Title: Remote detection in abelian fracton phases - Quantum Matter Seminar

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Abstract:

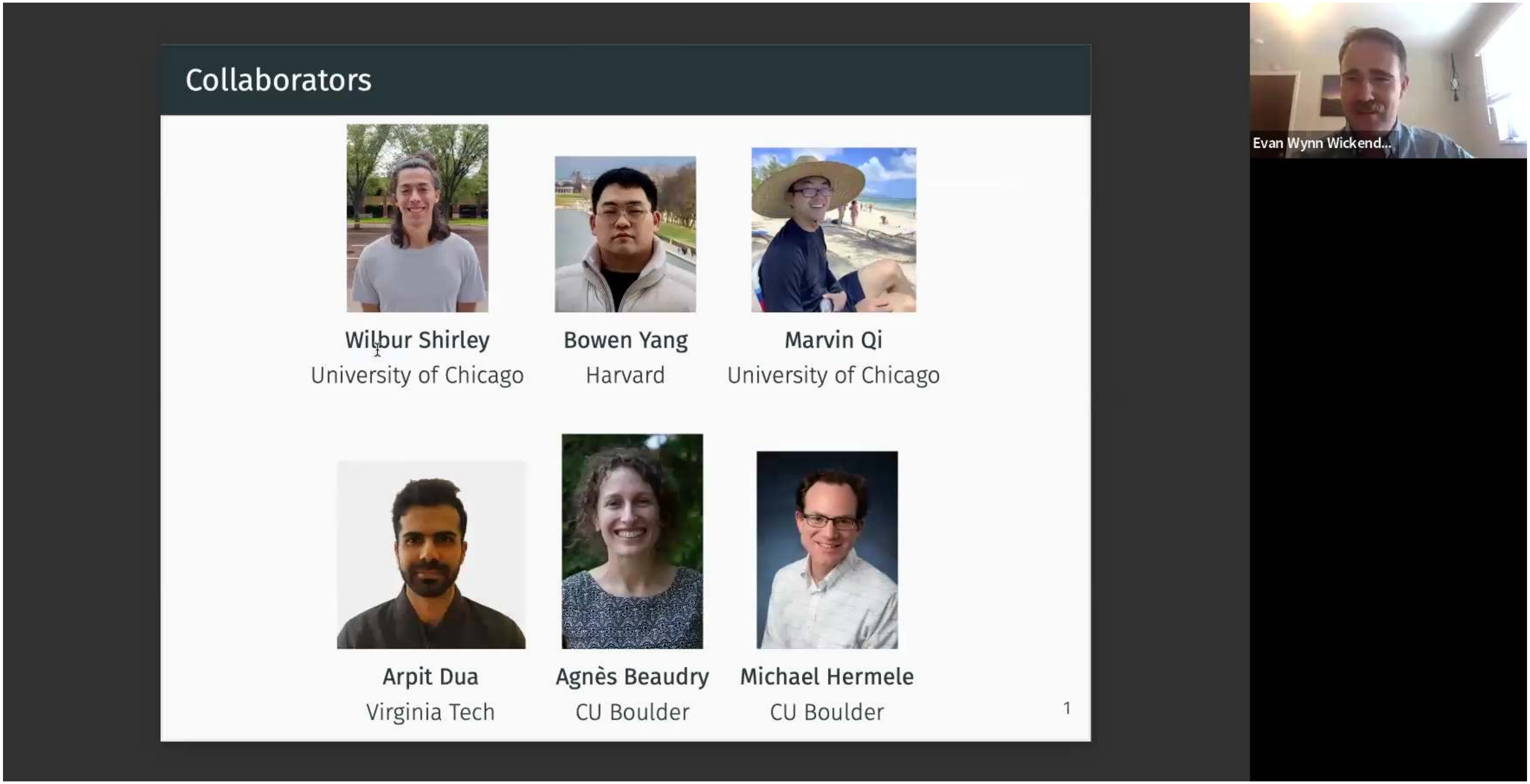
Gapped phases without symmetry are largely characterized by the fusion and statistics of their fractionalized quasiparticles. This is best understood for 2D topological phases. An important constraint on statistical data in this case is the principle of remote detectability, which implies that any nontrivial anyon braids nontrivially with another anyon in the system.

