg action come from "merge at stop"
input
① g
S

ategory.

(q)

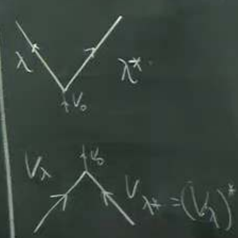
$\mathcal{U}_q(\mathfrak{n}_-)$

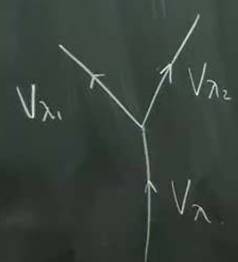
p of $\mathcal{U}_q(\mathfrak{g})$
h wt ω.

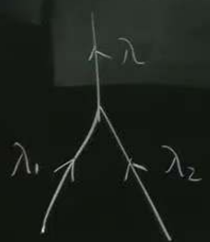
Functors:

① isotopy of "punctures" and "stops" induces equivalence of cats

② merge & split "punctures" ⇝ functors







$\lambda_1, \lambda_2, \lambda$: min. wts

$\phi : V_\lambda \hookrightarrow V_{\lambda_1} \otimes V_{\lambda_2}$

$\Phi : F(\ \lambda \times\ ) \longrightarrow F(\ \lambda_1 \times \ \ \lambda_2 \times\ )$

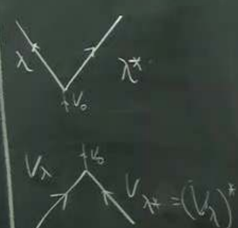
ategory.

(q).

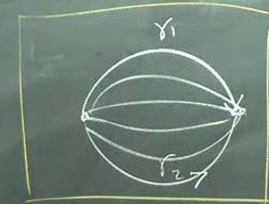
$U_q(n_-)$

p of $U_q(q)$

h wt ω.



Functors:



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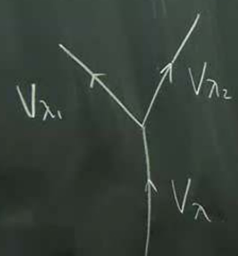

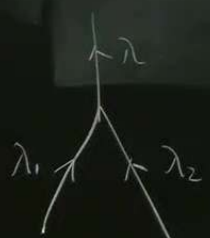

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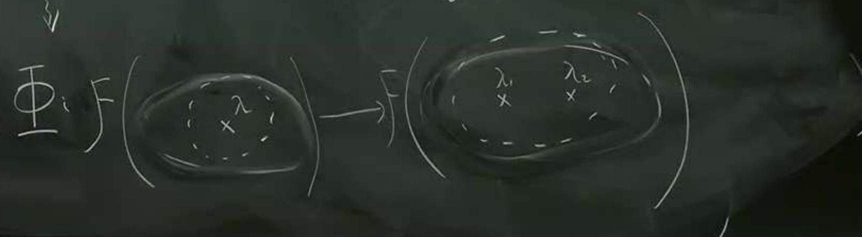

tum Group for

① Drinfeld - Jimbo : $U(\mathfrak{g}) \overset{\text{deform as Hopf alg}}{\rightsquigarrow} U_q(\mathfrak{g})$

② Ringel : consider quiver rep cat of $\vec{Q}$ $\bullet \to \bullet \to \bullet$

- $\operatorname{Rep}(\vec{Q})_{\mathbb{F}_q}$
- Hall alg
  - element : isom classes of reps $[M]$
  - prod : $[M] * [N] = \sum_{M \hookrightarrow L \twoheadrightarrow N} \#_q\left(\frac{1}{\operatorname{Aut}(\cdots)}\right) [L]$
- Ringel Hall Alg $(\operatorname{Rep}(Q)) \simeq U_q(\mathfrak{g})_-$

