Title: Lecture - Quantum Field Theory III, PHYS 777

Speakers: Jaume Gomis

Collection/Series: Quantum Field Theory III, PHYS 777, February 23 - March 27, 2026

Subject: Quantum Fields and Strings

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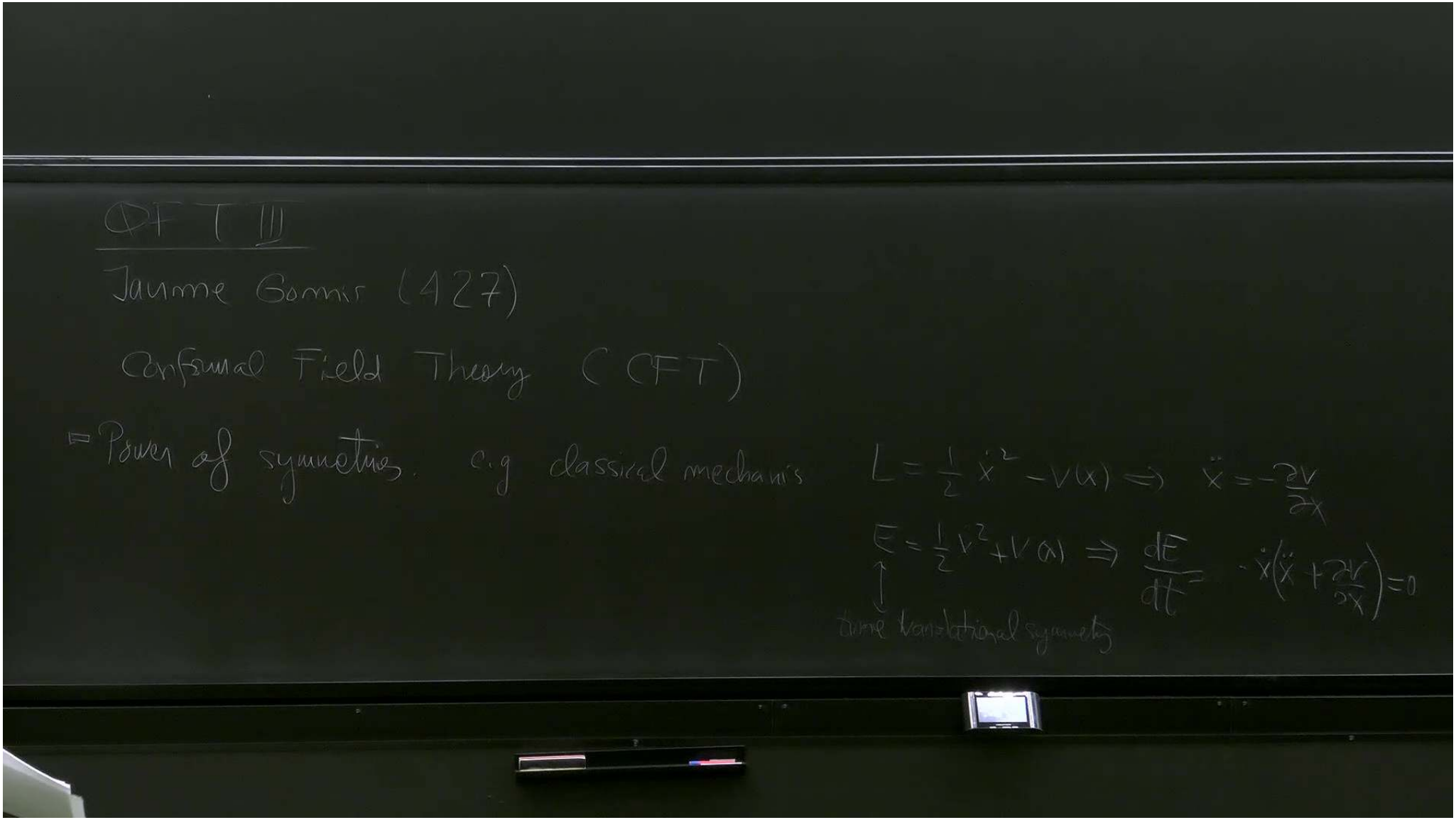
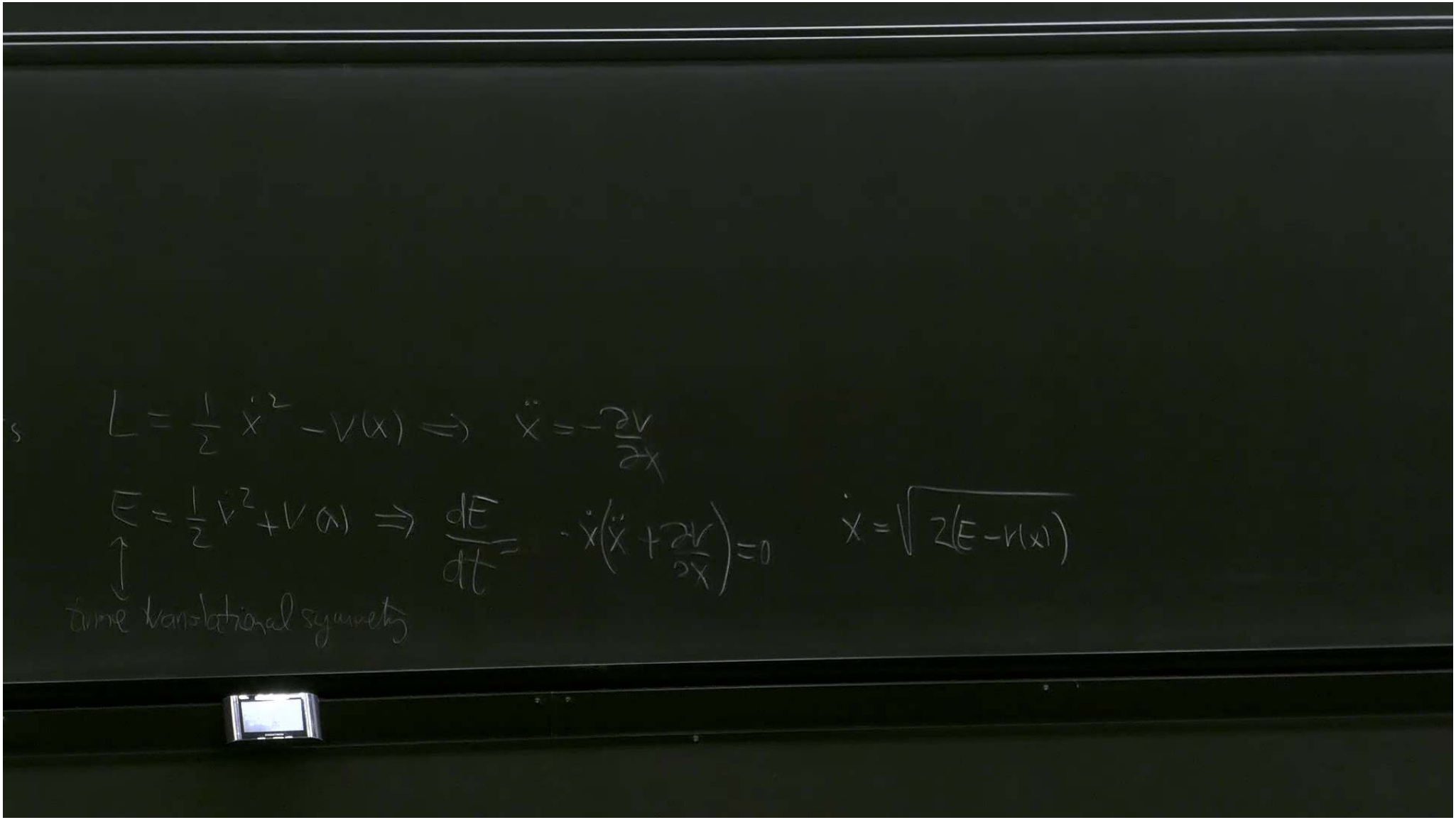

