Title: Intro to the BV formalism

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Collection/Series: Training Programs (TEOSP)

Subject: Other

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Intro to BV

Motivation:

- Need quantize gauge theories
- Prove Renormalizability
- Unifies e.o.m's and gauge sym's: Indispensable for SUGRA
- Makes (pert) CS (and HT theories) easy

Problem: $\nexists!$ Propagator

$$S_{MW} = \int F^2$$

$$= \int A(\eta\Box - \partial\partial)A$$

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$$S_{MW} = \int F^2$$

$$= \int A\underbrace{(\eta\Box - \partial\partial)}_{D}A$$

$DP = \delta$

$P + \partial\alpha$

B.V as deformation of integrals:

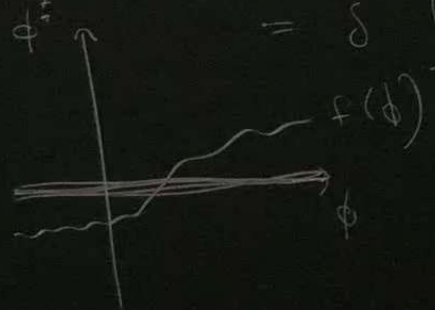
$S(\phi, 0) = S_0(\phi)$

$$\delta\langle \mathcal{O} \ldots \rangle_f = \int d\phi \, \underbrace{e^{-S(\phi, \phi^\dagger = f)} \, \mathcal{O} \ldots}_{F} \, \cancel{\delta(\phi^\dagger - f(\phi))} \quad ; \quad f = \frac{\partial \psi}{\partial \phi}$$

$$= \delta \int d\phi \, F(\phi, \phi^\dagger = f) = \int d\phi \, \delta f \, \frac{\partial F}{\partial \phi^\dagger} \qquad \boxed{\Delta F = 0}$$

$$= \int d\phi \, \frac{\partial \delta\psi}{\partial \phi} \frac{\partial F}{\partial \phi^\dagger}$$

$$= \int d\phi \, \delta\psi \, \Delta F \, , \quad \Delta = \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi^\dagger}$$



• Prove Renormalizability

• Unifies e.o.m.s and gauge sym's: Indispensable for SUGRA

• Makes (pert) CS (and HT theories) easy

$DP = \delta$

$P + \partial\alpha$

$\int e^{-\frac{1}{\hbar}S}$

Classical Master Eq

$$0 = \Delta e^{-\frac{1}{\hbar}S} = \frac{1}{\hbar^2}\Big(\boxed{\frac{\partial S}{\partial \phi}\frac{\partial S}{\partial \phi^+}} - \hbar\Delta S\Big)e$$

$\frac{\partial S}{\partial \phi}\frac{\partial S}{\partial \phi^+} = 0$

$$F = e^{-\frac{1}{\hbar}S}\,\mathcal{O}$$

$$\Delta F = \frac{-1}{\hbar}\Big(\underbrace{\frac{\partial S}{\partial \phi}\frac{\partial \mathcal{O}}{\partial \phi^+} + \frac{\partial S}{\partial \phi^+}\frac{\partial \mathcal{O}}{\partial \phi}}_{=\{S,\mathcal{O}\}=0} - \hbar\Delta\mathcal{O}\Big)$$

$0 = \{S,\mathcal{O}\} \Leftrightarrow \mathcal{O}$ is gauge inv.

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$e^{-\frac{1}{\hbar}S}$

Classical Master Eq $\Leftrightarrow$ $\{S,S\} = 0$ $\Leftrightarrow$ $Q^2 = 0$

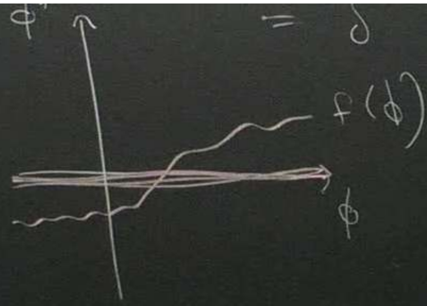
$$0 = \Delta e^{-\frac{1}{\hbar}S} = \frac{1}{\hbar^2}\left(\boxed{\frac{\partial S}{\partial\phi}\frac{\partial S}{\partial\phi^{\dagger}}} - \hbar\Delta S\right)e^{\cdots}$$

