Title: Lecture - Standard Model (Elective), PHYS 622

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Collection/Series: Standard Model (Elective), PHYS 622, January 5 - February 6, 2026

Subject: Particle Physics

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Cancellation of gauge anomalies $\frac{1-\gamma^5}{2}$

1) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}\,(i\partial\!\!\!/ - eA\!\!\!/)\, P_L \psi$

$J_L^\mu = \bar{\psi} \gamma^\mu P_L \psi$

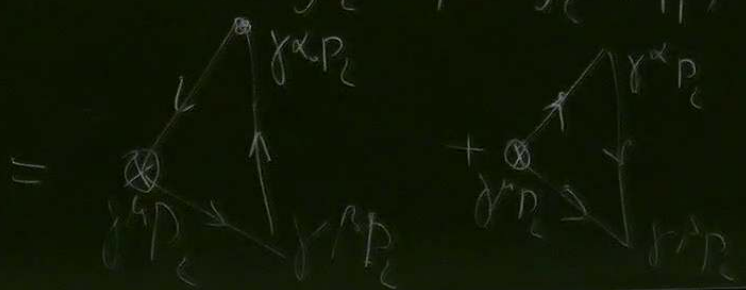
Check: $\partial_\mu J_L^\mu = 0$

γ^5

$$\langle \partial_\mu J_L^\mu(x) \rangle_A = \frac{ie^2}{2} \int \frac{d^4p\, d^4q_1\, d^4q_2}{(2\pi)^{12}} A_\alpha(q_1) A_\beta(q_2)\, e^{-ipx}$$

$$(2\pi)^4\, \delta(p - q_1 - q_2)\, p_\mu M_L^{\mu\alpha\beta}(p, q_1, q_2)$$

$$M_L^{\mu\alpha\beta} = \langle J_L^\mu(-p)\, J_L^\alpha(q_1)\, J_L^\beta(q_2) \rangle =$$

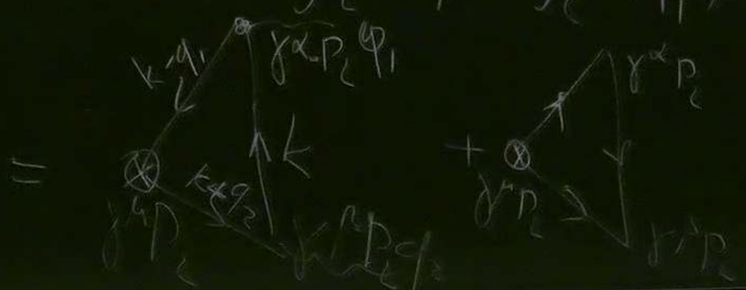


γ^5

$$\langle \partial_\mu j_L^\mu(x) \rangle_A = \frac{ie^2}{2} \int \frac{d^4p \, d^4q_1 \, d^4q_2}{(2\pi)^{12}} A_\alpha(q_1) A_\beta(q_2) e^{-ipx}$$

$$(2\pi)^4 \delta(p - q_1 - q_2) \, p_\mu M_L^{\mu\alpha\beta}(p, q_1, q_2)$$

$$M_L^{\mu\alpha\beta} = \langle j_L^\mu(-p) \, j_L^\alpha(q_1) \, j_L^\beta(q_2) \rangle =$$



$$M_L^{\mu\alpha\beta} = \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{tr\left(\gamma^\alpha \not k \gamma^\beta (\not k + \not q_2) \gamma^\mu P_L (\not k - \not q_1)\right)}{k^2 (k+q_2)^2 (k-q_1)^2} + (\alpha, q_1 \leftrightarrow \beta, q_2) \right\}$$

$$P_L = \frac{1-\gamma^5}{2}$$

$$p_\mu M_L^{\mu\alpha\beta} = p_\mu \frac{1}{2}\left(\cancel{M_V^{\mu\alpha\beta}} - M_5^{\mu\alpha\beta}\right) = -\frac{1}{2} p_\mu M_5^{\mu\alpha\beta} \neq 0$$

$$\langle J_V^\mu J_V^\alpha J_V^\beta \rangle \qquad \langle J_5^\mu J_V^\alpha J_V^\beta \rangle$$

Cancellation of gauge anomalies

$$P_L = \frac{1-\gamma^5}{2}$$

1) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}\,(i\not{\partial} - e\not{A})\,P_L\,\psi$

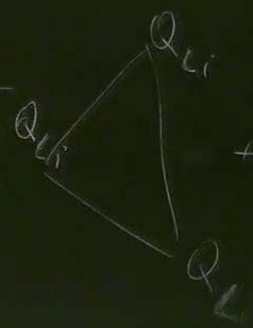
$$J^\mu_L = \bar{\psi}\,\gamma^\mu P_L\,\psi$$

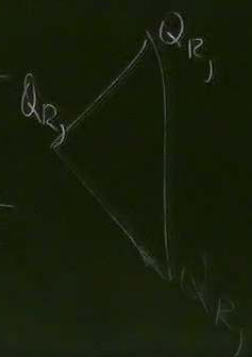
Check: $\partial_\mu J^\mu_L = 0$

$$\boxed{\langle \partial_\mu J^\mu_L \rangle_A = \frac{e^2}{32\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu}}$$