QFT III
Jaume Gomis (427)
Conformal Field Theory (CFT)
- Power of symmetries. e.g classical mechanics
$L = \frac{1}{2}\dot{x}^2 - V(x) \Rightarrow \ddot{x} = -\frac{\partial V}{\partial x}$
$E = \frac{1}{2}v^2 + V(x) \Rightarrow \frac{dE}{dt} = \dot{x}\left(\ddot{x} + \frac{\partial V}{\partial x}\right) = 0$
time translational symmetry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$\dot{x} = \sqrt{2(E - V(x))}$
time translational symmetry

time translational symmetry

$$V(x) = \frac{\alpha}{x^2}$$

additional symmetry : scaling symmetry $\Longrightarrow$ Q conserved quantity $\frac{dQ}{dt} = 0$

$Q(\dot{x}, x)$ $\quad$ $X(E, Q)$

CFT? QFT with additional spacetime symmetries $(\phi(x) \to e^{i\alpha}\phi(x))$

Beyond

- Relativistic : Poincaré + conformal symmetry
- Nonrelativistic : Galilean invariance + conformal symmetry

- Relativistic : Poincaré
- Nonrelativistic : Galilean invariance

limit

+ conformal symmetries

Conformal Transformations

$x^\mu = (t, x^i)$ scale transformation

$\| \quad \vec{x} \to \lambda \vec{x}$ $\quad z$: critical dynamical exponent

$\vec{x}$ $\quad t \to \lambda^z t$ $\quad z=1$ relativistic

$z=2$ non-relativistic

$x^\mu \to \tilde{x}^\mu$ that preserve angles

$$ds^2 \longrightarrow e^{2\sigma(x)} ds^2$$

Why CFTs?

1. Asymptotic / extreme / low energy (IR limit) of any system is a scale invariant QFT

   a "Trivial", a theory with no local operators in IR $\langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle \sim e^{-m|x|}$

   non local operators ⟆ that survive

   extreme IR $|x| \to \infty \longrightarrow \mathcal{O} \to 0$

   described by topological quantum field theory (TQFT)

   b. or CFT

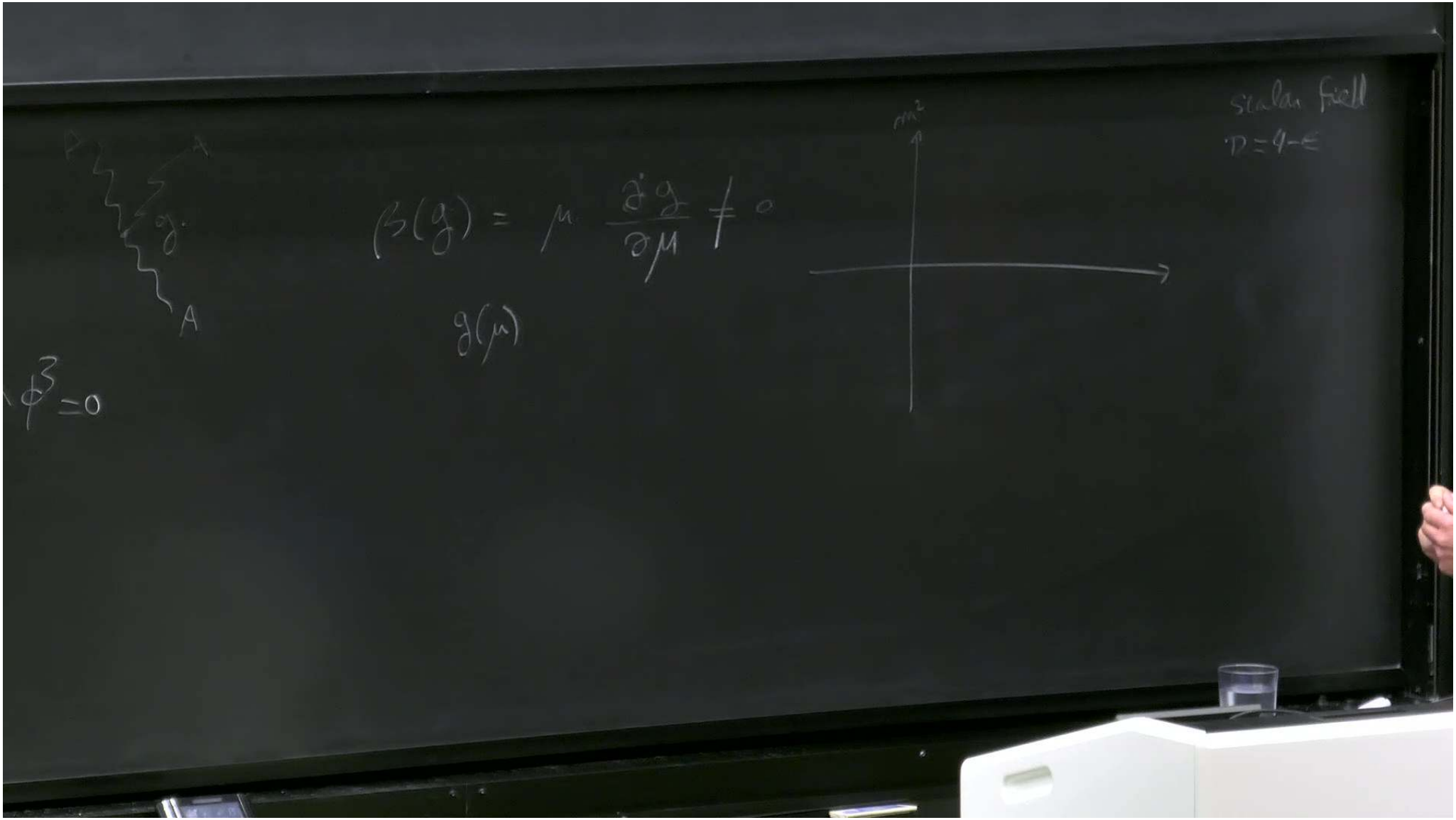
2. Any massive QFT is a deformation of a CFT.

