Title: Lecture - Numerical Methods (Core), PHYS 777

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Collection/Series: Numerical Methods (Core), PHYS 777-006, Jan 5 - Feb 6, 2026

Subject: Other

Date: January 05, 2026 - 2:00 PM

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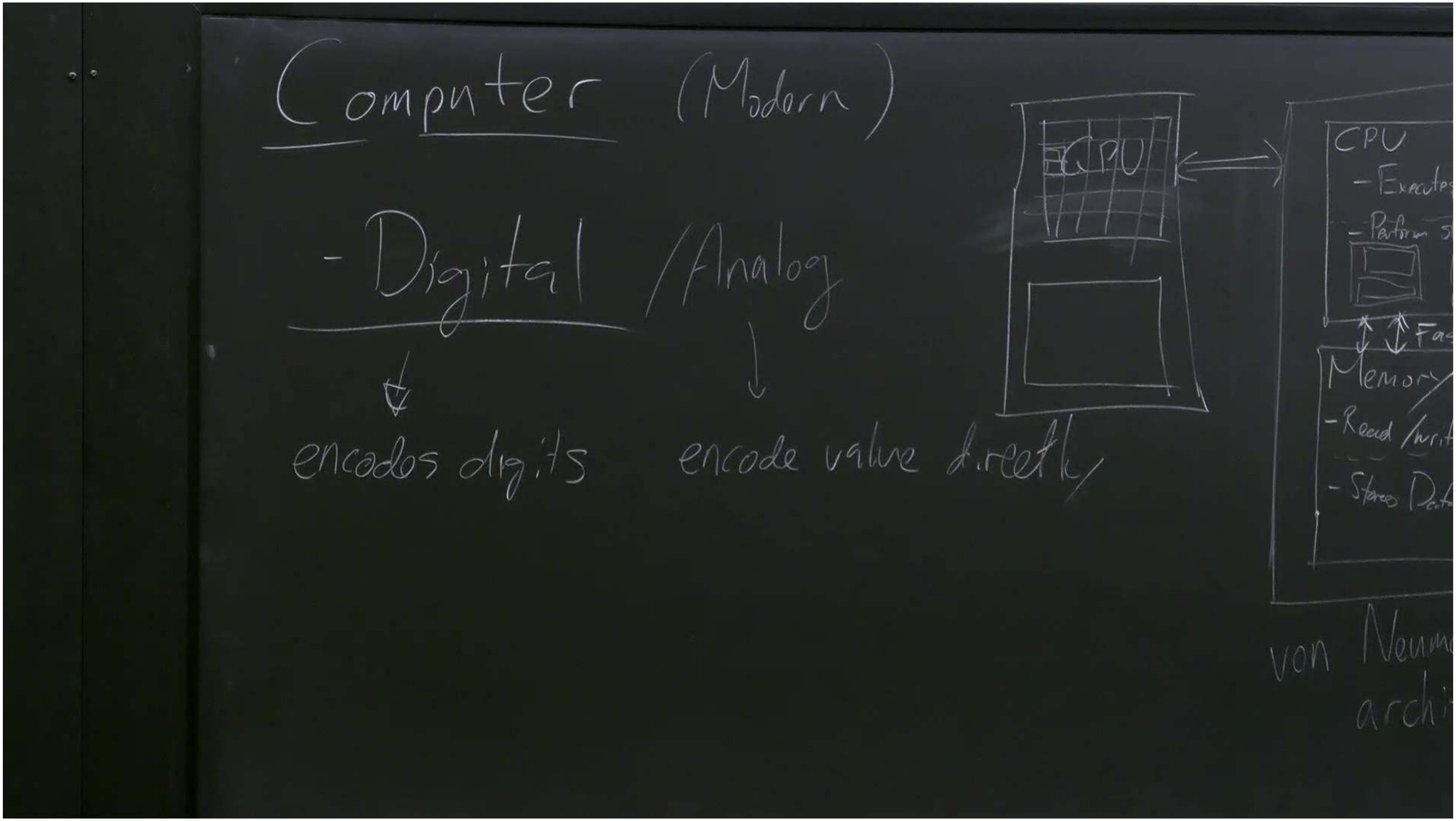
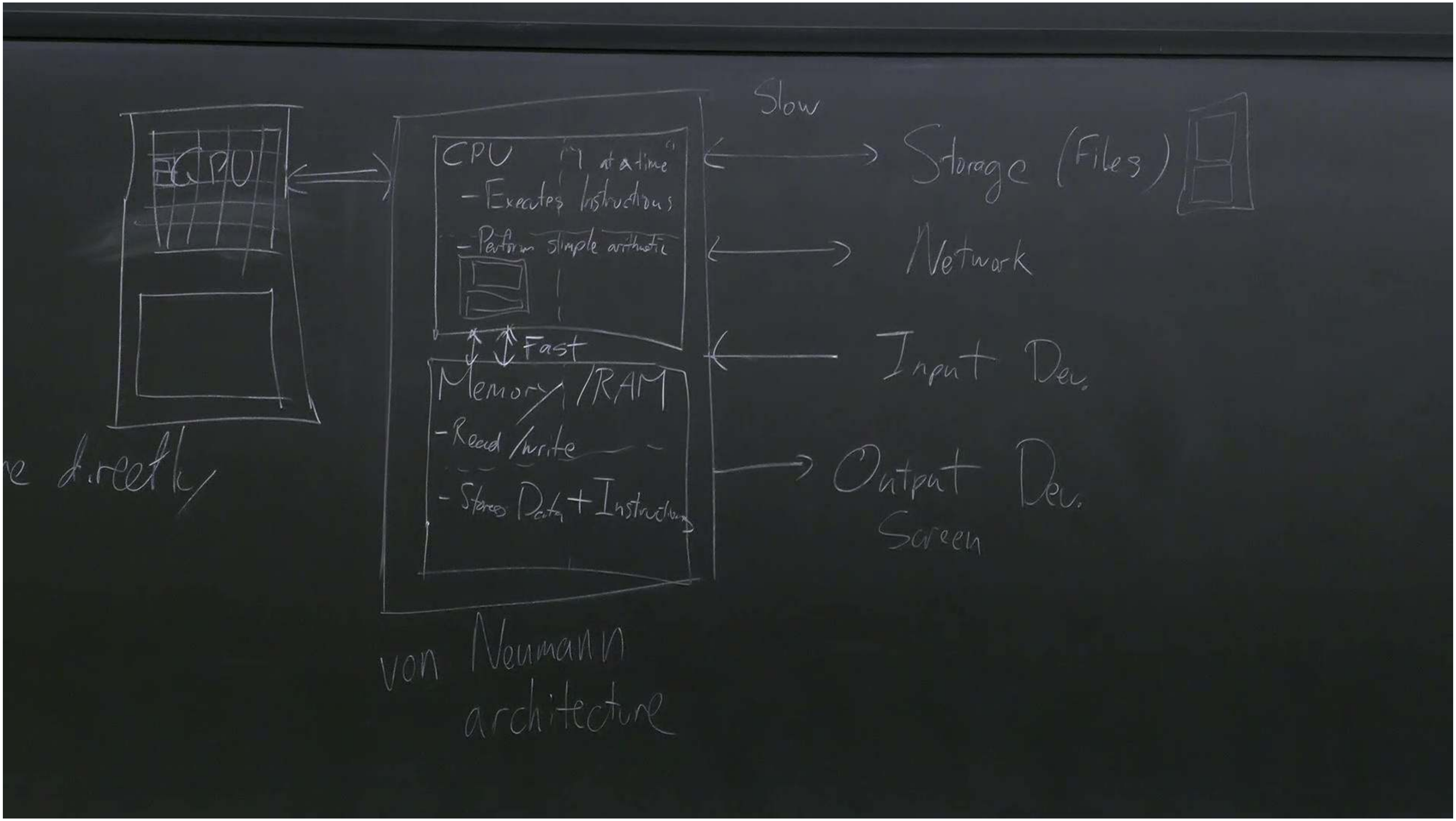