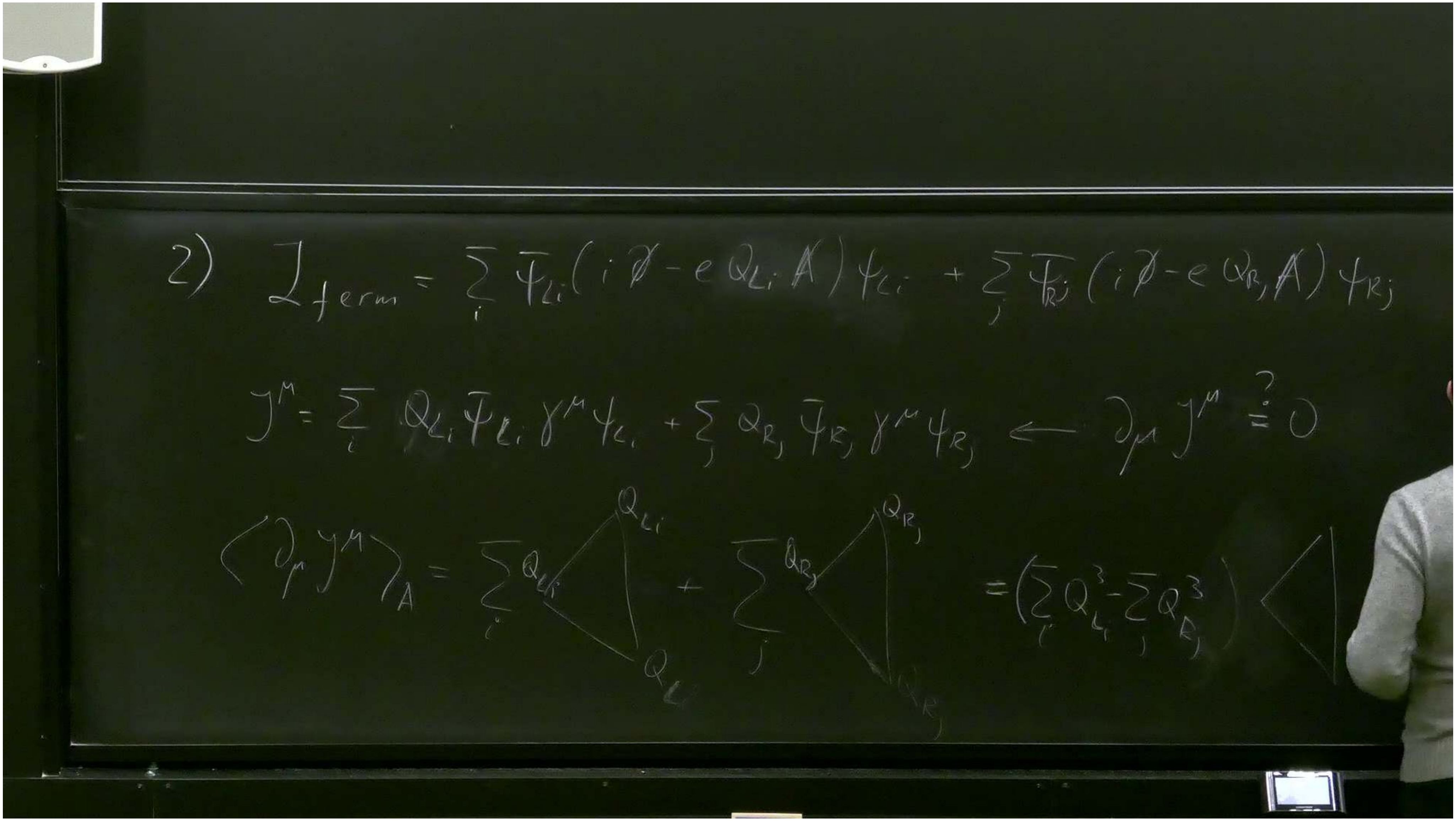
2) $\mathcal{L}_{ferm} = \sum_i \bar{\psi}_{L_i}\left(i\not{\partial} - e Q_{L_i} \not{A}\right)\psi_{L_i} + \sum_j \bar{\psi}_{R_j}\left(i\not{\partial} - e Q_{R_j} \not{A}\right)\psi_{R_j}$

$J^\mu = \sum_i Q_{L_i}\bar{\psi}_{L_i}\gamma^\mu\psi_{L_i} + \sum_j Q_{R_j}\bar{\psi}_{R_j}\gamma^\mu\psi_{R_j} \longleftarrow \partial_\mu J^\mu \overset{?}{=} 0$

$\langle \partial_\mu J^\mu \rangle_A = \sum_i$ 

$+ \sum_j$ 

$= \left(\sum_i Q_{L_i}^3\right.$ 


2) $\mathcal{L}_{ferm} = \sum_i \bar{\psi}_{Li}(i\not{\partial} - eQ_{Li}\not{A})\psi_{Li} + \sum_j \bar{\psi}_{Rj}(i\not{\partial} - eQ_{Rj}\not{A})\psi_{Rj}$
$J^\mu = \sum_i Q_{Li}\bar{\psi}_{Li}\gamma^\mu\psi_{Li} + \sum_j Q_{Rj}\bar{\psi}_{Rj}\gamma^\mu\psi_{Rj} \leftarrow \partial_\mu J^\mu \overset{?}{=} 0$
$\langle \partial_\mu J^\mu \rangle_A = \sum_i$
Q_{Li}
Q_{Li}
Q_{Li}
$+ \sum_j$
Q_{Rj}
Q_{Rj}
Q_{Rj}
$= \left(\sum_i Q_{Li}^3 - \sum_j Q_{Rj}^3\right)$

$\ldots_R, A) \psi_{Rj}$

$\ldots \overset{?}{=} 0$

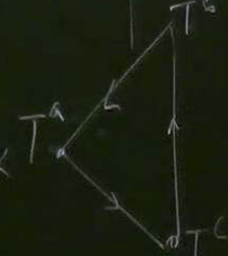
- $Q_L = Q_R \qquad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

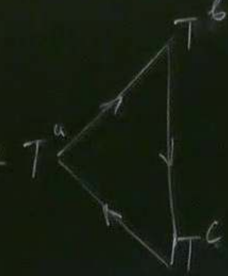
- $Q_L = 2\,, \quad Q_{R_1} = \ldots = Q_{R_8} = 1$

3) $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - g A\!\!\!/^a T^a) P_L \psi$ | $SU(N)$