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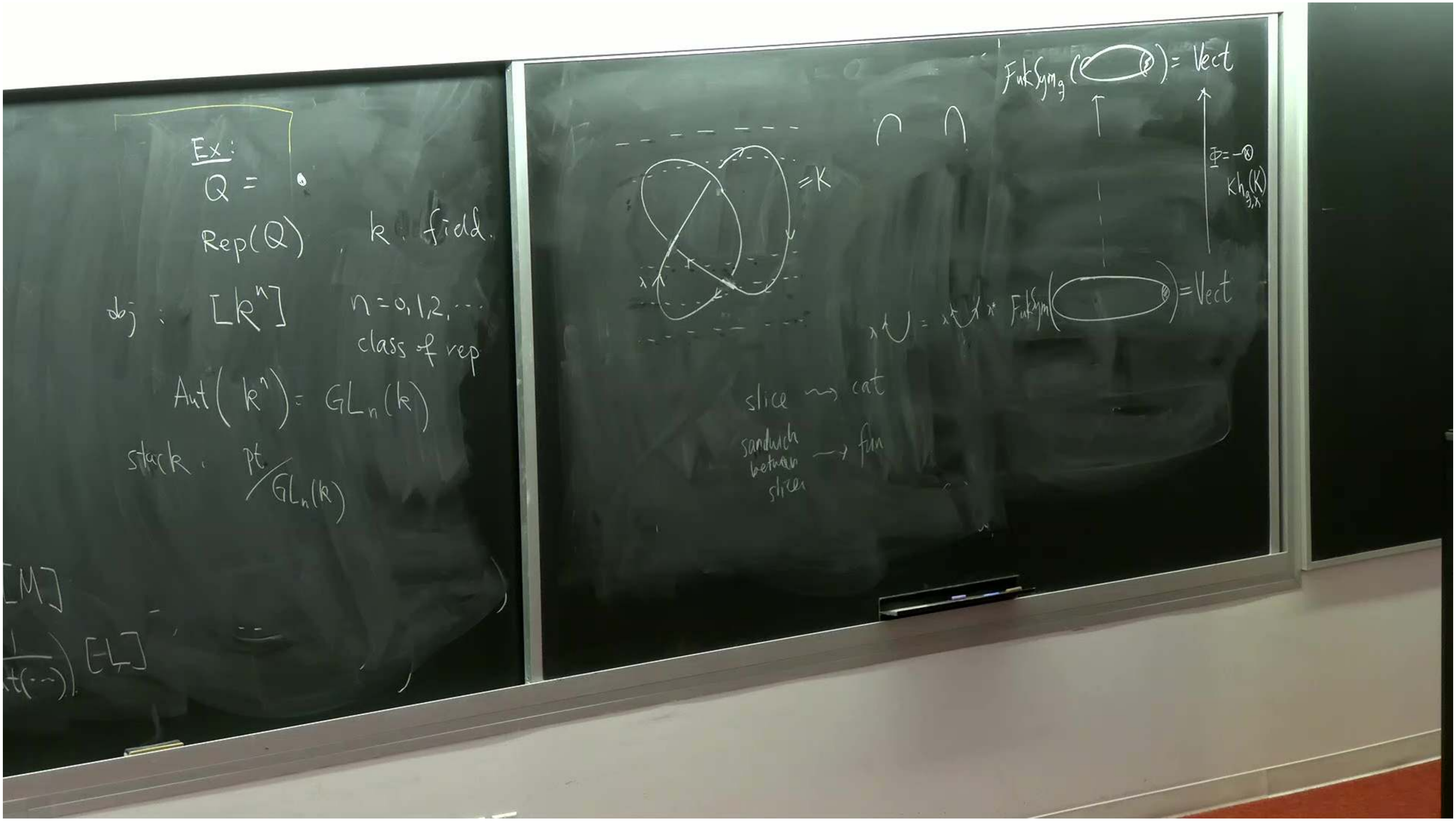

Ex:
Q = •
Rep(Q)
k field.
obj : [k^n]
n = 0,1,2, ...
class of rep
Aut(k^n) = GL_n(k)
stack : pt/GL_n(k)
[M]
[L]
= K
slice ⇝ cat
sandwich between slices → fun
FukSym_g( ) = Vect
Φ = −⊗ Kh_{g,λ}(K)
FukSym( ) = Vect

...ntum Group for

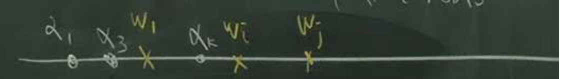
categorification of $\mathcal{U}_q(\mathfrak{g})_-$ $\rightsquigarrow$ KLRW - category