The principle of remote detectability can be applied to more general gapped phases. In 3D topological phases, for example, fully mobile point particles cannot braid nontrivially with each other, but the principle is rescued by the existence of loop excitations. In 3D fracton phases, by contrast, there need not exist loop excitations, but point quasiparticles have restricted mobility, so the principle can still hold. However, fracton phases exhibit many possible patterns of mobility restriction, and it is not yet understood how to parameterize the inequivalent “braidings” that may be compatible with a given fusion theory.

In this talk, I will describe recent progress on this problem. I will introduce the class of planon- modular fracton orders—phases in which every excitation can be detected by braiding with a planon—and highlight several structural consequences of this definition. I will then focus on the case with only abelian planon excitations, where remote detectability is strengthened to a more rigid statistical constraint that we call the excitation-detector principle. I will present new classi- fication results that follow from this principle. Finally, I will describe how these ideas extend to a framework for characterizing braiding and remote detection in general abelian fracton orders, and in abelian gapped phases more broadly.

Remote Detection in Abelian Fracton Phases

Evan Wickenden
December 4, 2025
University of Colorado Boulder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Outline

Introduction

Remote detection in fracton phases

Upcoming work

Summary

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Motivation

Overarching goal: characterize and classify gapped phases of matter without symmetry.

Homogeneous lattice systems, f.d. local Hilbert spaces, thermodynamic limit

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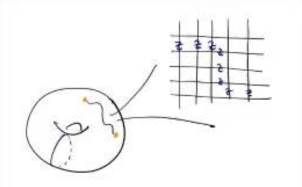
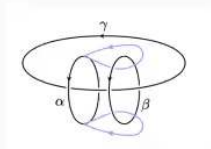
Low-dimensional gapped phases



2D TO

Anyonic quasiparticles (UMTC)



3D TO

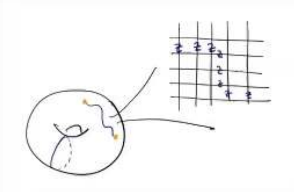
Mobile point & loop excitations (braided fusion 2-category)

3D fracton

Restricted-mobility point excitations (??)

4

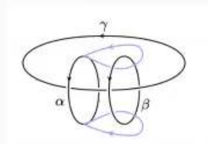
Low-dimensional gapped phases



2D TO

Anyonic quasiparticles (UMTC)

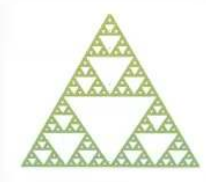
TQFT + renormalization group description



3D TO

Mobile point & loop excitations (braided fusion 2-category)

TQFT / RG paradigm



3D fracton

Restricted-mobility point excitations (??)

Bifurcating entanglement renormalization

4

Fracton organizing principles

Quasiparticle mobility:
Type I — has a mobile quasiparticle.
Type II — does not.

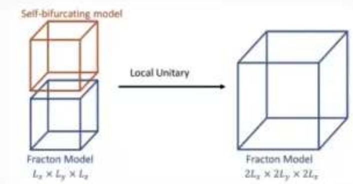
Bifurcating entanglement renormalization:

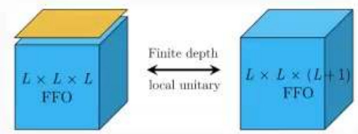
$$A \longrightarrow A \oplus B$$

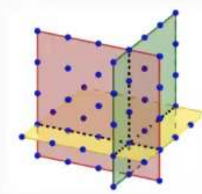
Foliated — B is decoupled layers.
Self-bifurcating — $B = A$.

Construction:
Gauging — subsystem symmetries (SPTs).
Coupling — p-string condensation.