$\frac{\partial S}{\partial\phi}\frac{\partial S}{\partial\phi^{\dagger}} = 0$

$F = e^{-\frac{1}{\hbar}S}\cdot\mathcal{O}$

$$\Delta F = \frac{-1}{\hbar}\left(\underbrace{\frac{\partial S}{\partial\phi}\frac{\partial\mathcal{O}}{\partial\phi^{\dagger}} + \frac{\partial S}{\partial\phi^{\dagger}}\frac{\partial\mathcal{O}}{\partial\phi}}_{=\{S,\mathcal{O}\}=0} - \hbar\Delta\mathcal{O}\right)$$

$0 = \underbrace{\{S,\mathcal{O}\}}_{Q\mathcal{O}} \Leftrightarrow \mathcal{O}$ is gauge inv.



$$= \delta \int d\phi\, F(\phi, \phi^\dagger = f) = \int d\phi\, \ldots \frac{\ldots}{\partial \phi^\dagger}$$

$$\boxed{\Delta F = 0}$$

$$= \int d\phi\, \frac{\partial \delta\psi}{\partial \phi} \frac{\partial F}{\partial \phi^\dagger}$$

$$= \int d\phi\, \delta\psi\, \Delta F \,, \quad \Delta = \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi^\dagger}$$

$$\delta_\alpha \phi = \alpha^i \phi$$

$$\downarrow$$

$$Q \phi = c^i \phi$$

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Classical Master Eq

$$\{S,S\} = 0 \quad Q^2 = 0$$

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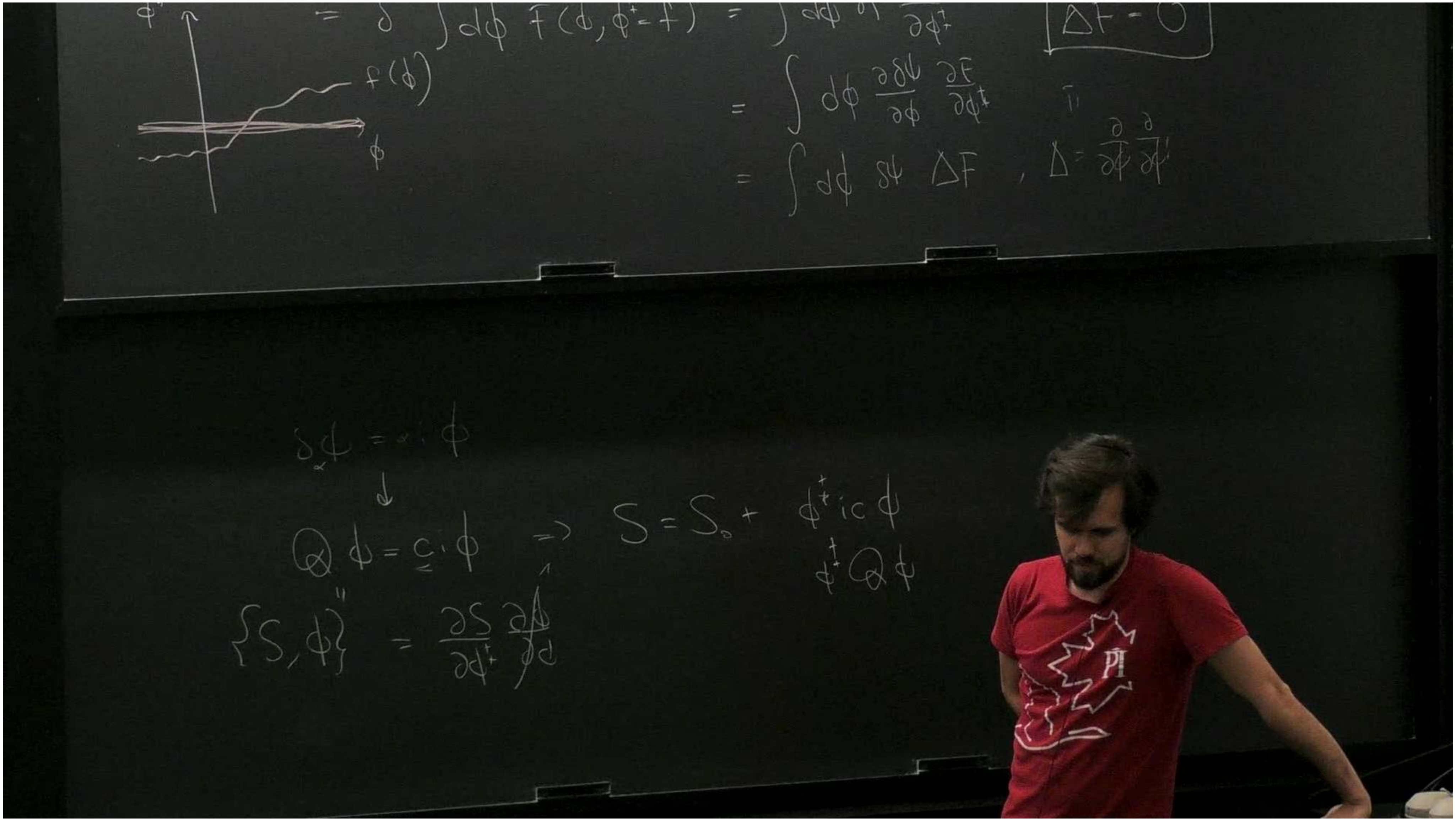
$$F = e^{-\frac{1}{\hbar}S}\,\mathcal{O}$$