Examples of CFTs: classically $D_\mu F^{\mu\nu} = 0$ Yang-Mills

- scalar field $[\phi]$ $\partial^2\phi + \lambda\phi^3 = 0$

- $\not{D}\psi = 0$

$\mathcal{L}(\phi, \psi)$ $\mathcal{L}_{int} = \lambda\bar{\psi}\psi\phi$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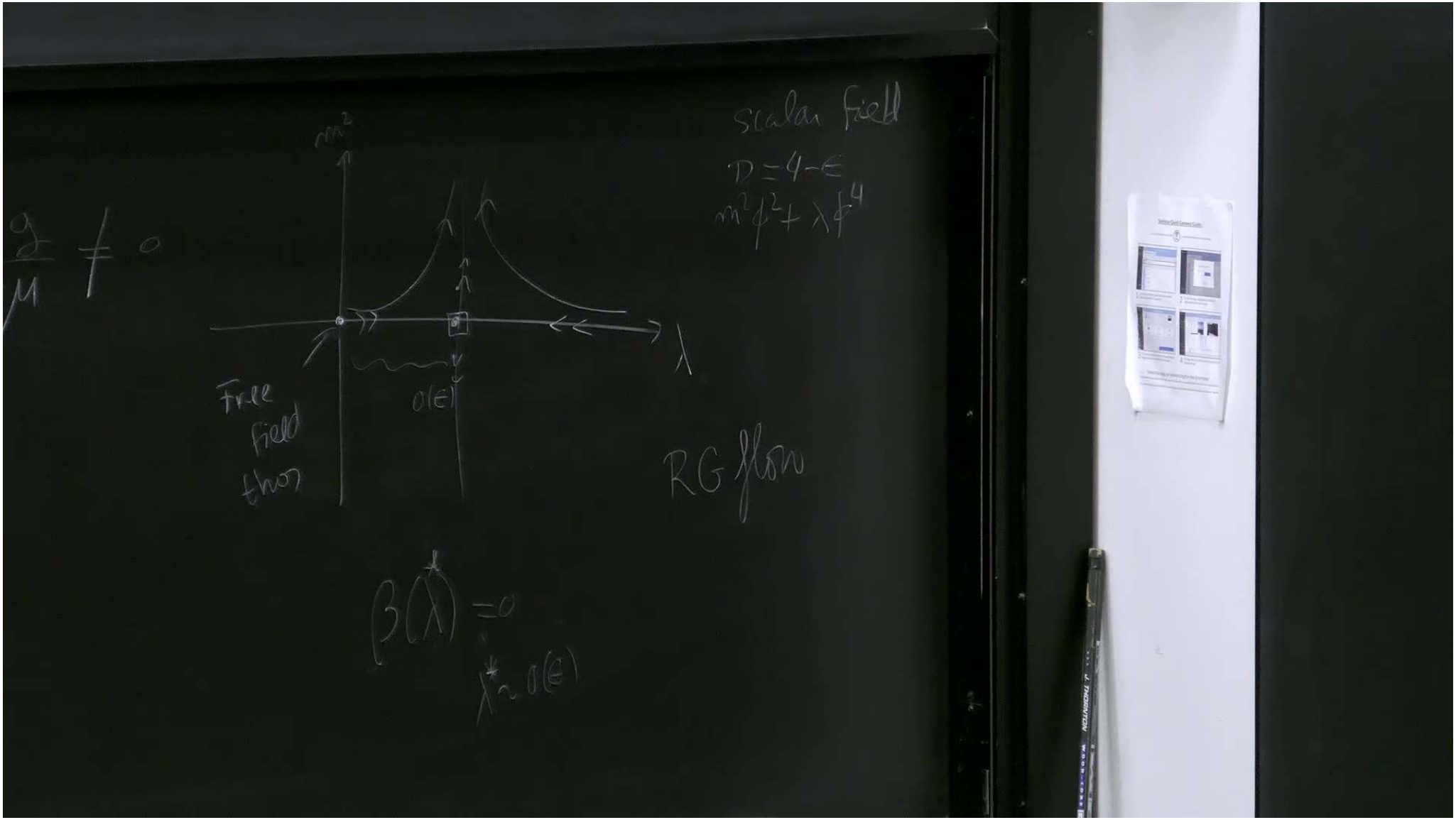
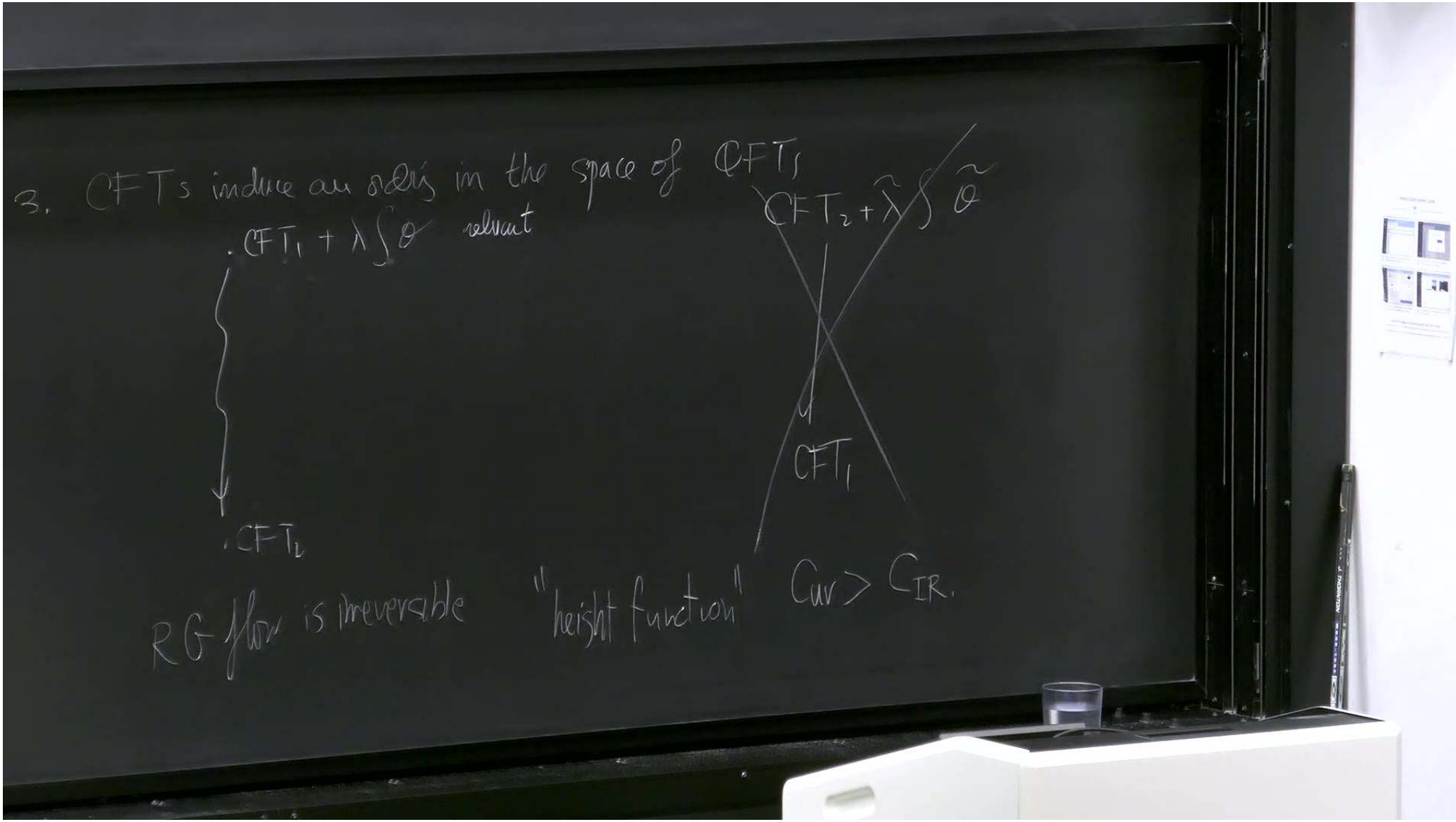
$\beta(g) = \mu \frac{\partial g}{\partial \mu} \neq 0$
$g(\mu)$
$\phi^3 = 0$
m^2
Scalar field
$D = 4 - \epsilon$


