Title: Lecture - Strong Gravity, PHYS 777

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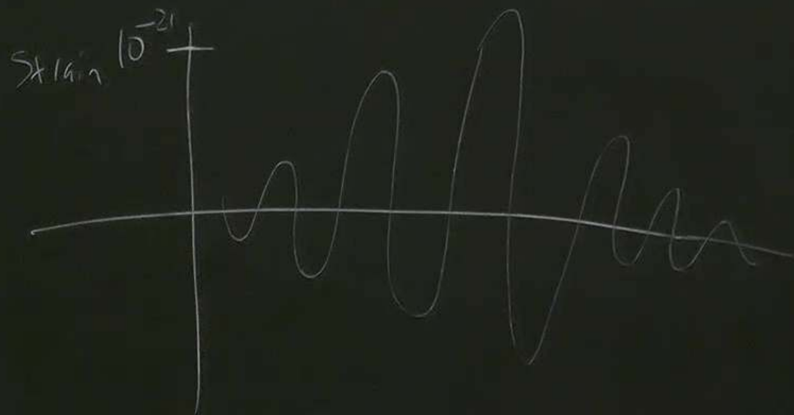
Collection/Series: Strong Gravity (Elective), PHYS 777, February 23 - March 27, 2026

Subject: Strong Gravity

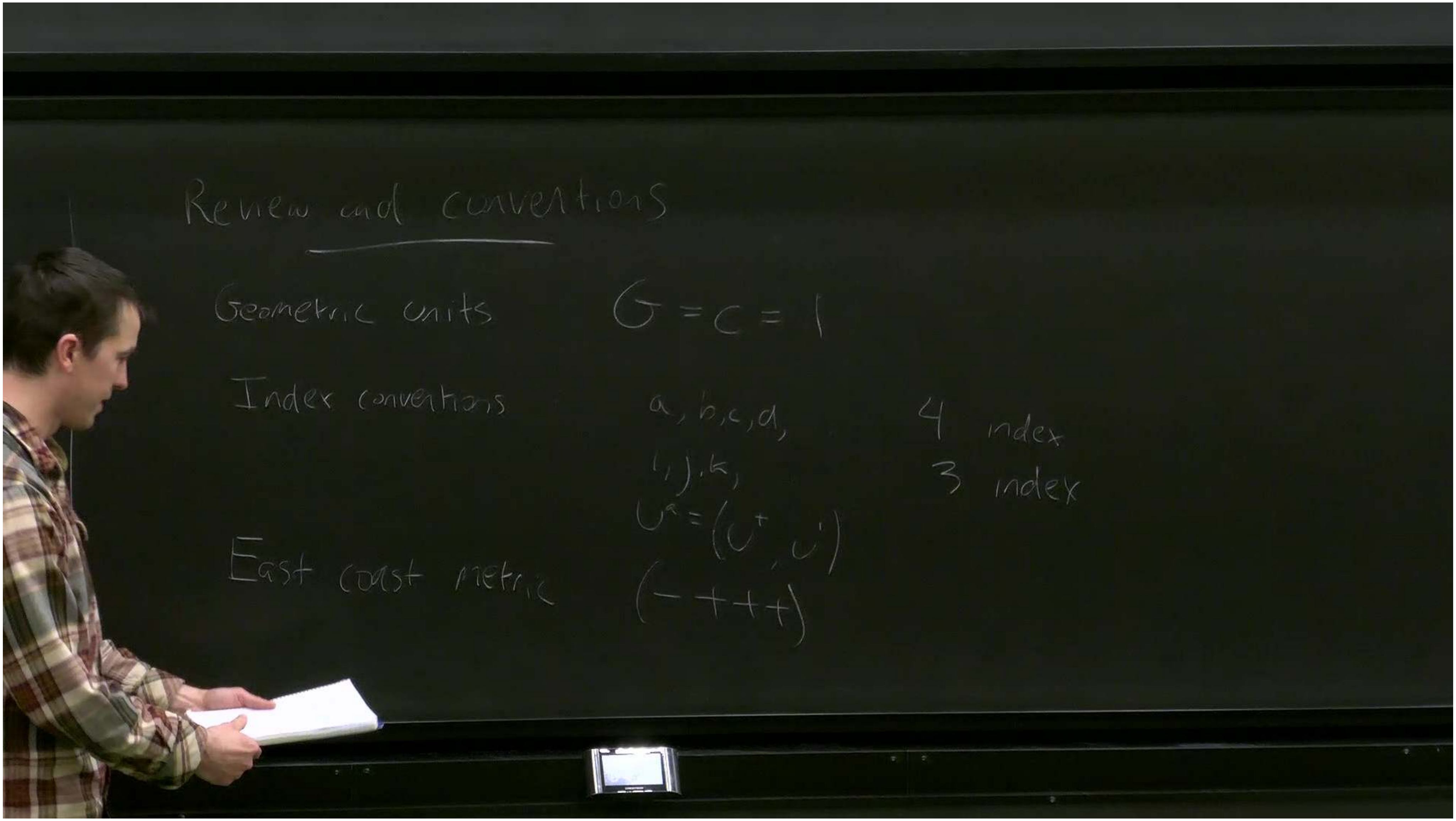
Date: February 23, 2026 - 4:30 PM

URL: https://pirsa.org/26020010

GW150914



GW250114 SNR ~ 80


Review and conventions
Geometric units $G = c = 1$
Index conventions
$a, b, c, d,$ 4 index
$i, j, k,$ 3 index
$u^a = (u^t, u^i)$
East coast metric
$(- + + +)$

Metric: $ds^2 = g_{ab}\, dx^a\, dx^b$

Christoffel Symbols: $\Gamma^a_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc} \right)$

$$\nabla_a T^b{}_c = \partial_a T^b{}_c + \Gamma^b{}_{ad} T^d{}_c - \Gamma^d{}_{ca} T^b{}_d$$

Metric compatible: $\nabla_a g_{bc} = 0$

Riemann tensor

$$(\nabla_a \nabla_b - \nabla_b \nabla_a) V^c = R^c{}_{dab} V^d$$

$$R^c{}_{dab} = \partial_a \Gamma^c{}_{bd} - \partial_b \Gamma^c{}_{db} + \Gamma\,\Gamma - \Gamma\,\Gamma$$