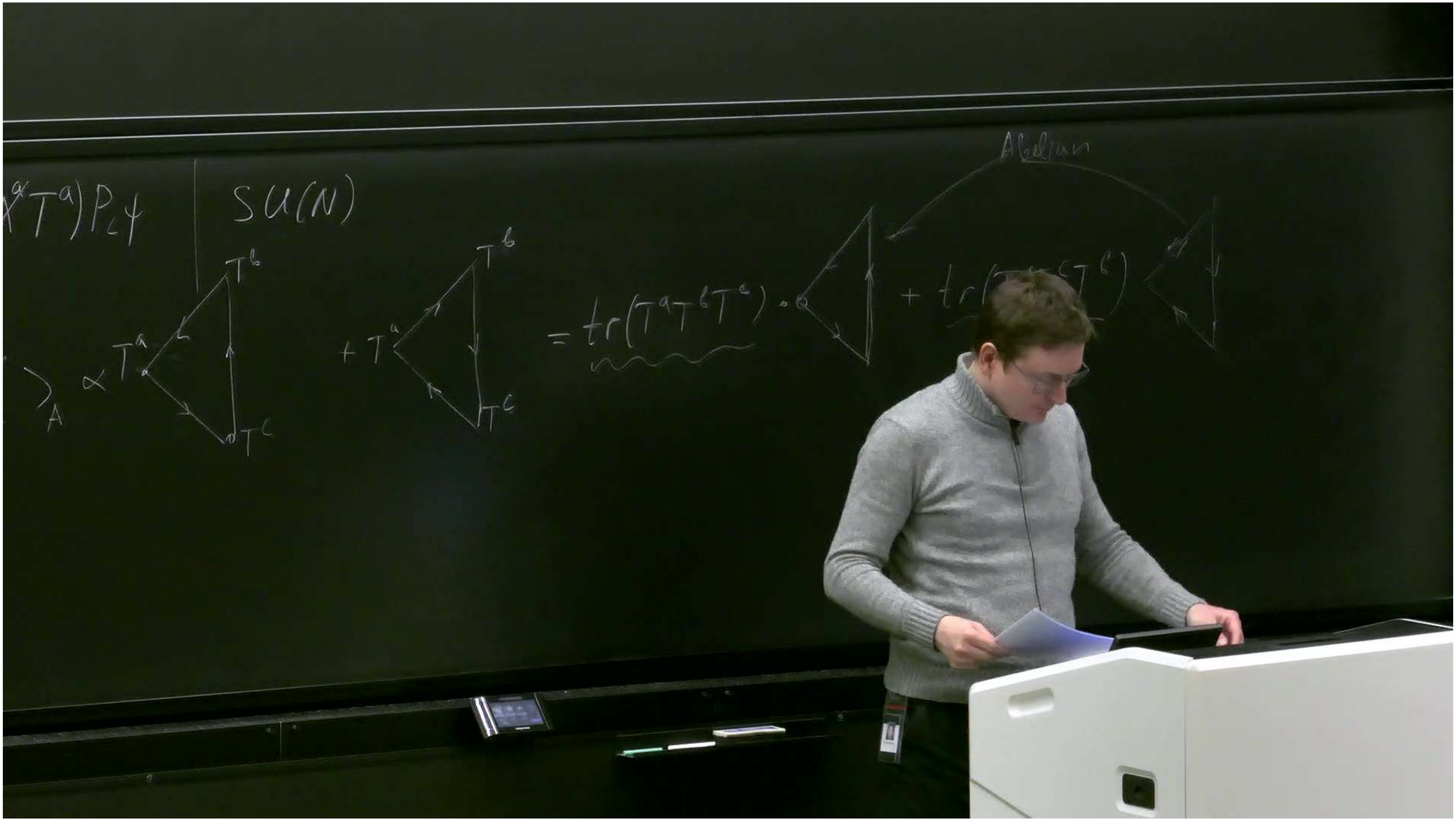
$J^{a\mu}_L = \bar{\psi}_i T^a_{ij} \gamma^\mu P_L \psi_j$

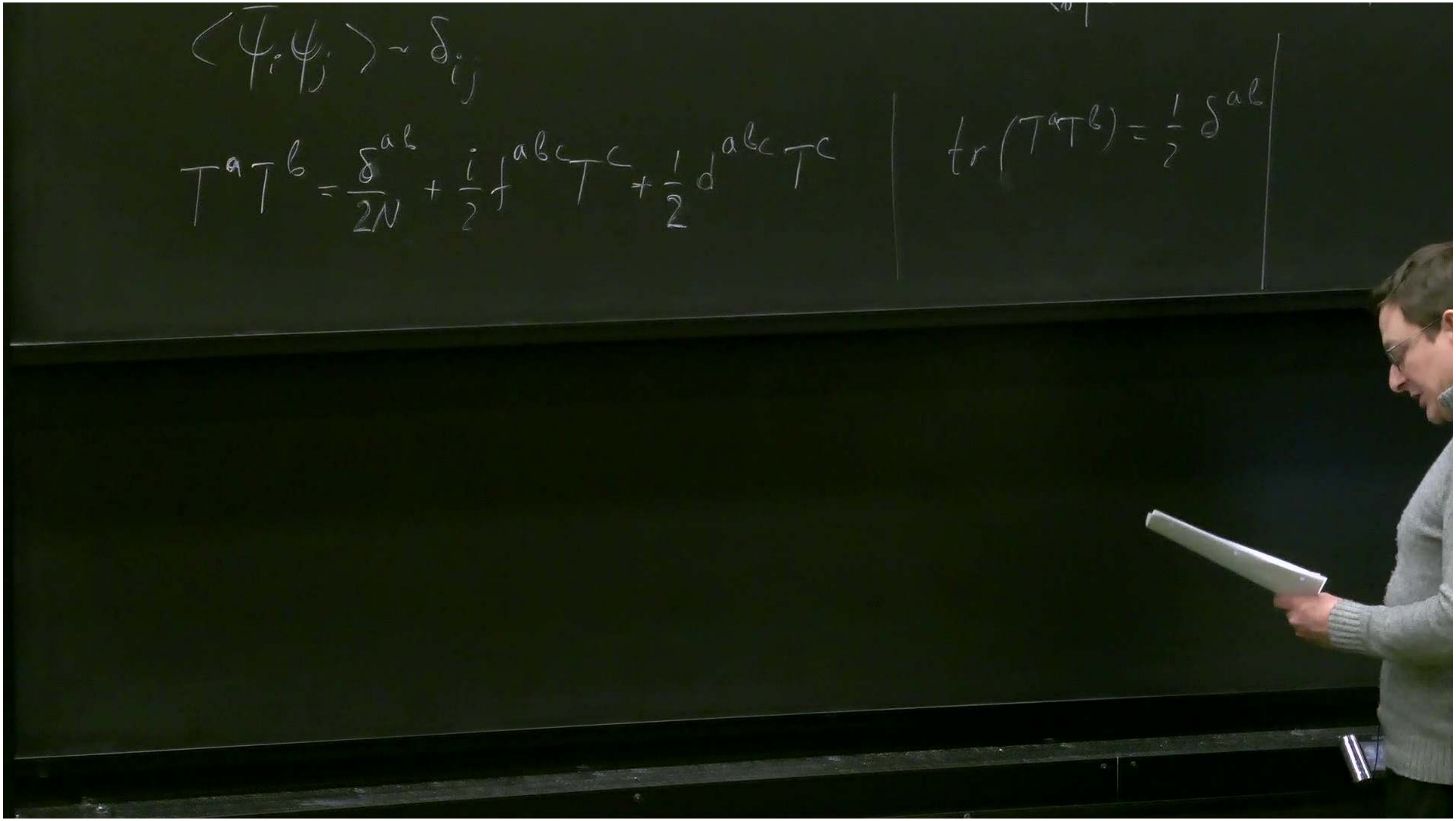
$\langle \bar{\psi}_i \psi_j \rangle \sim \delta_{ij}$