$\begin{cases} w_i : \text{wts} \\ \alpha_i : \text{roots} \end{cases}$

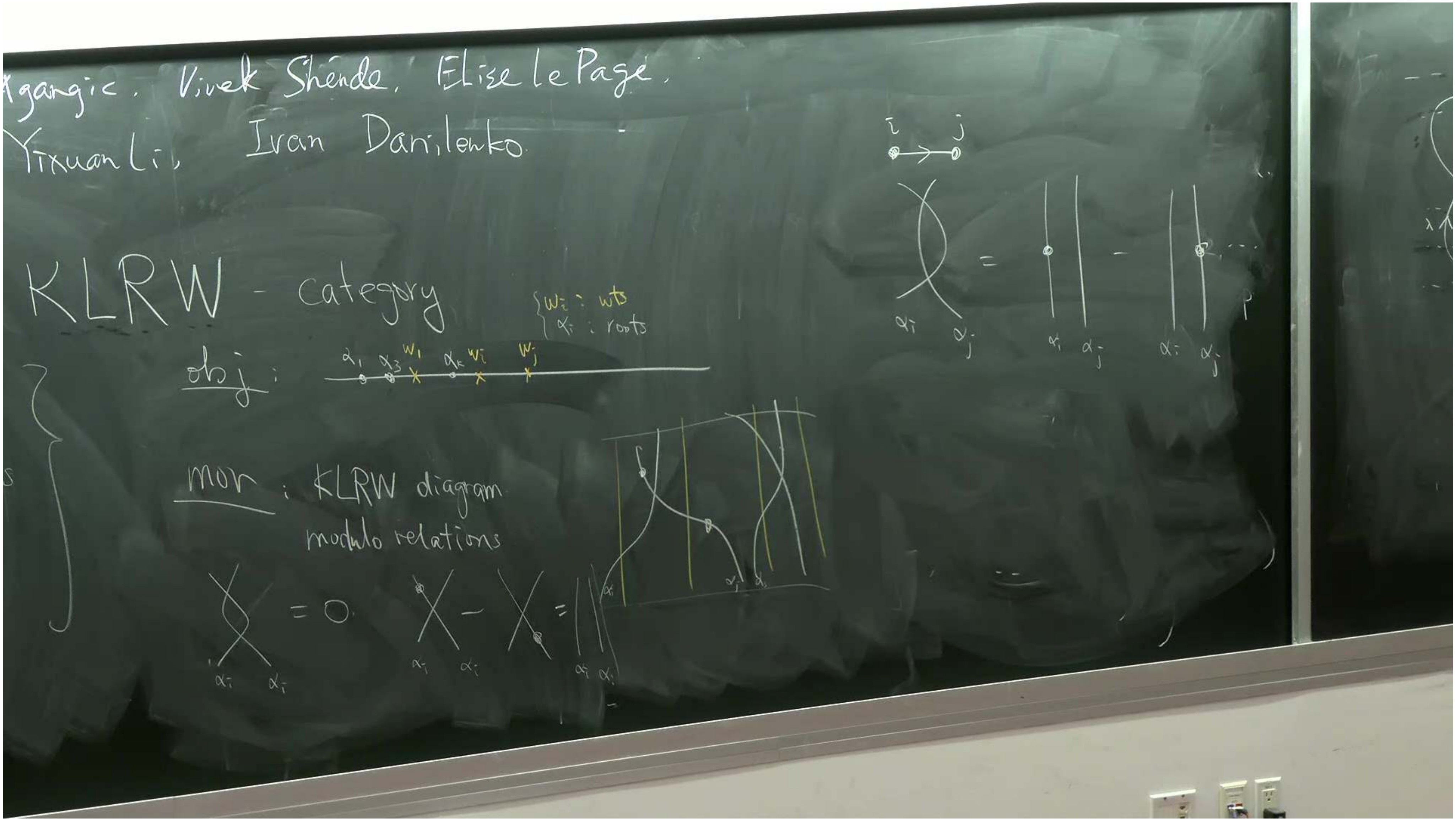
<u>input</u> :