$$\Delta F = \frac{-1}{\hbar}\Big(\underbrace{\frac{\partial S}{\partial\phi}\frac{\partial\mathcal{O}}{\partial\phi^\dagger} + \frac{\partial S}{\partial\phi^\dagger}\frac{\partial\mathcal{O}}{\partial\phi}}_{=:\{S,\mathcal{O}\}=0} - \hbar\Delta\mathcal{O}\Big)$$

$$\boxed{0 = \underbrace{\{S,\mathcal{O}\}}_{Q\mathcal{O}} \Leftrightarrow \mathcal{O} \text{ is gauge inv.}}$$

$$Q = Q^\phi\frac{\partial}{\partial\phi} + Q^{\phi^\dagger}\frac{\partial}{\partial\phi^\dagger}$$



BV as deformation of integrals

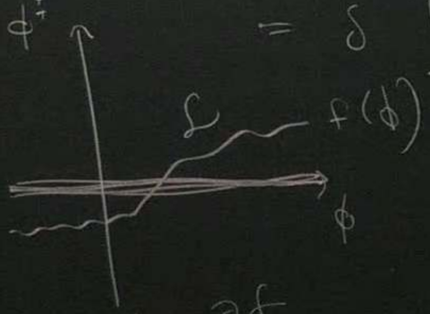
$S(\phi, 0) = S_0(\phi)$

$$\delta\langle O \ldots \rangle_f = \int d\phi\, \underbrace{e^{-S(\phi, \cancel{\phi^\ddagger}\, f)}\, O}_{F} \ldots \cancel{\delta(\phi^\ddagger - f(\phi))} \quad ; \quad f = \frac{\partial \psi}{\partial \phi}$$

$$= \delta \int d\phi\, F(\phi, \phi^\ddagger = f) = \int d\phi\, \delta f \frac{\partial F}{\partial \phi^\ddagger} \qquad \boxed{\Delta F = 0}$$

$$= \int d\phi\, \frac{\partial \delta\psi}{\partial \phi} \frac{\partial F}{\partial \phi^\ddagger}$$

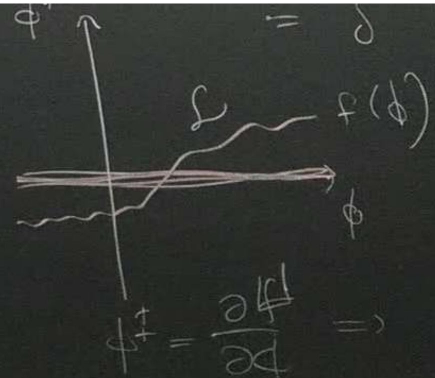
$$= \int d\phi\, \delta\psi\, \Delta F \ , \quad \Delta = \frac{\partial}{\partial \phi}\frac{\partial}{\partial \phi^\ddagger}$$



$$\phi^\ddagger = \frac{\partial f}{\partial \phi} \Rightarrow \underbrace{\omega = d\phi \wedge d\phi^\ddagger \Big|_{\mathcal{L}}}_{=0}$$