m^2
λ
Free field thon
$O(\epsilon)$
scalar field
$D = 4 - \epsilon$
$m^2\phi^2 + \lambda\phi^4$
RG flow
$\beta(\lambda^*) = 0$
$\lambda^* \sim O(\epsilon)$

$S = S_{free} + \lambda \int \mathcal{O}\, d^D x$ scaling dimension of $\mathcal{O} \Longrightarrow \Delta$

- $\Delta > D$: irrelevant operators.
- $\Delta < D$: relevant operator
- $\Delta = D$: marginal

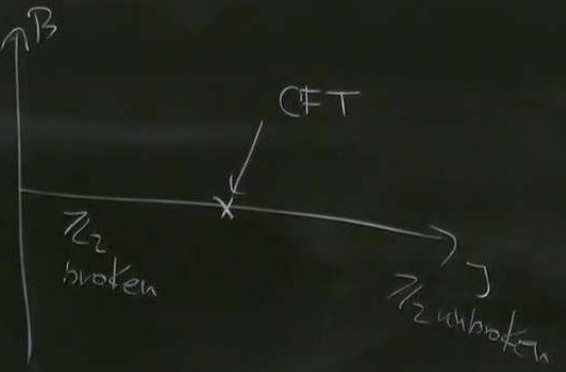

3. CFTs induce an order in the space of CFTs
. $CFT_1 + \lambda \int \mathcal{O}$ relevant
. CFT_2
$CFT_2 + \tilde{\lambda} \int \tilde{\mathcal{O}}$
CFT_1
RG flow is irreversible "height function" $C_{UV} > C_{IR}$.

CFT's directly relevant for nature

1. Thermal phase transitions (thermal)
2. Quantum phase transitions (quantum, $T=0$)

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j + B \sum_i \sigma_i$$

$\mathbb{Z}_2$ symmetry $\sigma_i \to -\sigma_i$



Einstein, Yang-Mills, $\beta(\lambda)=0$

conformal invariance of 2d QFT on string

AdS/CFT