Ricc. tensor $R_{ab} = R^c{}_{acb}$

Ricc. Scalar $R = R^a{}_a$

Einsten Tensor $G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}$

Einstein Eqns:

$$G_{ab} = 8\pi T_{ab} \qquad \nabla_a G^{ab} = 0$$

Perfect fluid S.E. tensor:

$$T_{ab} = (\rho + P) u_a u_b + P g_{ab}$$

$$\nabla_a T^{ab} = 0$$

Null energy condition

$$T_{ab}\, k^a k^b \geq 0$$

k^a future pointing null vector

$$(\rho + P)(u^a k_a)^2 \geq 0$$

Metric in Boyer-Lindquist coordinates

$$ds^2 = -\left(\frac{\Delta - a^2 \sin^2\theta}{\Sigma}\right) dt^2 - \frac{2 a \sin^2\theta \,(r^2 + a^2 - \Delta)}{\Sigma}\, dt\, d\phi$$

where $\Sigma = r^2 + a^2 \cos^2\theta$

$\Delta = r^2 + a^2 - 2Mr$

$a = J/M$

$\bar{a} = J/M^2$

$a = J/M$

$\bar{a} = J/M^2$

$a = 0$ Schwarzschild metric

$M \to 0$ Fix $\bar{a}$

$$ds^2 = -dt^2 + \left(\frac{r^2 + a^2\cos^2\theta}{r^2 + a^2}\right) dr^2 + (r^2 + a^2\cos^2\theta)\, d\theta^2 + (r^2 + a^2)\sin^2\theta\, d\phi^2$$

$$x = (r^2+a^2)^{1/2} \sin\theta \cos\phi$$
$$y = (r^2+a^2)^{1/2} \sin\theta \sin\phi$$
$$z = r\cos\theta$$