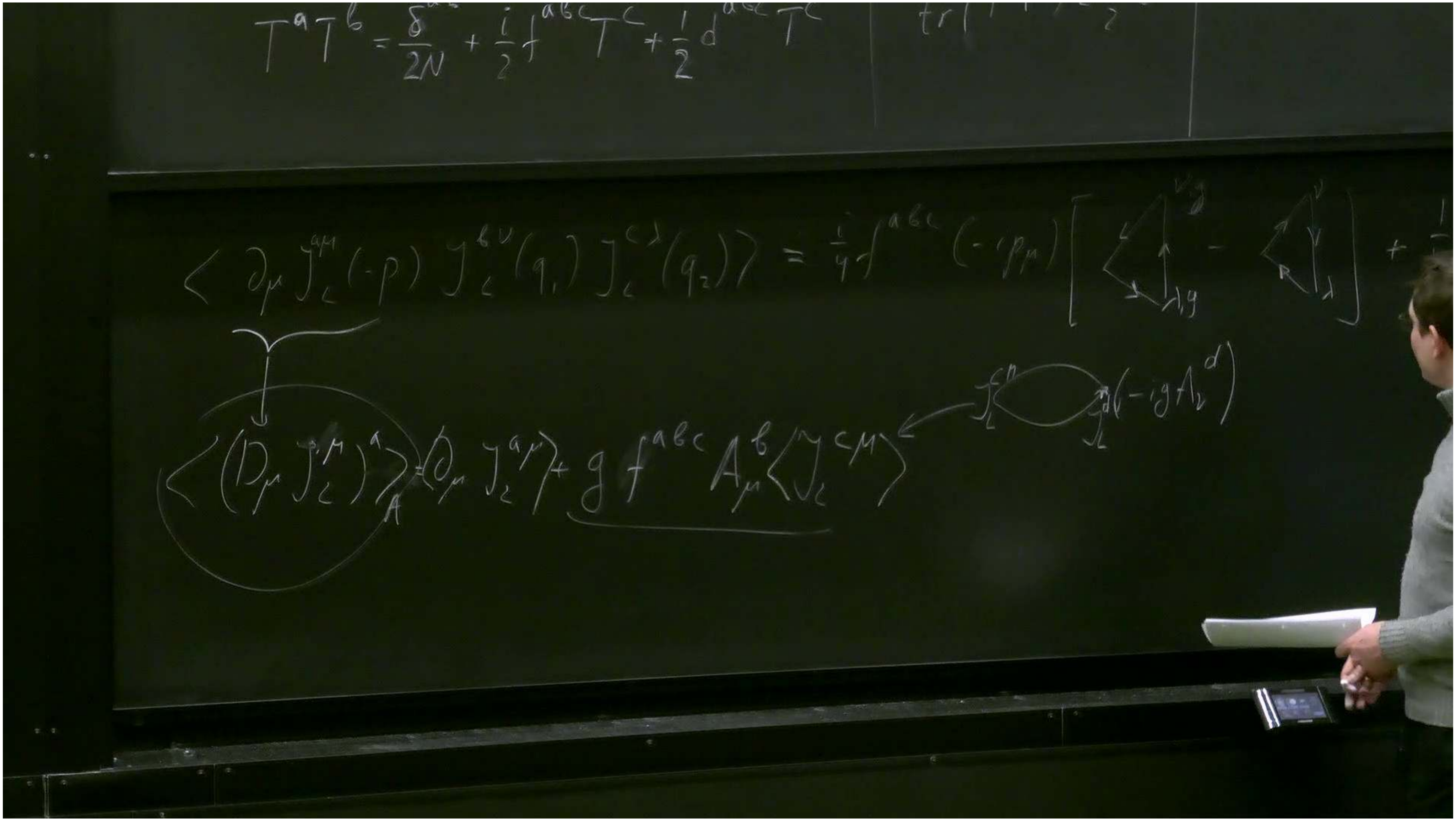
$\langle \partial_\mu J^{a\mu}_L \rangle_A \propto$ 

$+$ 

$= tr(T^a T$


SU(N)
Abelian


$\langle \bar{\psi}_i \psi_j \rangle \sim \delta_{ij}$
$T^a T^b = \frac{\delta^{ab}}{2N} + \frac{i}{2} f^{abc} T^c + \frac{1}{2} d^{abc} T^c$
$tr(T^a T^b) = \frac{1}{2} \delta^{ab}$