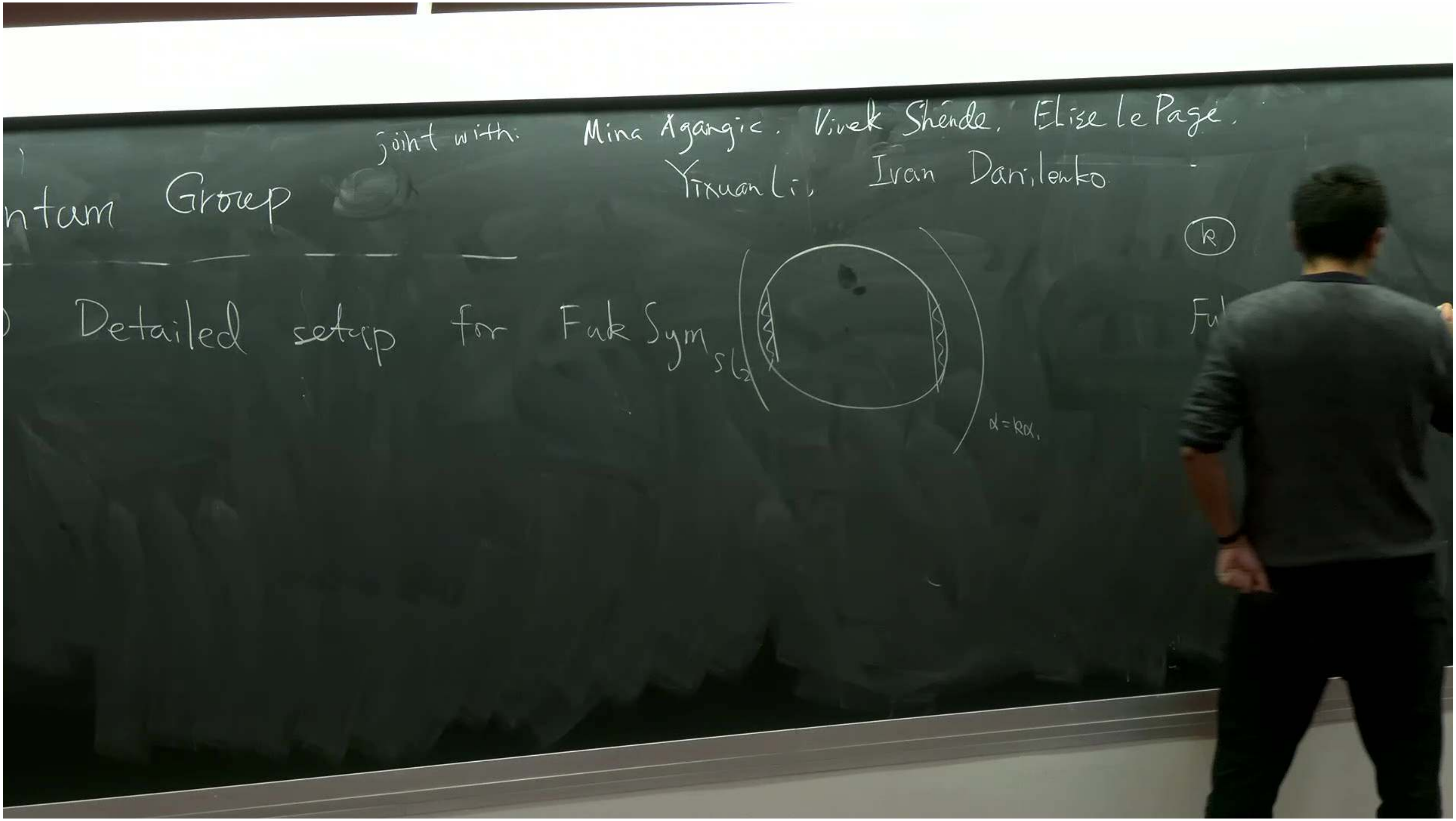
- $\mathfrak{g}$: ADE
- a fixed set of minuscule wts
- $\alpha = \sum_{i=1}^{r} k_i \alpha_i$