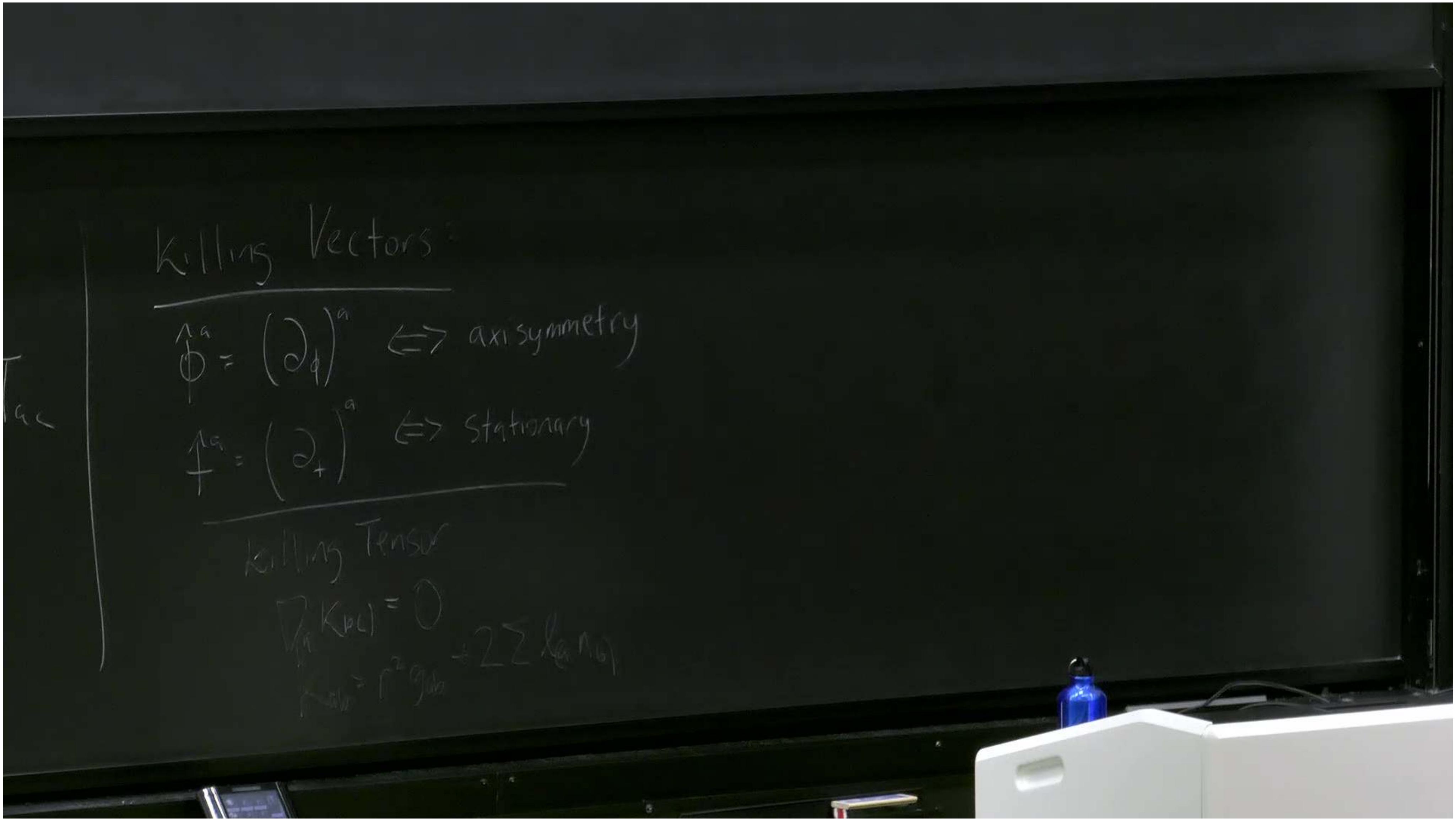
$d\phi^2$

Killing $\qquad \mathcal{L}_k \, g_{ab} = 0$

$$\mathcal{L}_k T_{ab} = k^c \partial_c T_{ab} + (\partial_a k^c) T_{cb} + (\partial_b k^c) T_{ac}$$

$$\partial_a \to \nabla_a$$

$$\mathcal{L}_k g_{ab} = 0 + \nabla_a k_b + \nabla_b k_a = 2 \nabla_{(a} k_{b)}$$


Killing Vectors:
φ^a = (∂_φ)^a ⟺ axisymmetry
t^a = (∂_t)^a ⟺ stationary
Killing Tensor
∇_(a K_bc) = 0
K_ab = r^2 g_ab + 2Σ l_(a n_b)