$\delta\phi = Q\phi \Rightarrow S = S_0 + \phi^\ddagger Q \phi$

$$\{S, \phi\} = \frac{\partial S}{\partial \phi^\ddagger}\frac{\partial \phi}{\partial \phi}$$



$$= \delta \int d\phi\, F(\phi, \phi^+ = f) = \int d\phi \ldots \frac{\ldots}{\partial \phi^+} \qquad \boxed{\Delta F = 0}$$

$$= \int d\phi \frac{\partial \delta\psi}{\partial \phi} \frac{\partial F}{\partial \phi^+}$$

$$= \int d\phi\, \delta\psi\, \Delta F , \quad \Delta = \frac{\partial}{\partial \phi}\frac{\partial}{\partial \phi^+}$$

$$\phi^+ = \frac{\partial \psi}{\partial \phi} \Rightarrow \underset{=0}{\omega = d\phi \wedge d\phi^+}\Big|_{\mathcal{L}}$$

Steps:

- Introduce fermionic gauge parameter fields
- " antifields for everything
- Choose Lagrangian sub-mfld

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• Makes (pert) CS (and HT theories) easy

$D P = \delta$

$p + \partial\alpha$

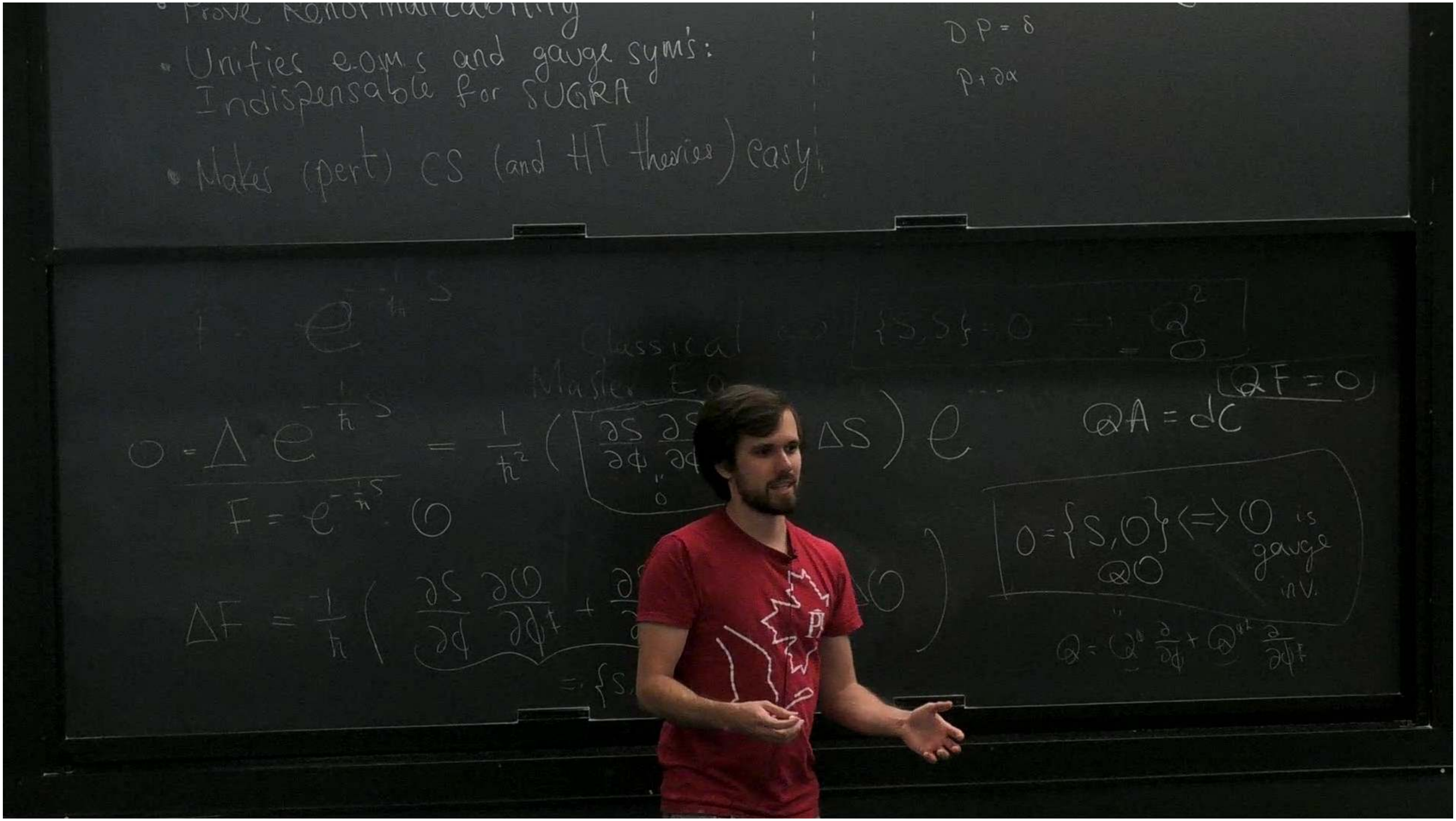
$$S = \int F^2 + \int d^\dagger b \; \underbrace{dc}_{\partial A} + \int c^\dagger \cancel{\partial}^{\,0} c$$