<u>obj</u> :



<u>mor</u> : KLRW diagram


gangic, Vivek Shende, Elise le Page
Yixuan Li, Ivan Danilenko
KLRW - category
wᵢ : wts
αᵢ : roots
obj :
mor : KLRW diagram
modulo relations
= 0


joint with: Mina Aganagic, Vivek Shende, Elise le Page,
Yixuan Li, Ivan Danilenko
ntum Group
Detailed setup for Fuk Sym
sl2
α = kα,
k

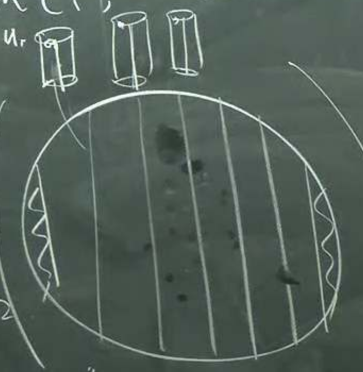
Mina Aganagic, Vivek Shende, Elise Le Page,
Yixuan Li, Ivan Danilenko

$x_1, \dots, x_k$: fiber coord on $\mathbb{C}^*$.
$y_1, \dots, y_k$: base coord

$$W\Big|_{\substack{\text{open loci}\\ y_i \neq y_j}} = \sum_{i=1}^{k} \underbrace{\frac{1}{x_i} \frac{1}{\prod_{j\neq i}(y_i - y_j)}}_{\boxed{u_i}} = \sum_i u_i$$

k

$$\mathrm{Fuk\,Sym}_{sl_2}\left(\quad \right)$$



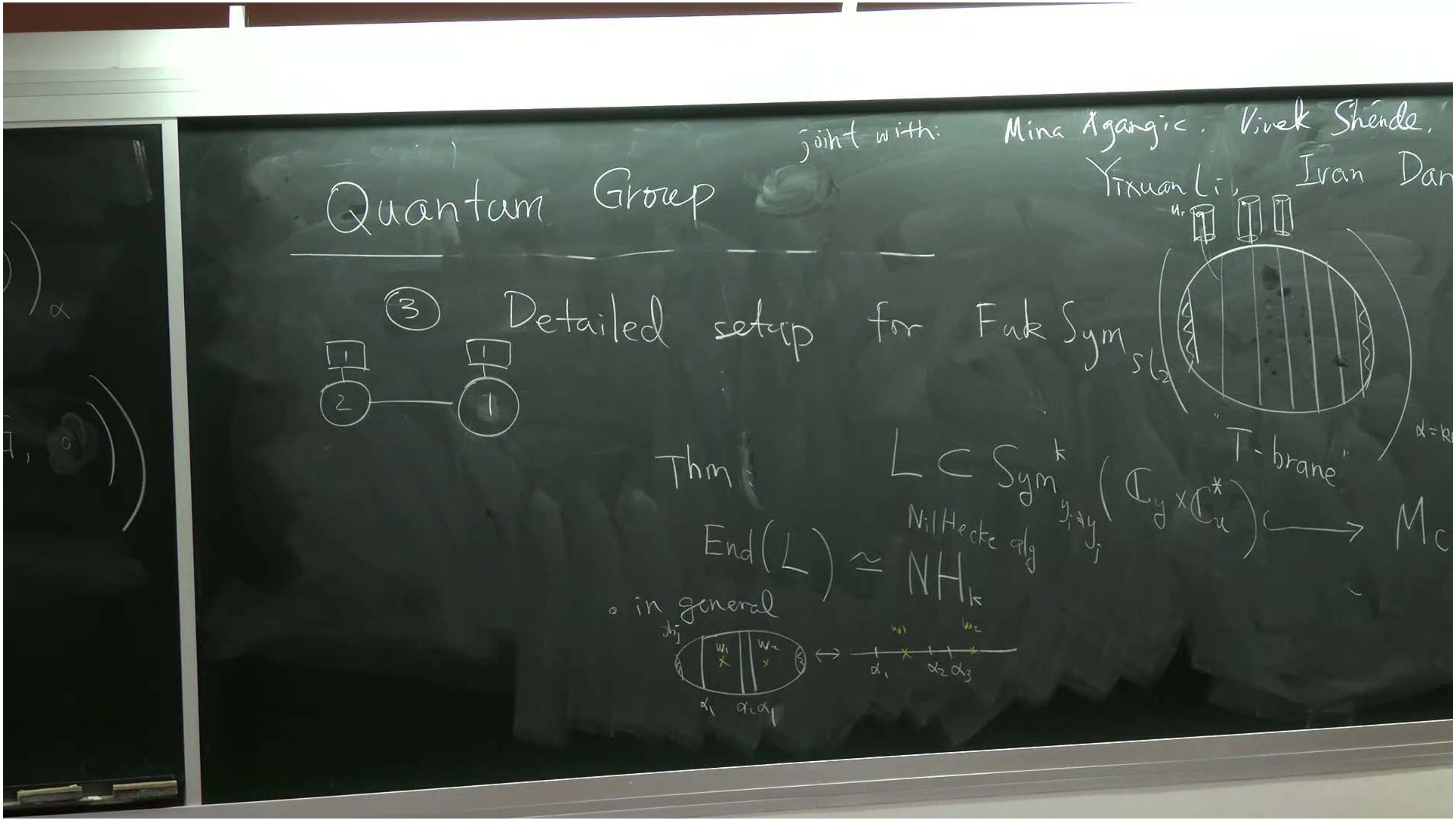
"T-brane"