Computer (Modern)
- Digital / Analog
encodes digits
encode value directly
CPU
CPU
Memory

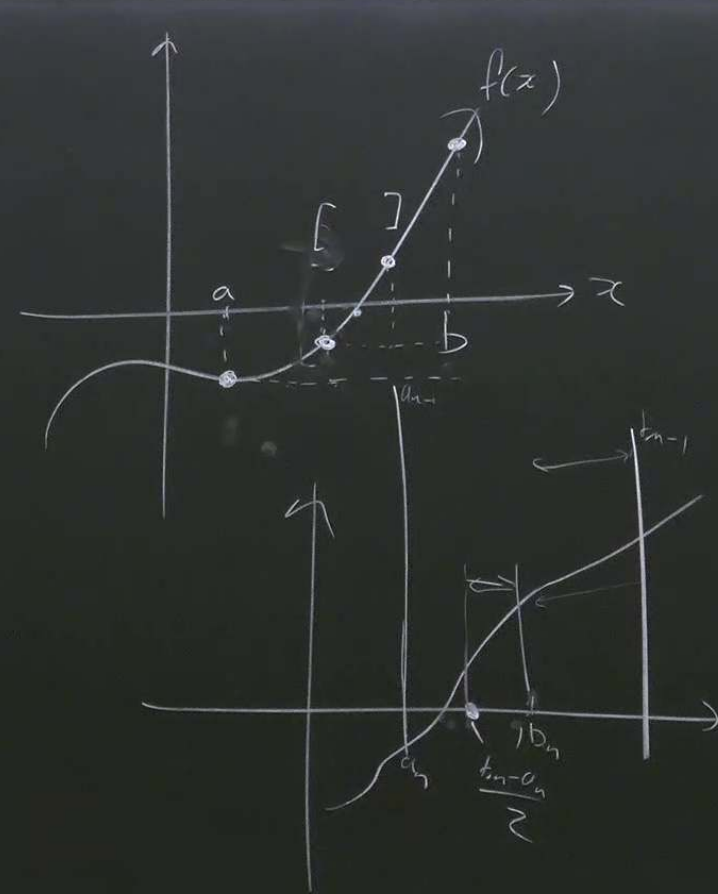

CPU
CPU
1 at a time
- Executes Instructions
- Perform simple arithmetic
Fast
Memory /RAM
- Read /write
- Stores Data + Instructions
Slow
Storage (Files)
Network
Input Dev.
Output Dev.
Screen
directly
von Neumann
architecture