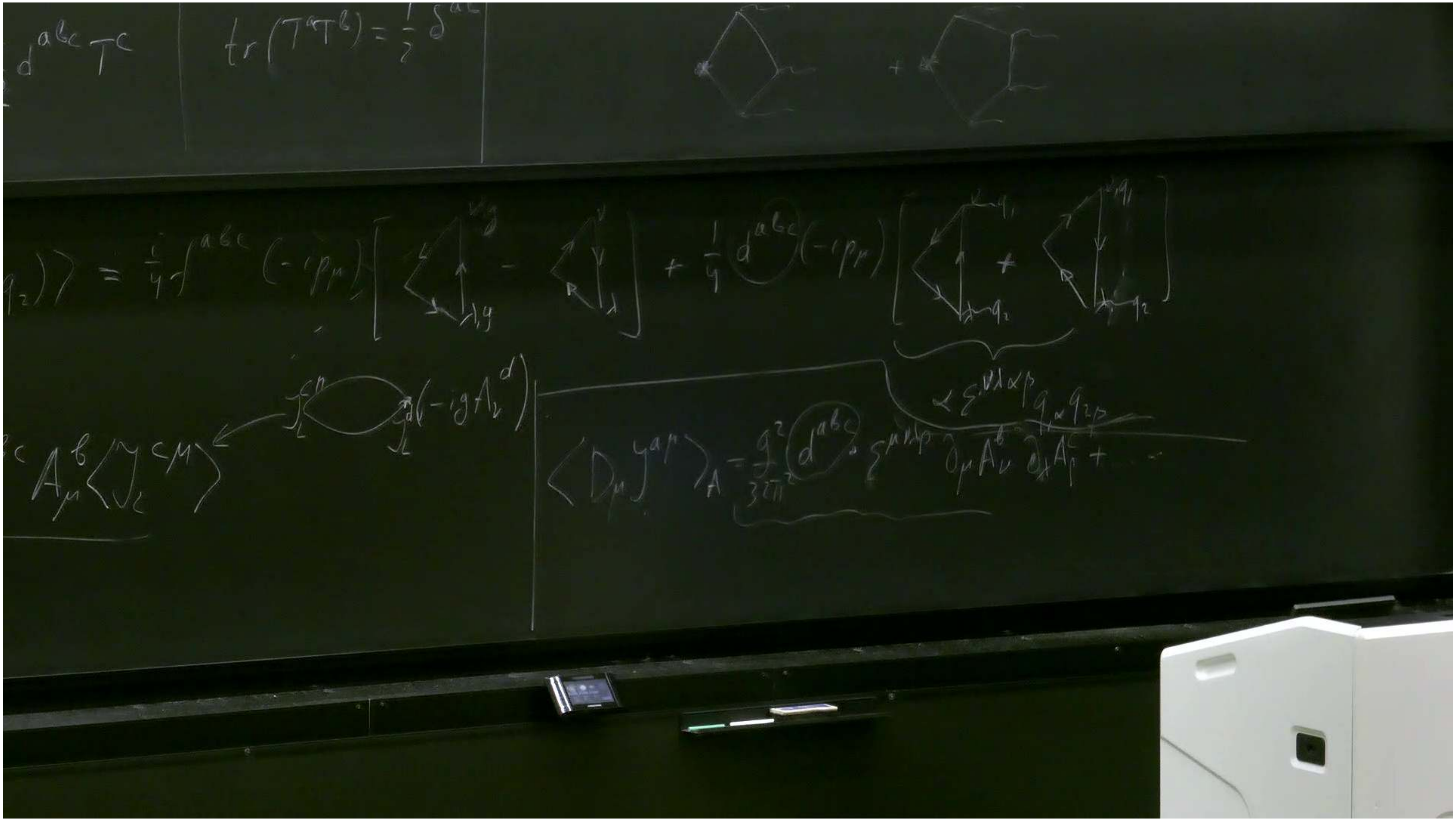

$T^a T^b = \frac{\delta^{ab}}{2N} + \frac{i}{2} f^{abc} T^c + \frac{1}{2} d^{abc} T^c$
$\langle \partial_\mu J_L^{a\mu}(-p)\, J_L^{b\nu}(q_1)\, J_L^{c\lambda}(q_2) \rangle = \frac{i}{4} f^{abc} (-ip_\mu) [\ \ -\ \ ] +$
$\langle (D_\mu J_L^\mu)^a \rangle_A = \partial_\mu J_L^{a\mu} + g f^{abc} A_\mu^b \langle J_L^{c\mu} \rangle$
$J_L^{c\mu}$
$J_L^d (-igA_\nu^d)$

$$T^a T^b = \frac{\delta^{ab}}{2N} + \frac{i}{2} f^{abc} T^c + \frac{1}{2} d^{abc} T^c \qquad \mathrm{tr}(\ldots$$

$$\langle \partial_\mu J_L^{a\mu}(-p)\, J_L^{b\nu}(q_1)\, J_L^{c\lambda}(q_2) \rangle = \frac{i}{4} f^{abc} (-ip_\mu) \Big[\ldots - \ldots \Big] + \frac{1}{4}\ldots$$

$J_L^{c\mu}$ $J_L^d(-igA_2^d)$

$$\langle (D_\mu J_L^\mu)^a \rangle_A = \partial_\mu \langle J_L^{a\mu} \rangle + \underline{g f^{abc} A_\mu^b} \langle J_L^{c\mu} \rangle$$


tr(T^a T^b) = 1/2 δ^ab
∝ ε^νλαρ q_1α q_2ρ
⟨D_μ J^aμ⟩_A = g²/32π² d^abc ε^μνλρ ∂_μ A_ν^b ∂_λ A_ρ^c + . . .