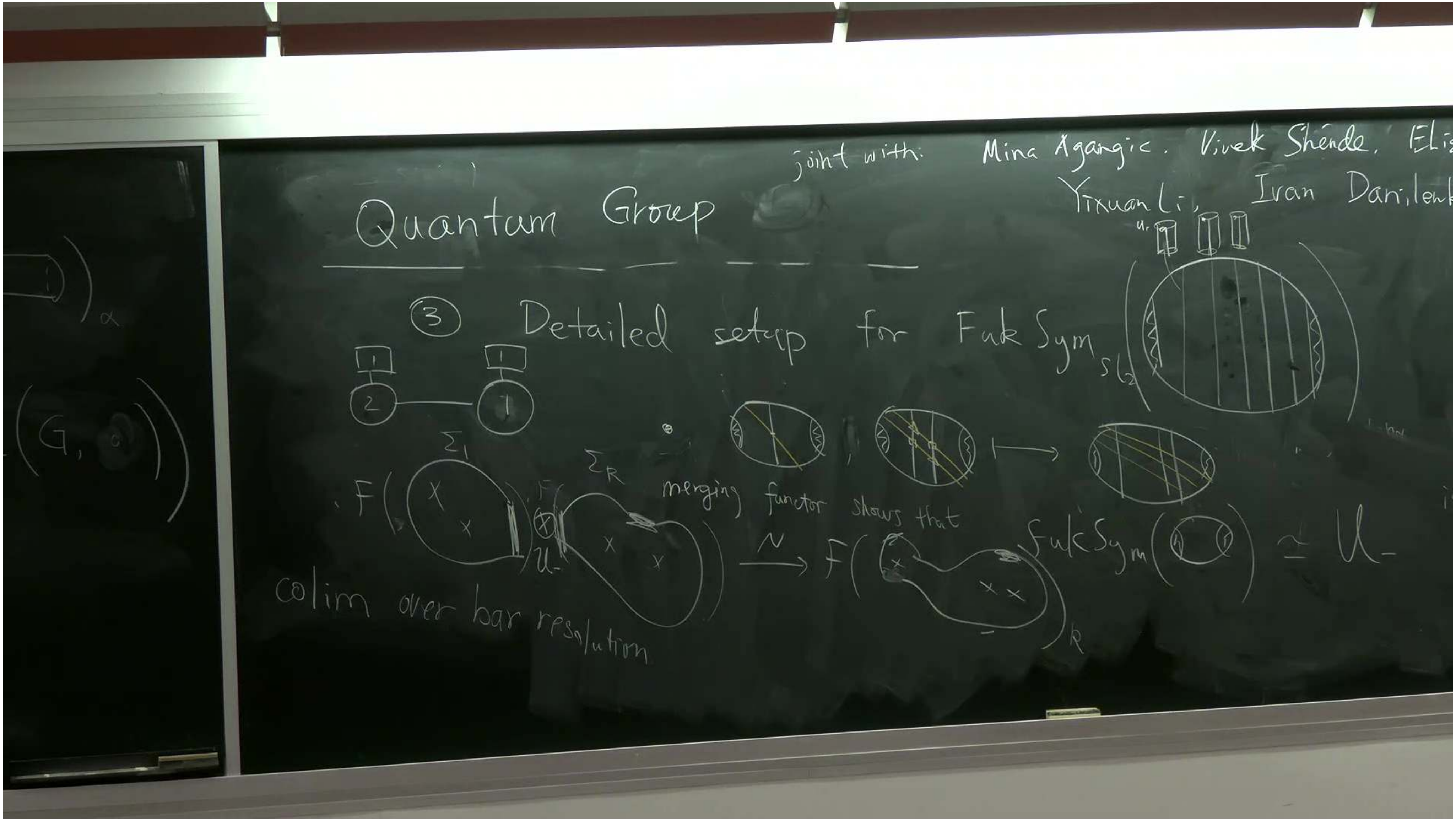
$\alpha = k\alpha.$

$$\mathrm{Fuk\,Sym}(\quad)_k = \mathrm{Fuk}\left(\mathcal{M}_C(GL_k), W\right)$$

$$\mathcal{M}_C(GL_k) \overset{\text{"roughly"}}{\approx} \frac{T_{GL_k} \times T^\vee_{GL_k}}{S_k} = \mathrm{Sym}^k(\mathbb{C}^* \times \mathbb{C}^*)$$

$$\mathrm{Sym}^k_{y_i \neq y_j}(\mathbb{C}_y \times \mathbb{C}^*_u) \hookrightarrow \mathcal{M}_C(GL_k) \overset{\text{BFM}}{\llcorner} \mathrm{Hilb}^k_{hor}\begin{pmatrix} \mathbb{C}_y \times \mathbb{C}^*_x \\ \downarrow \\ \mathbb{C}_y \end{pmatrix} \subset \mathrm{Hilb}^k(\quad)$$


joint with: Mina Aganagic, Vivek Shende,
Yixuan Li, Ivan Dan
Quantum Group
③ Detailed setup for Fuk Sym
"T-brane"
Thm: L ⊂ Sym
End(L) ≃ NH
NilHecke alg
in general


joint with: Mina Aganagic, Vivek Shende, Eli
Yixuan Li, Ivan Danilenk
Quantum Group
③ Detailed setup for Fuk Sym sl_2
merging functor shows that