$d^\dagger \delta b \qquad c^\dagger = 0$

$$0 = \omega\big|_{\mathcal{L}} = \int \delta A\, \delta A^\dagger + \delta c\, \delta c^\dagger \qquad (\; d\phi \wedge d\phi^\dagger)$$

$$\mathcal{L} = \left\{ c^\dagger = 0,\; A^\dagger = d^\dagger b \quad {*d*} \atop A \in \ker d^\dagger \right\}$$

$d^\dagger A = \partial_\mu A^\mu = 0$



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Master Eq
QF = 0
QA = dC
F = e^{-S/ħ} O
0 = {S, O} ⟺ O is gauge inv.

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$$0 = \omega\big|_{\mathcal{L}} \int \delta A\, \delta A^\dagger + \delta c\, \cancel{\delta} c^\dagger \qquad (d\phi \wedge d\phi^\dagger)$$

$$\mathcal{L} = \left\{ c^\dagger = 0,\ A^\dagger = d^\dagger b \qquad *d* \right\} \qquad \boxed{A \in \ker d^\dagger}$$

$d^\dagger A = \partial_\mu A^\mu = 0 \ ; \quad \partial_\mu P^{\mu\nu} = 0$

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$$D P = \delta$$

$$p + \partial\alpha$$

$$S = \int F^2 + \int \underset{d^\dagger \delta b}{d^\dagger b} \, \underset{Q(b\, \partial_\mu A^\mu)}{dc} + \int c^\ddagger \overset{0}{\cancel{\phi}} c$$

$$0 = \omega\big|_{\mathcal{L}} \int \delta A \, \delta A^\ddagger + \delta c \, \cancel{\delta} c^\ddagger \qquad (d\phi \wedge d\phi^\ddagger)$$

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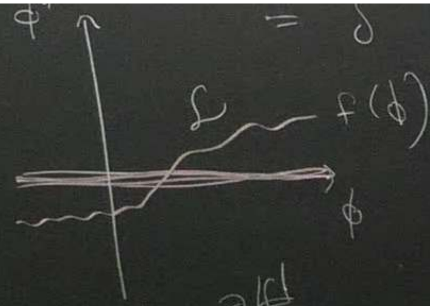

$$= \delta \int d\phi\, F(\phi, \phi^{+} = f) = \int d\phi\, \ldots \frac{\partial}{\partial \phi^{+}} \qquad \boxed{\Delta F = 0}$$

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$$\phi^{+} = \frac{\partial \psi}{\partial \phi} \Rightarrow \omega = \underbrace{d\phi \wedge d\phi^{+}\Big|_{\mathcal{L}}}_{=0}$$