$R_\ell \, , \quad R_r$

$$\langle D_\mu J^{\mu a} \rangle_A \propto \left(\sum_{\text{left}} d^{abc}_{R_\ell} - \sum_{\text{right}} d^{abc}_{R_r} \right) \times \underline{\qquad\qquad} = \left(\sum_{\text{left}} A(R_\ell) - \right.$$

$$2\,\mathrm{Tr}\left[T^a_{R_\ell} \left(T^b_{R_\ell} T^c_{R_\ell} + T^c_{R_\ell} T^b_{R_\ell} \right) \right] = \underbrace{A(R_\ell)}_{\text{anomaly coefficient}} d^{abc}$$

e

$\left(\sum_{left} A(R_l) - \sum_{right} A(R_r) \right) d^{abc} = $

bc

efficient

- $d^{abc} = 0 \leftarrow SU(2)$

$tr(\sigma^a \{\sigma^b, \sigma^c\}) = 2\delta^{bc} tr(\sigma^a) = 0$

- vector coupling

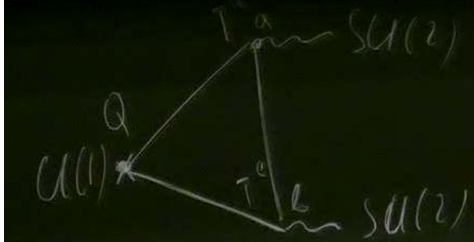
	$L=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$	e_R	$Q=\begin{pmatrix}u_L\\ d_L\end{pmatrix}$	u_R	d_R
SU(3)	—	—	fund	fund	fund
SU(2)	fund	—	fund	—	—
U(1)	Y_L	Y_e	Y_Q	Y_u	Y_d

- $SU(3)^3$ vector ✓
- $SU(2)^3$ no d^{abc} ✓
  even # of fermions
- $U(1)^3$ $\sum_{\text{left}} Y_i^3 - \sum_{\text{right}} Y_j^3 = 0$

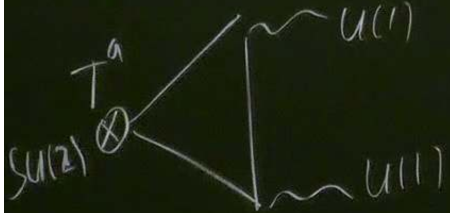
$$\boxed{2Y_L^3 + 3\cdot 2\, Y_Q^3 - Y_e^3 - 3Y_u^3 - 3Y_d^3 = 0}$$

$$T^a T^b = \frac{\delta^{ab}}{2N} + \frac{i}{2} f^{abc} T^c + \frac{1}{2} d^{abc} T^c$$

$$\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta$$



$$\langle \partial_\mu J^\mu \rangle_W = \mathrm{Tr}(T^a T^b) \propto Q \cdot \frac{g^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^a_{\mu\nu}$$



$$\propto \mathrm{Tr}(T^a) = 0$$

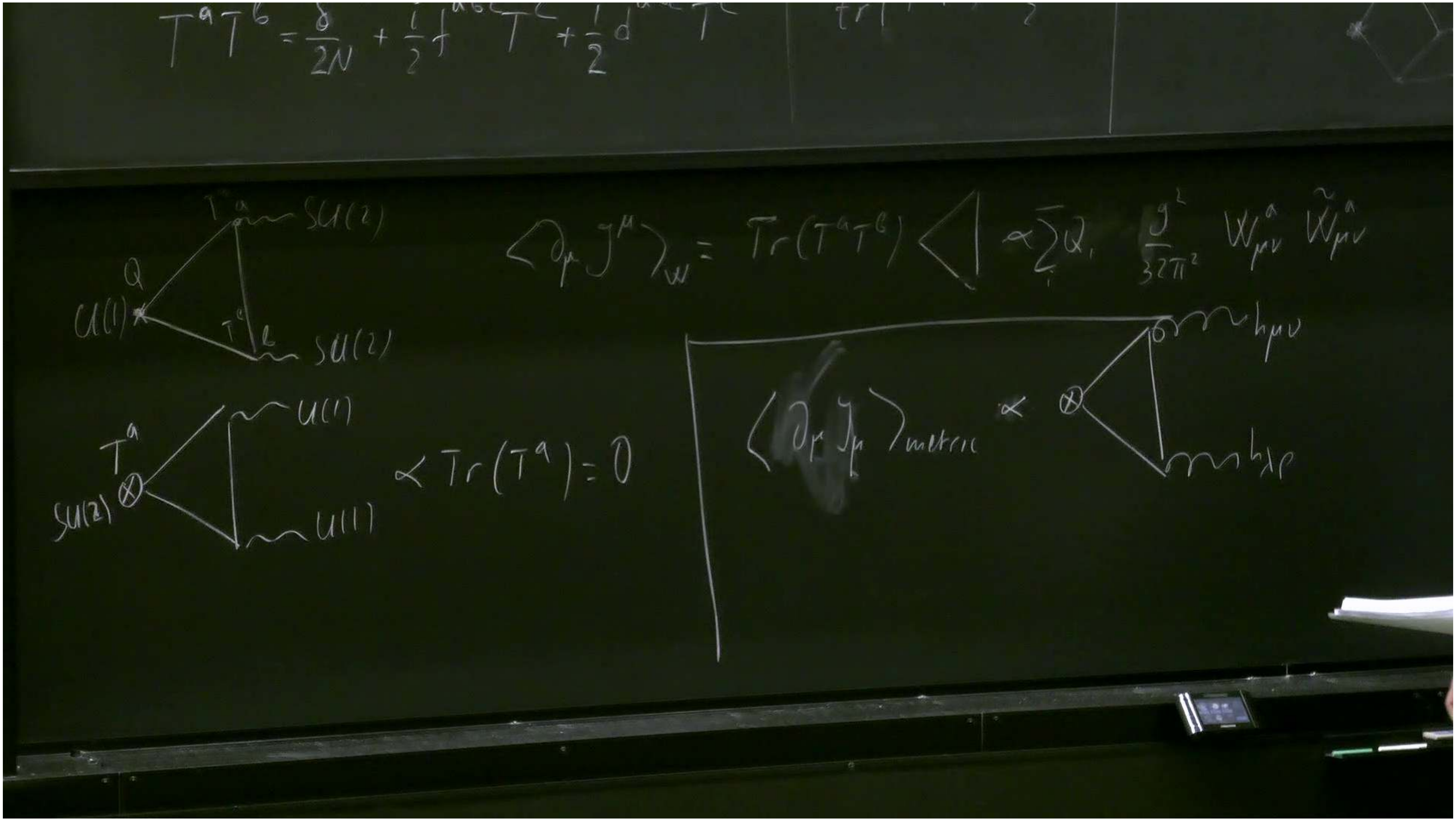
- $SU(3)^3$ vector ✓
- $SU(2)^3$ no d^{abc} ✓