FukSym( ) ≃ U_-
colim over bar resolution

...gangic. Vivek Shende. Elise le Page.

Yixuan Li, Ivan Danilenko

$x_1, \dots, x_k$: fiber coord on $\mathbb{C}^*$

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$$W\Big|_{\substack{\text{open loci}\\ y_i \neq y_j}} = \sum_{i=1}^{k} \underbrace{\frac{1}{x_i} \frac{1}{\prod_{j \neq i} (y_i - y_j)}}_{\boxed{u_i}} = \sum_i u_i$$

$\textcircled{k}$

$$\downarrow W = z^2$$

$$\mathbb{C}$$

$$\mathrm{Fuk\,Sym}(\mathbb{C})_k = \mathrm{Fuk}\left(M_{\mathbb{C}}(GL_k), W\right)$$

$W^{-1}(+\infty)$ "roughly $\cong$"

$$\frac{\mathfrak{t}_{GL_k} \times T^\vee_{GL_k}}{S_k} = \mathrm{Sym}^k(\mathbb{C}^* \times \mathbb{C}^*)$$

Sym_{sl_2}

is a monoidal cat :

or $\left(\begin{array}{c} \mathbb{C}_y \times \mathbb{C}^*_x \\ \downarrow \pi \\ \mathbb{C}_y \end{array}\right) \subset \mathrm{Hilb}^k$

$$\mathrm{Fuk\,Sym}(\dots) \simeq U_-$$