Root Finding

$f:\mathbb{R}\to\mathbb{R}$, continuous

find x s.t. $f(x)=0$

Kepler: $M=E-e\sin E$

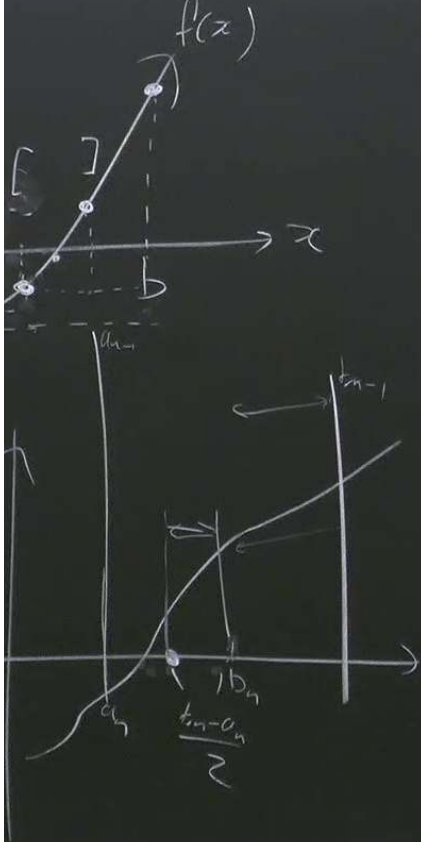


B

Gi

a

Give



Bisection

Given a, b s.t. $f(a)f(b) < 0$

$a_0 = a$, $b_0 = b$

Given a_n, b_n

$$x_n = \frac{1}{2}(a_n + b_n)$$

if $f(x_n)$ same sign $f(a)$

$$a_{n+1} = x_n$$
$$b_{n+1} = b_n$$

else

$$a_{n+1} = a_n$$
$$b_{n+1} = x_n$$

error $\varepsilon_n \equiv x_n - x_*$

$$\varepsilon_n \le \frac{1}{2}(b_n - a_n)$$
$$= \frac{1}{2}\varepsilon_{n-1}$$

$$\varepsilon_n \le 2^{-n}\varepsilon_0$$

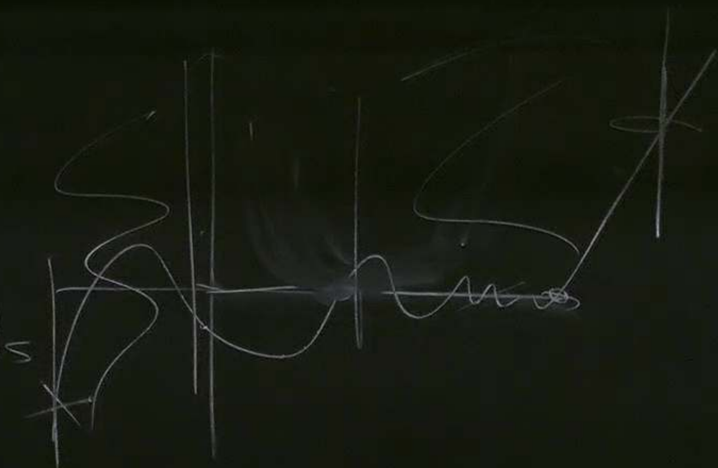
linear convergence

Bisection

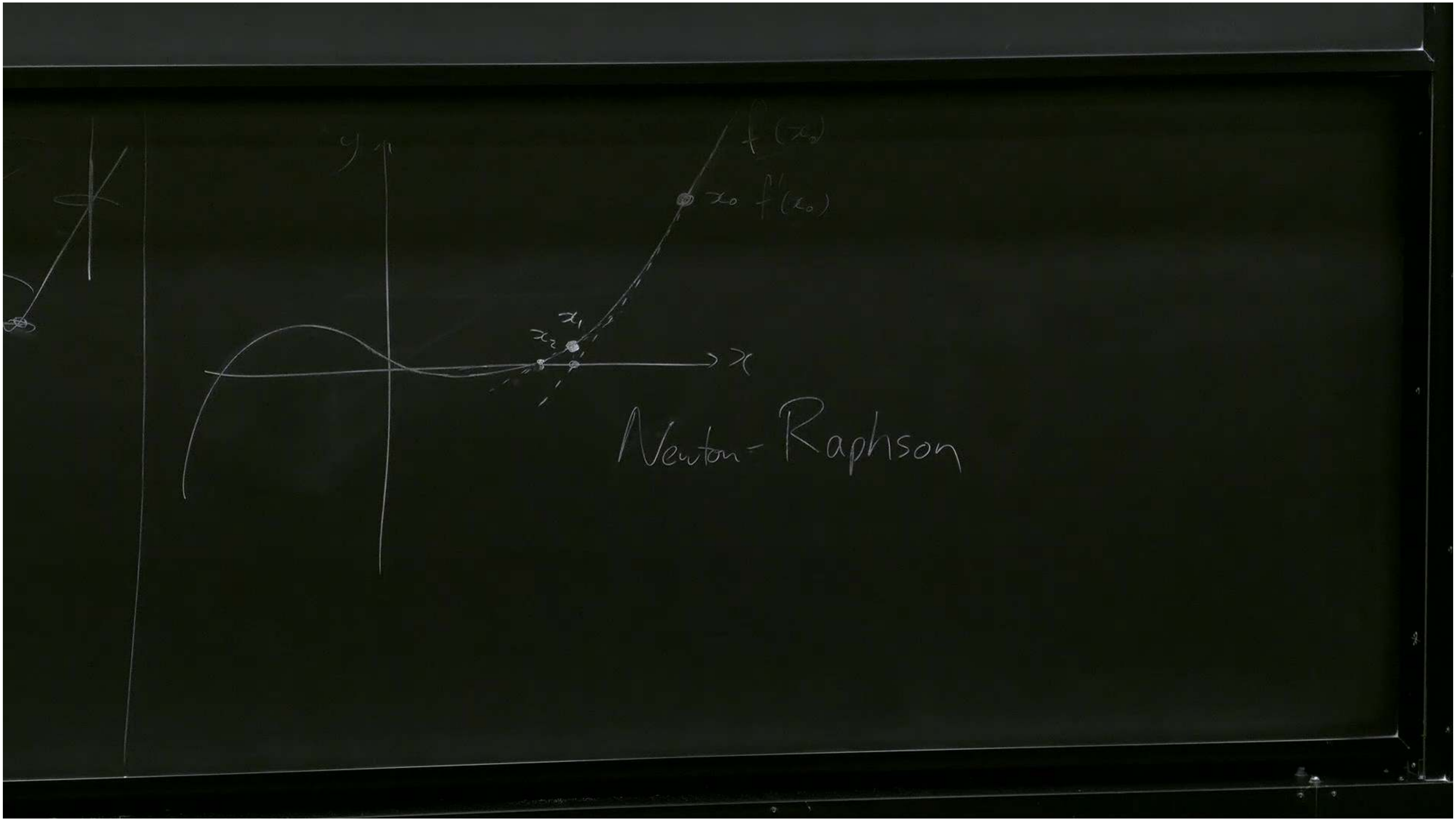
→ Always Converges