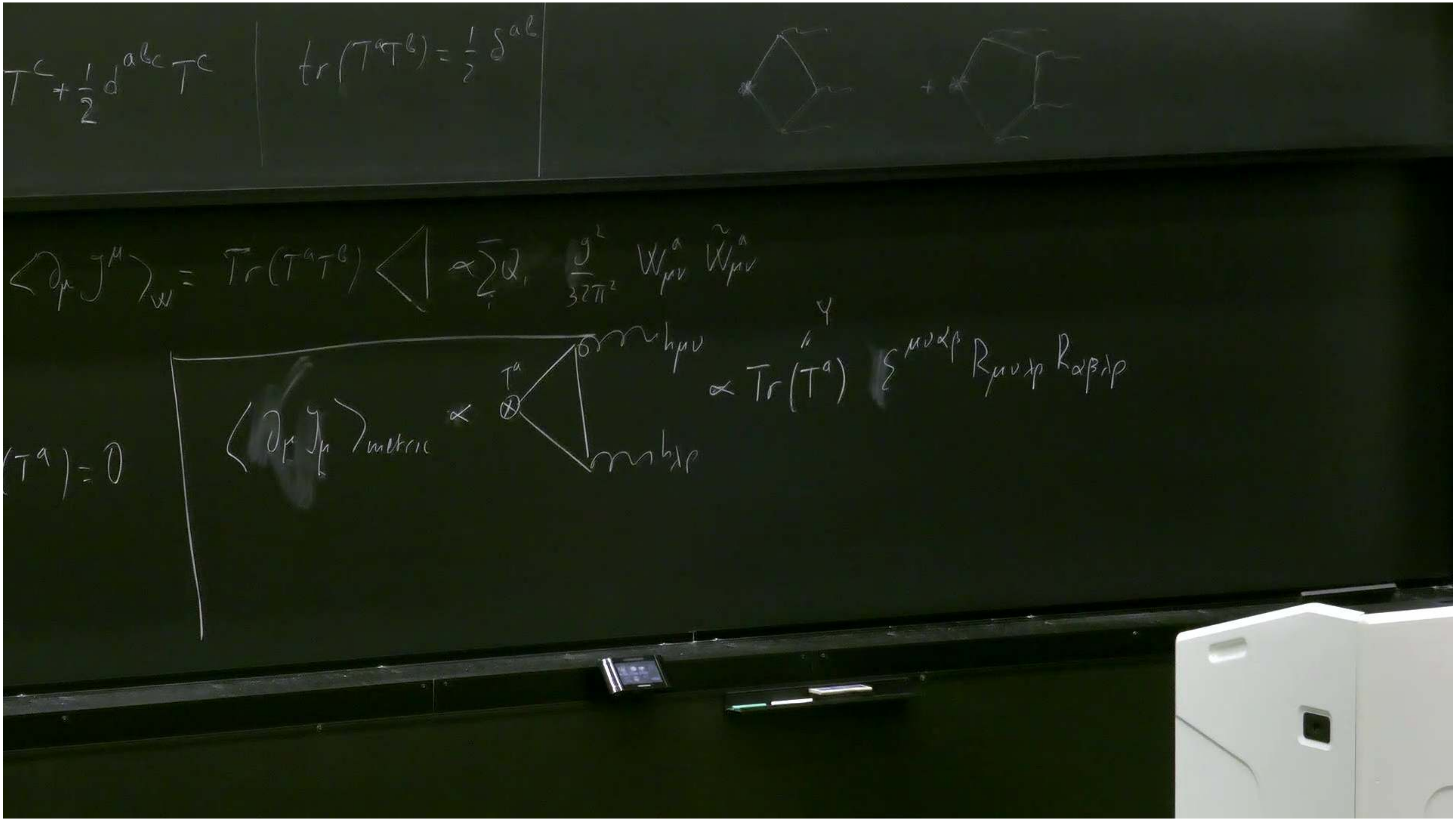
  even # of fermions
- $U(1)^3$ $\sum_{\text{left}} Y_i^3 - \sum_{\text{right}} Y_j^3 = 0$

- $\cancel{SU(N)} \times U(1)^2$ — vanishes
- $U(1) \times SU(2)^2$ $\boxed{Y_L + 3Y_Q = 0}$
- $U(1) \times SU(3)^2$ $\boxed{2Y_Q - Y_u - Y_d = 0}$

$$\boxed{2Y_L^3 + 3\cdot 2\, Y_Q^3 - Y_e^3 - 3Y_u^3 - 3Y_d^3 = 0}$$

- $U(1) \times \text{grav}^2$


$T^a T^b = \frac{\delta}{2N} + \frac{i}{2} f^{abc} T^c + \frac{1}{2} d^{abc} T^c$
$\langle \partial_\mu J^\mu \rangle_W = Tr(T^a T^b) \propto \sum_i Q_i \frac{g^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^a_{\mu\nu}$
U(1)
Q
T^a
SU(2)
T^b
SU(2)
T^a
SU(2)
U(1)
U(1)
$\propto Tr(T^a) = 0$
$\langle \partial_\mu J_\mu \rangle_{metric} \propto$
$h_{\mu\nu}$
$h_{\lambda\rho}$



$T^c + \frac{1}{2} d^{abc} T^c$
$tr(T^a T^b) = \frac{1}{2}\delta^{ab}$
$\langle \partial_\mu J^\mu \rangle_W = Tr(T^a T^b) \propto \sum_i Q_i \frac{g^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^a_{\mu\nu}$
$(T^a) = 0$
$\langle \partial_\mu J_\mu \rangle_{metric} \propto$
T^a
$h_{\mu\nu}$
$h_{\lambda\rho}$
$\propto Tr(T^a)\, \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta\lambda\rho}$
Y

- $SU(3)^3$ vector ✓
- $SU(2)^3$ no d^{abc} ✓
  even # of fermions
- $U(1)^3$ $\sum_{\text{left}} Y_i^3 - \sum_{\text{right}} Y_i^3 = 0$