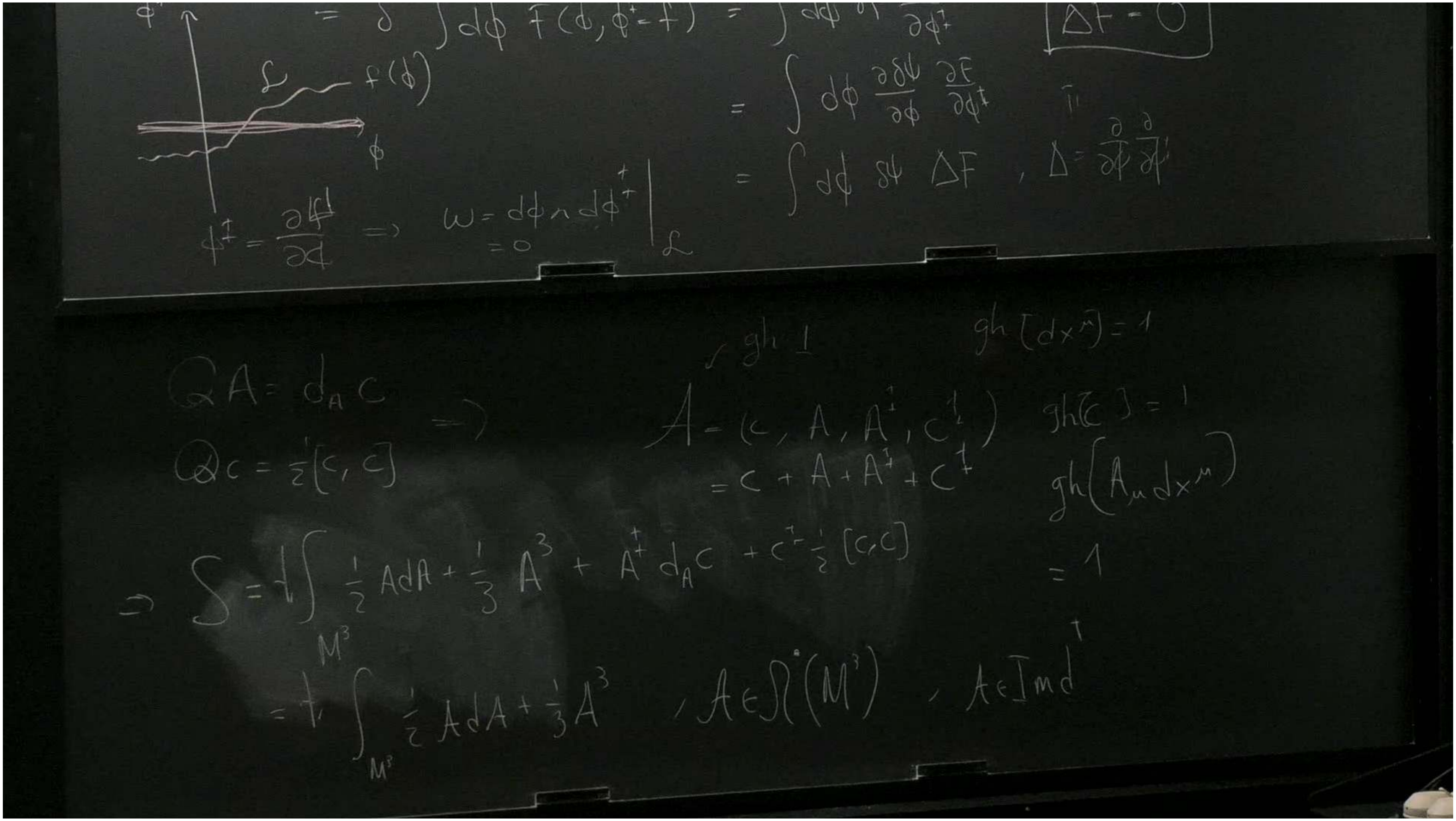
$$QA = d_A c$$

$$Qc = \tfrac{1}{2}[c, c] \quad \Rightarrow \quad \mathcal{A} = (\overset{\text{gh } 1}{c}, A, A^{+}, c^{+})$$

$$= c + A + A^{+} + c^{+}$$

$$\Rightarrow S = \mathrm{tr} \int_{M^3} \tfrac{1}{2} A dA + \tfrac{1}{3} A^3 + A^{+} d_A c + c^{+} \tfrac{1}{2}[c, c]$$

$$= \mathrm{tr} \int_{M^3} \tfrac{1}{2} A dA + \tfrac{1}{3} A^3$$


= δ ∫ dφ F(φ, φ† = f) = ∫ dφ ∂/∂φ†
ΔF = 0
f(φ)
= ∫ dφ ∂δψ/∂φ ∂F/∂φ†
= ∫ dφ δψ ΔF , Δ = ∂/∂φ ∂/∂φ†
φ† = ∂f/∂φ ⟹ ω = dφ ∧ dφ†|_L = 0
QA = d_A c
Qc = 1/2 [c, c]
gh 1
gh [dx^μ] = 1
𝒜 = (c, A, A†, c†)
= c + A + A† + c†
gh[c] = 1
gh(A_μ dx^μ)
= 1
⟹ S = ∫_{M^3} 1/2 AdA + 1/3 A^3 + A† d_A c + c† 1/2 [c, c]
= ∫_{M^3} 1/2 𝒜d𝒜 + 1/3 𝒜^3 , 𝒜 ∈ Ω^•(M^3) , 𝒜 ∈ Im d†