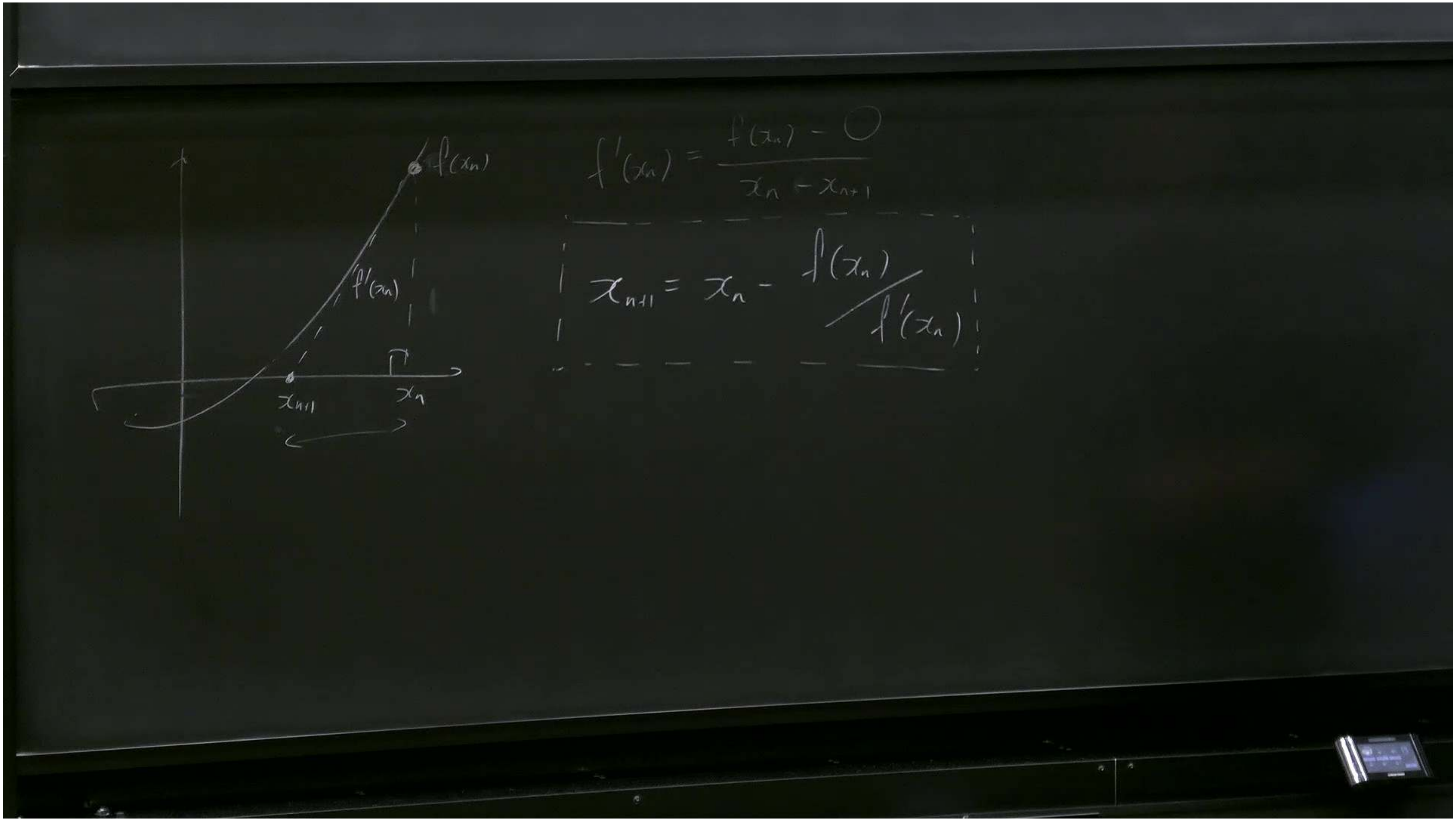
→ Cannot handle: - even-repeated roots

- multiple dimensions

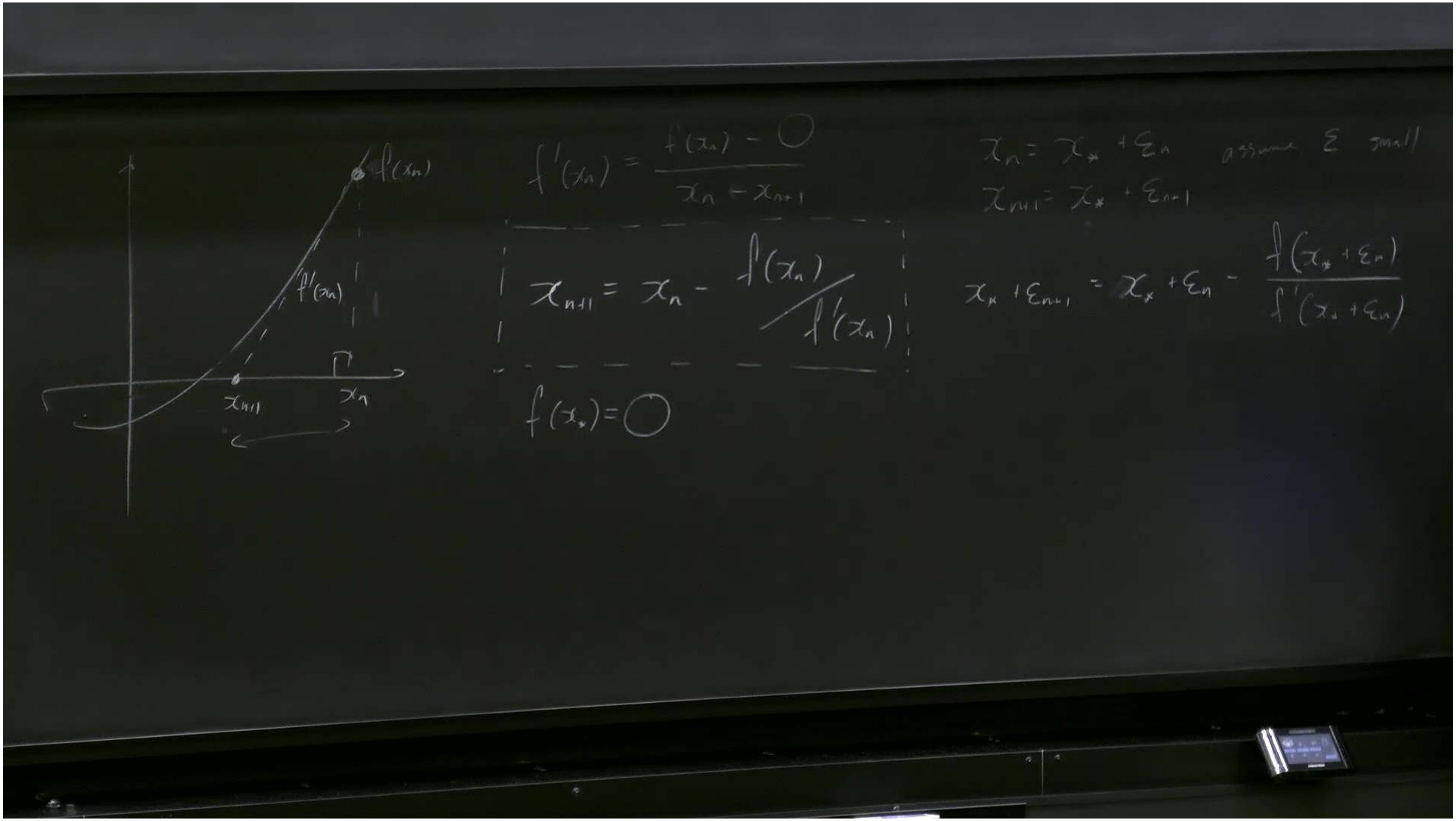


$f: \mathbb{R}^n \to \mathbb{R}$


y
x
x_0
$f'(x_0)$
x_1
x_2
Newton-Raphson



$f(x_n)$
$f'(x_n)$
x_{n+1}
x_n
$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}}$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$


$f(x_n)$
$f'(x_n)$
x_{n+1}
x_n
$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}}$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
$f(x_*) = 0$
$x_n = x_* + \varepsilon_n$ assume ε small
$x_{n+1} = x_* + \varepsilon_{n+1}$
$x_* + \varepsilon_{n+1} = x_* + \varepsilon_n - \frac{f(x_* + \varepsilon_n)}{f'(x_* + \varepsilon_n)}$

$f(x_n)$

$$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$f(x_*) = 0$

$$\boxed{\varepsilon_{n+1} \approx \frac{1}{2}\frac{f''(x_*)}{f'(x_*)}\varepsilon_n^2}$$

Quadratic

$x_n = x_* + \varepsilon_n$ assume ε small

$x_{n+1} = x_* + \varepsilon_{n+1}$

$$\cancel{x_*} + \varepsilon_{n+1} = x_* + \varepsilon_n - \frac{f(x_* + \varepsilon_n)}{f'(x_* + \varepsilon_n)}$$

$$\approx x_* + \varepsilon_n - \frac{\cancel{f(x_*)} + \varepsilon_n f'(x_*) + \frac{1}{2}\varepsilon_n^2 f''(x_*)}{f'(x_*) + \varepsilon_n f''(x_*)}$$

$$\approx \cancel{x_*} + \cancel{\varepsilon_n} - \left(\cancel{\varepsilon_n} - \frac{1}{2}\varepsilon_n^2 \frac{f''(x_*)}{f'(x_*)}\right)$$