$\bar{a} = J/M^2$

Coordinates breakdown $\Sigma = 0,\ \Delta = 0$

$\Delta = (r - r_+)$

$R_{abcd} R^{abcd}$ blows up $\Sigma \to 0$

$r^2 + a^2 \cos^2\theta = 0$

$r =$ constant $\quad g^{ab} \nabla_a r \nabla_b r = g^{rr} = 0$ null $\Rightarrow g^{rr} = \dfrac{\Delta}{\Sigma}$

$$\Delta = (r - r_+)(r - r_-) \qquad r_\pm = M \pm \sqrt{M^2 - a^2}$$

Kerr-Schild metric

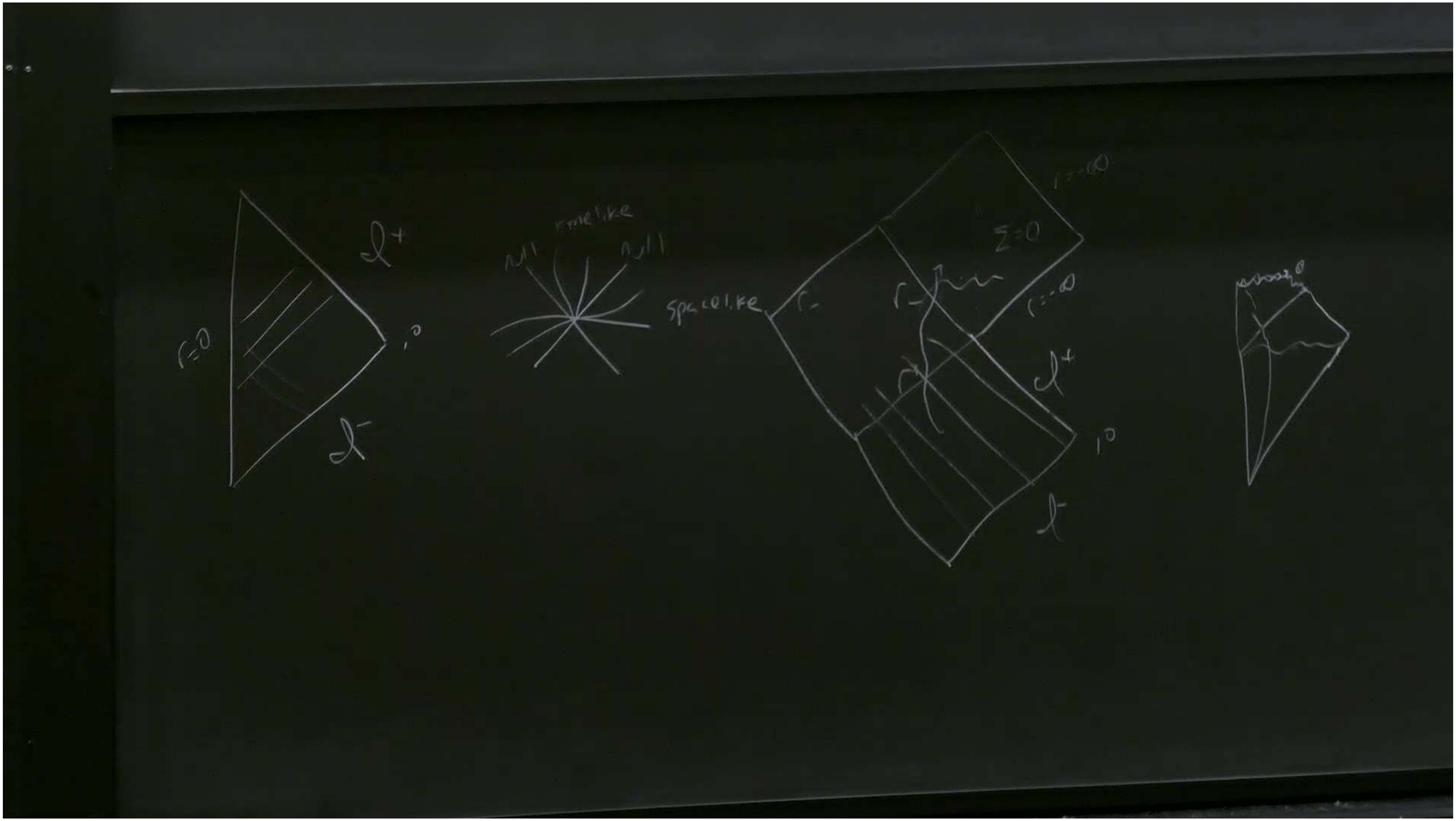
$$t_{KS} = t_{BL} + 2M \int \frac{r}{\Delta}\, dr$$

$$\phi_{KS} = \phi_{BL} + a \int \frac{dr}{\Delta}$$

$$x + iy = (r - ia) e^{i\phi_{KS}} \sin\theta$$

$$z = r\cos\theta$$

$x^2 + y^2 = a^2$ is singular



timelike
null
null
spacelike