Title: Lecture - Cosmology, PHYS 621

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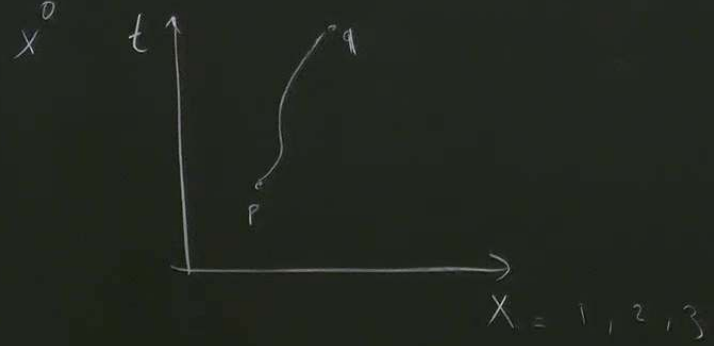
Collection/Series: Cosmology (Elective), PHYS 621, February 23 - March 27, 2026

Subject: Cosmology

Date: February 24, 2026 - 2:00 PM

URL: https://pirsa.org/26020016

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^\mu dx^\nu \equiv g_{\mu\nu}\, dx^\mu dx^\nu$$



Minkowski : $ds^2 = -dt^2 + (dx^2 + dy^2 + dz^2)$

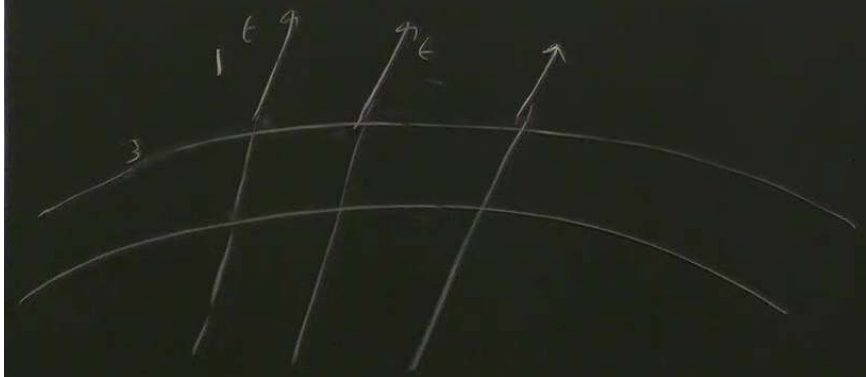
$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

$(- + + +)$

Cosmological principle (C.P.) :