- $\cancel{SU(N)\times U(1)^2}$ — vanishes
- $U(1)\times SU(2)^2$ $\boxed{Y_L + 3Y_Q = 0}$
- $U(1)\times SU(3)^2$ $\boxed{2Y_Q - Y_u - Y_d = 0}$

$$\boxed{2Y_L^3 + 3\cdot 2\,Y_Q^3 - Y_e^3 - 3Y_u^3 - 3Y_d^3 = 0}$$

- $U(1)\times \text{grav}^2$

$$\boxed{2Y_L + 6Y_Q - Y_e - 3Y_u - 3Y_d = 0}$$

—	—	fund	fund	fund
fund	—	fund	—	—
Y_L	Y_e	Y_Q	Y_u	Y_d

• $SU(2)^3$ no d^{abc} ✓
even # of fermions

• $U(1)^3$ $\sum_{\text{left}} Y_i^3 - \sum_{\text{right}} Y_i^3 = 0$

• $U(1) \times SU(2)^2$ Y_L …

• $U(1) \times SU(3)^2$ $2Y$…

$2Y_L^3 + 3 \cdot 2 Y_Q^3 - Y_e^3 - 3$…

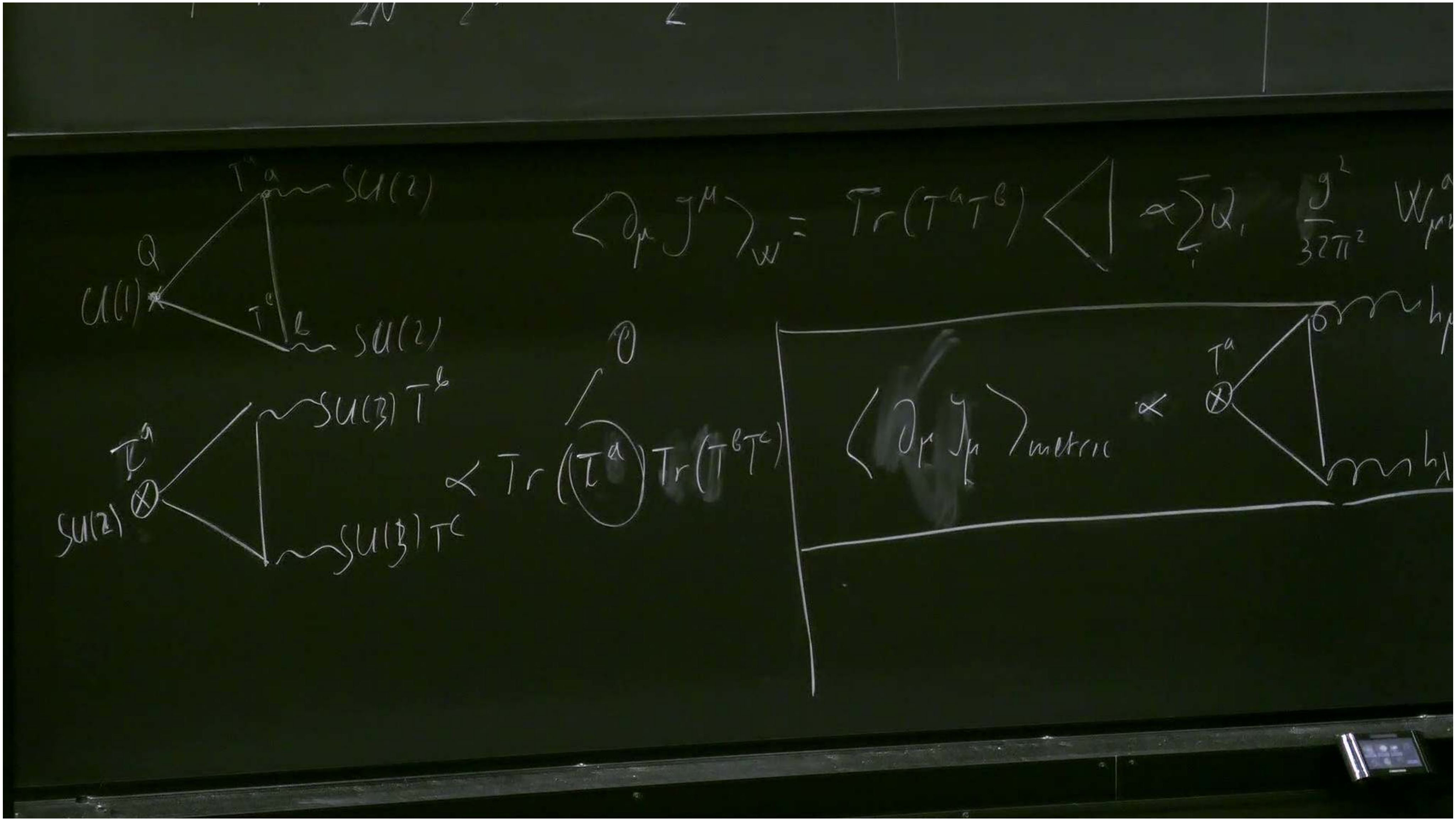
• $U(1) \times \text{grav}^2$

$2Y_L + 6Y_Q - Y_e - 3Y$…

a) $Y_u = -Y_d$, $Y_L = Y_e = Y_Q = 0$

b) $Y_u = -2Y_d = 2a$, $Y_Q = \frac{a}{2}$, $Y_L = -\frac{3}{2}a$, $Y_e = -3a$ $\Rightarrow$ $a = \frac{1}{3}$: $Y_u = \frac{2}{3}$, $Y_d = -\frac{1}{3}$, $Y_Q = \frac{1}{6}$, $Y_L = -\frac{1}{2}$, $Y_e = -1$

c) $Y_d = -2Y_u$



$\langle \partial_\mu J^\mu \rangle_W = Tr(T^a T^b) \propto \sum_i Q_i \frac{g^2}{32\pi^2} W_{\mu\nu}^a$
U(1)
Q
SU(2)
SU(2)
SU(3) T^b
SU(3) T^c
SU(2)
$\propto Tr(T^a) Tr(T^b T^c)$
0
$\langle \partial_\mu J_\mu \rangle_{metric} \propto$