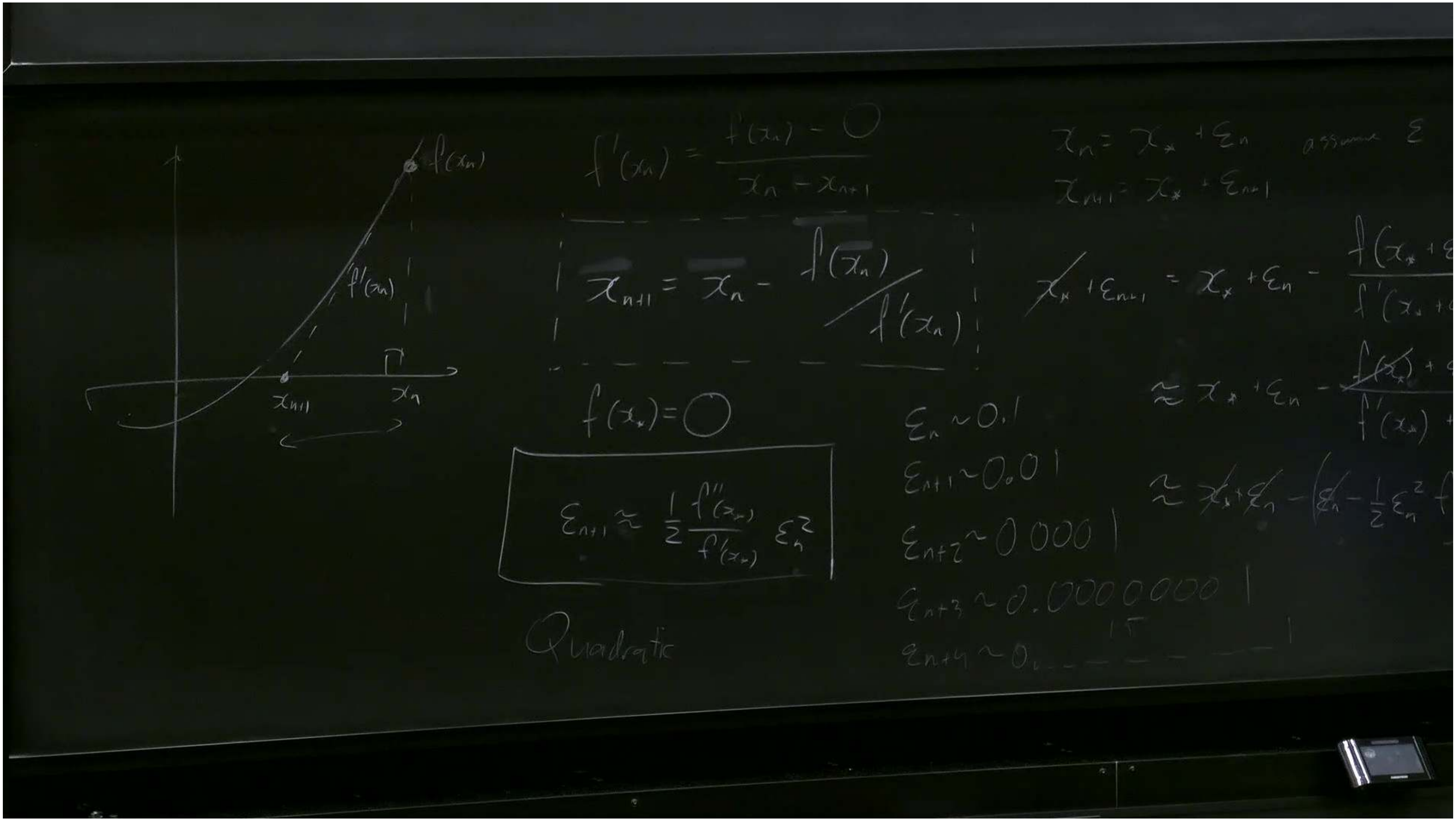
$\varepsilon_n \sim 0.1$

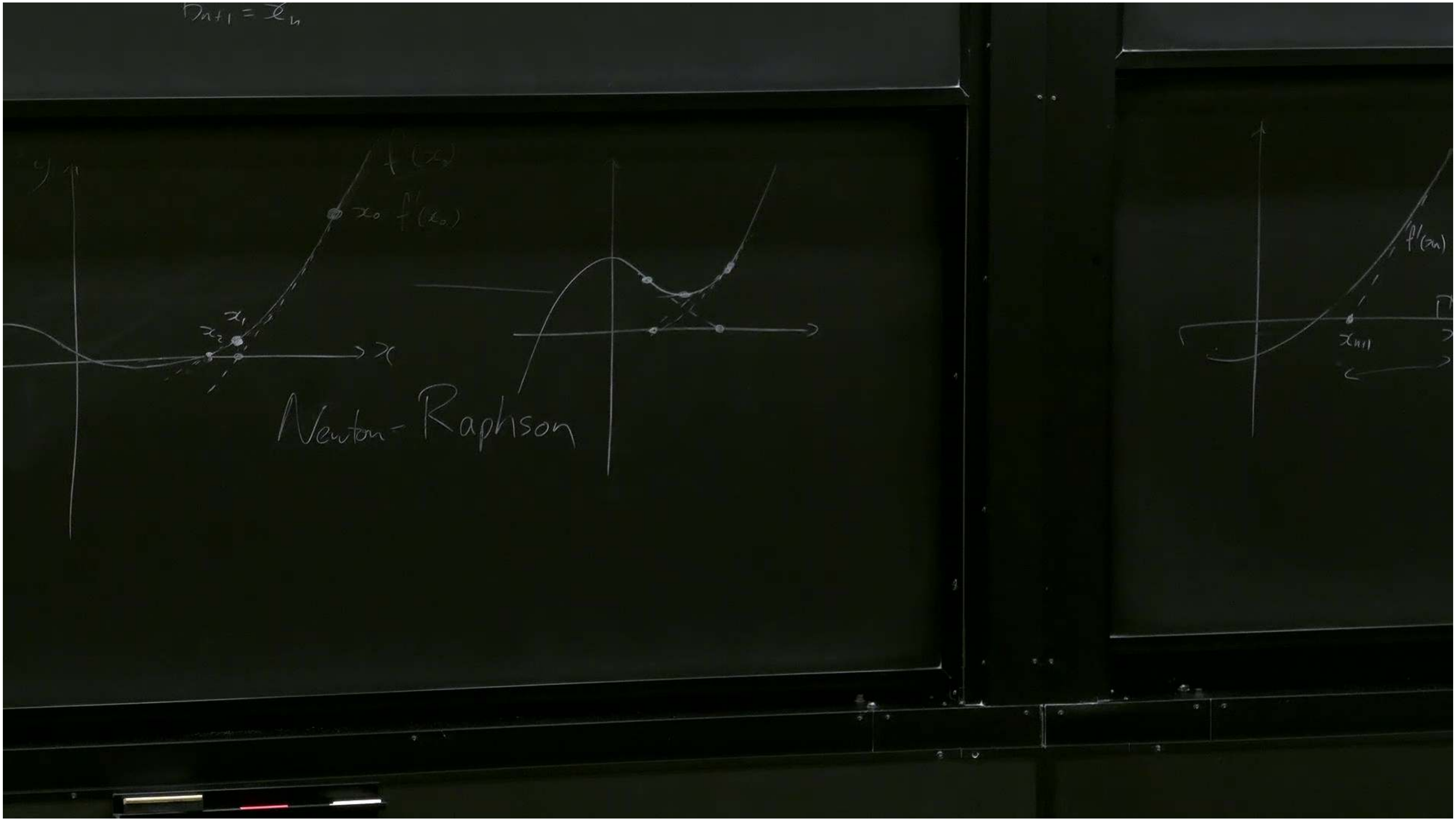
$\varepsilon_{n+1} \sim 0.01$

$\varepsilon_{n+2} \sim 0.0001$

$\varepsilon_{n+3} \sim 0.00000001$

$\varepsilon_{n+4} \sim 0.\underbrace{\ldots\ldots}_{15}1$


$f(x_n)$
$f'(x_n)$
x_{n+1}
x_n
$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}}$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
$f(x_*) = 0$
$\varepsilon_{n+1} \approx \frac{1}{2} \frac{f''(x_*)}{f'(x_*)} \varepsilon_n^2$
Quadratic
$x_n = x_* + \varepsilon_n$ assume ε
$x_{n+1} = x_* + \varepsilon_{n+1}$
$\cancel{x_*} + \varepsilon_{n+1} = x_* + \varepsilon_n -$
$\varepsilon_n \sim 0.1$
$\varepsilon_{n+1} \sim 0.01$
$\varepsilon_{n+2} \sim 0.0001$
$\varepsilon_{n+3} \sim 0.00000001$
$\varepsilon_{n+4} \sim 0.$


Newton-Raphson
x₀ f'(x₀)
x₁
x₂
f'(xₙ)
xₙ₊₁