at any given time, anywhere and in any direction the

where the description of the cosmos is on average

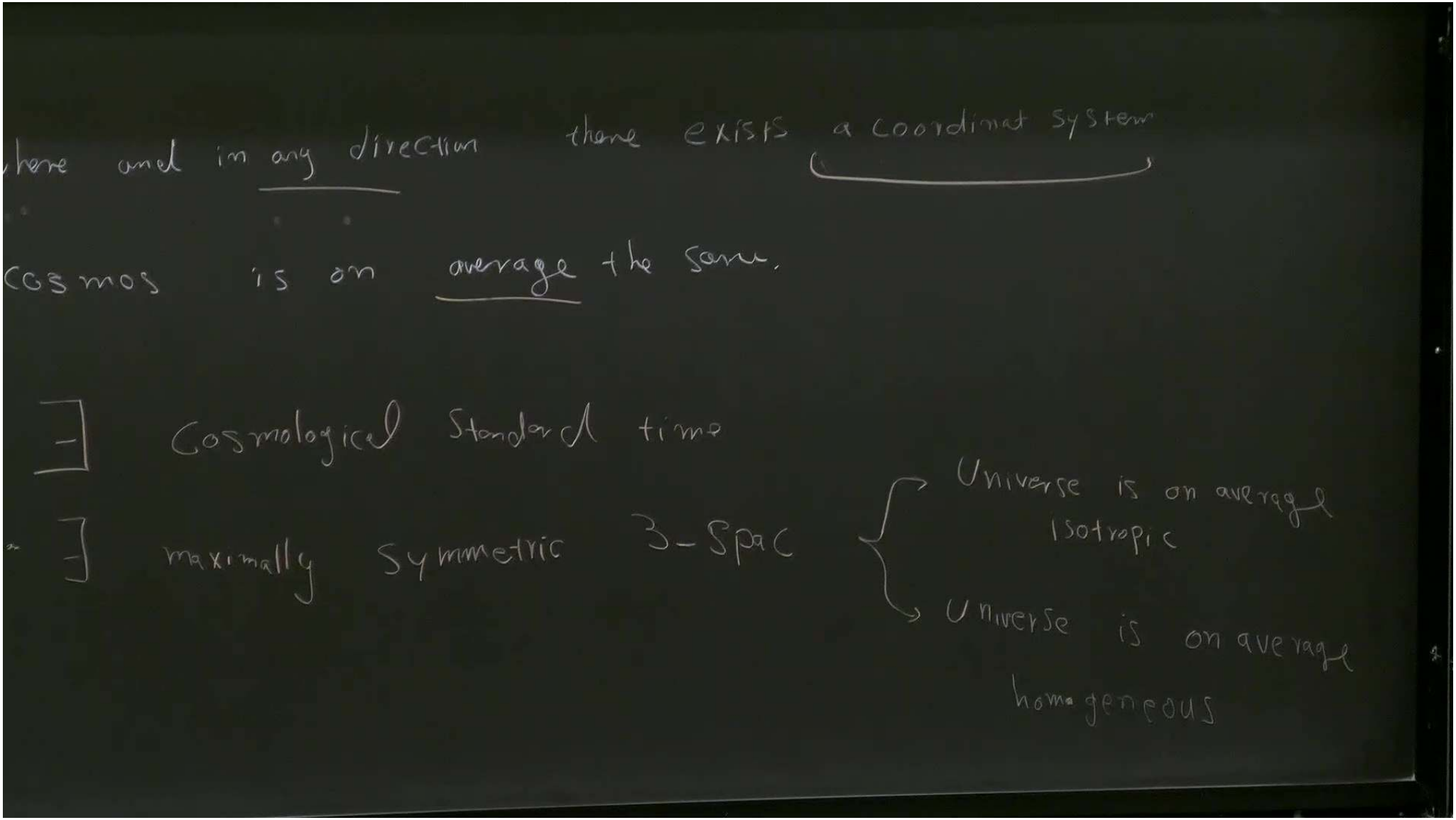


1)] Cosmological Standard

2)] maximally Symmetric

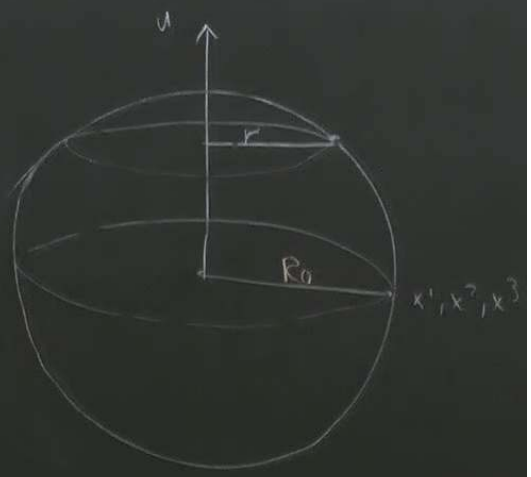
1) $\Rightarrow$ $ds^2 = dt^2 - a^2(t) \times d\ell^2$

2 $\Rightarrow$ FLRW


here and in any direction there exists a coordinat system
cosmos is on average the same.
∃ Cosmological Standard time
∃ maximally symmetric 3-Spac
Universe is on average isotropic
Universe is on average homogeneous

Friedmann - Lemaître - Robertson - Walker (FLRW) or (FRW)

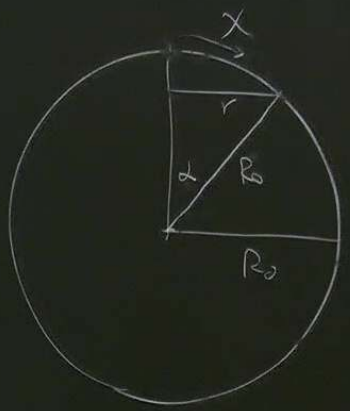
3-space
- I. zero curvature = E^3 (flat) → $dl^2 = \delta_{ij} dx^i dx^j = $
- II. positive " = S^3 (Spherical)
- III. negative " = H^3 (Hyperbolic)



FLRW metric :

FLRW metric : $ds^2 = dt^2 - a^2(t)\left[\frac{1}{1-k r^2/R_0^2}\,dr^2 + r^2 d\Omega^2\right]$

$k = 0$
$k = 1$
$k = -$



$k=1$

$\sin\alpha = \frac{r}{R_0}$ $\qquad \chi = \alpha R_0$ or $\alpha = \chi/R_0$

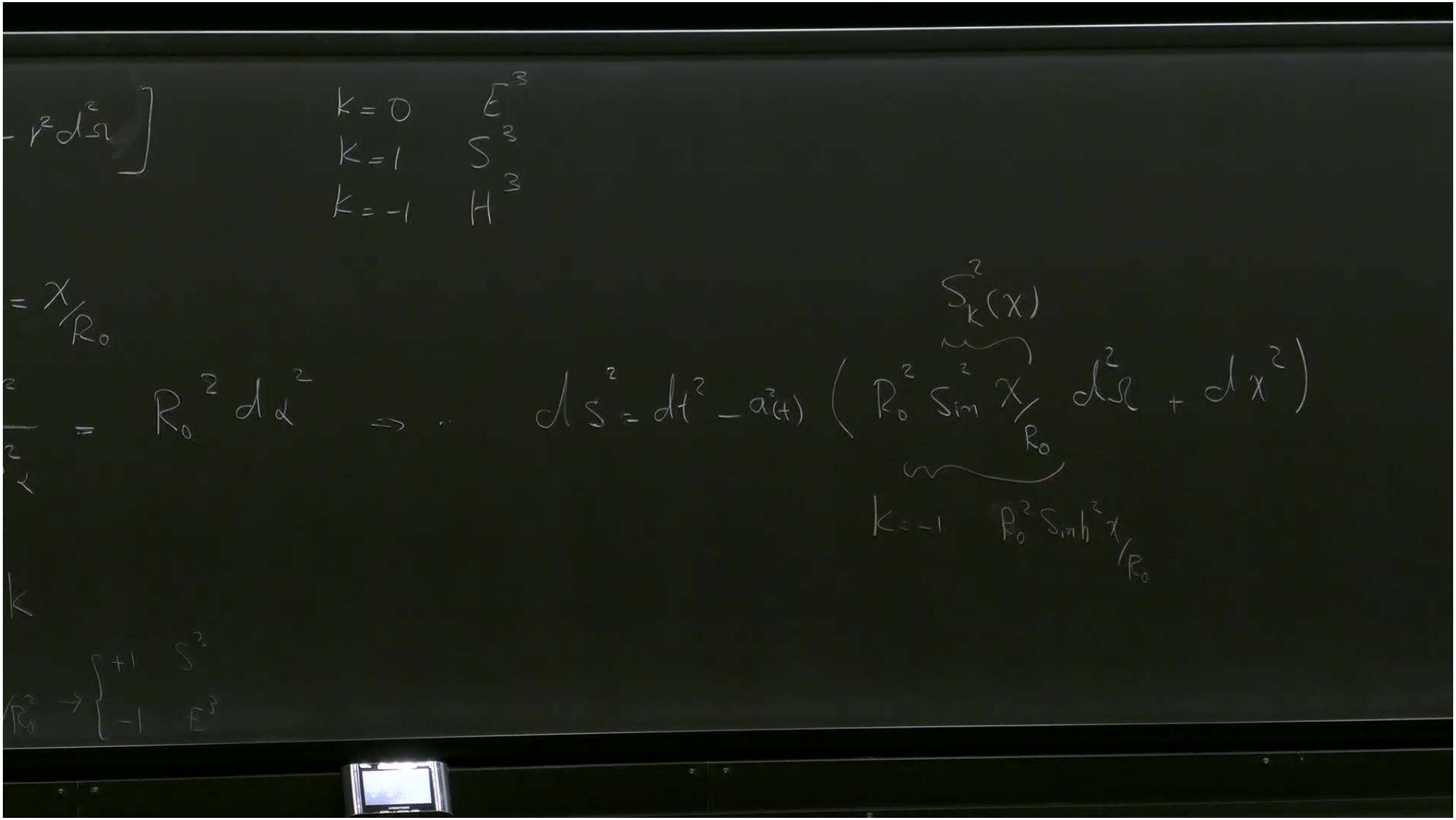
$\cos^2\alpha = 1 - r^2/R_0^2$ $\qquad \frac{dr^2}{1-r^2/R_0^2} = \frac{dr^2}{\cos^2\alpha} = R_0^2\, d\alpha^2$ $\Rightarrow$

$r\,dr = 2R_0^2 \sin\alpha\cos\alpha\, d\alpha$

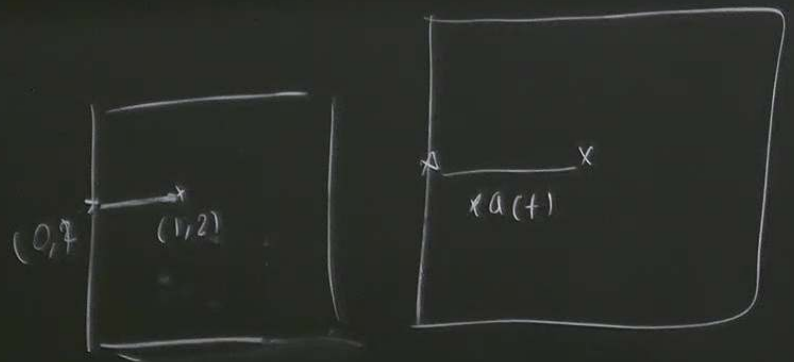
$1/R_0^2$

1) $a \to \lambda a$ $\qquad r \to r/\lambda$ $\qquad k \to \lambda^2 k$

Intrinsic curv $= 1/R^2(t) = 1/a^2(t)R_0^2$ $\qquad a(t_0) = 1$ $\qquad k \to R_0^2 k \to k/R_0^2 \to \begin{cases} +1 & S^3 \\ -1 & E^3 \end{cases}$


$k=0$ E^3
$k=1$ S^3
$k=-1$ H^3
$S_k^2(\chi)$
$k=-1$ $R_0^2 \sinh^2 \chi/R_0$

2) r is called comoving coordinat



$$r_{phys} = a(t)\, r_c$$

$$v_{phy} = \frac{d r_{phy}}{dt} = \dot{a} r_c + a \underbrace{\frac{d r_c}{dt}}_{v_{pec}} = \left(\frac{\dot{a}}{a}\right)(a r_c) + v_p = H r_{phyc} + v_p$$

$\frac{\dot{a}}{a} = H$

↳ Hubble Parameter