[Haah '11, Haah '14, Vijay–Haah–Fu '16, Williamson '16, Ma–Lake–Chen–Hermele '17, Shirley–Slagle–Chen '18, Shirley–Slagle–Chen '19, Dua–Sarkar–Williamson–Cheng '19]







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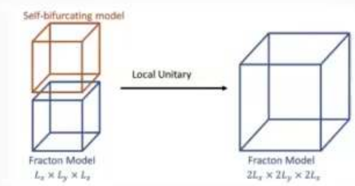
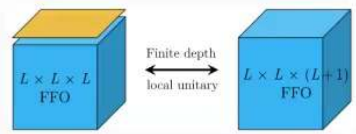


Bifurcating entanglement renormalization:

$$A \longrightarrow A \oplus B$$



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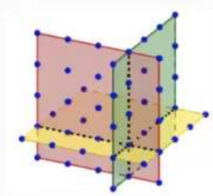

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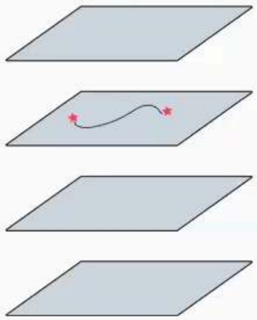
Examples

Toric Code Layers



- Type-I
- Foliated & self-bifurcating:

$$\mathrm{TCL}(a) \to \mathrm{TCL}(2a)^{\oplus 2}$$

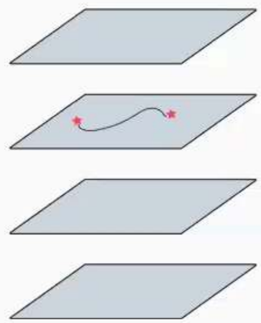
- $k = \log_2 \mathrm{GSD} = 2(L/a)$

6

Examples

couple $\Longrightarrow$

Toric Code Layers



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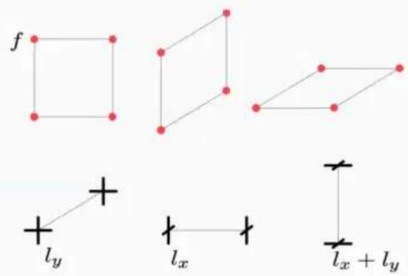
X-cube

[Vijay–Haah–Fu '16]



- Type-I
- Foliated:

$$\mathrm{XC}(a) \to \mathrm{XC}(2a) \oplus \mathrm{TCL}^{x,y,z}(2a)$$

- $k = 6(L/a) - 3$

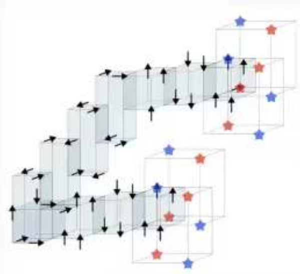
6

Examples

Four Color Cube

[Ma–Lake–Chen–Hermele '17,

EW–Qi–Dua–Hermele '24]



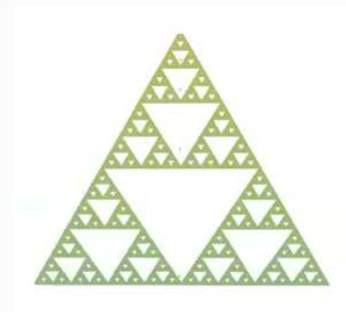
- Type-I
- Foliated:

$$\mathrm{FCC}(a) \to \mathrm{FCC}(5a) \oplus \mathrm{Layers}^7$$

- $k = 36(L/a) - 24$

Haah's Code

[Haah '11, Haah '14]



- Type-II
- AB-bifurcating:

$$\mathrm{CC}(a) \to \mathrm{CC}(2a) \oplus \mathrm{CCB}(2a)$$

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Questions

- What is space of abelian fracton fusion theories?
- How to parameterize possible inequivalent statistical data for given fusion theory?
- Theory of fracton condensation?
- Classification?

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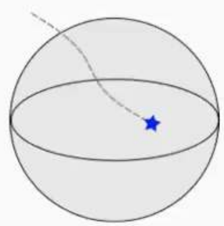
Principle of Remote Detection

Def. A **detection operator** is an operator supported on the boundary of an (infinitely) large ball that commutes with the Hamiltonian.



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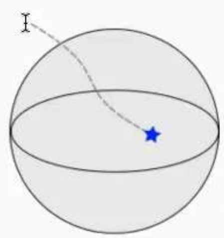

Principle of remote detection: every fractionalized excitation in a gapped phase can be detected by some detection operator.

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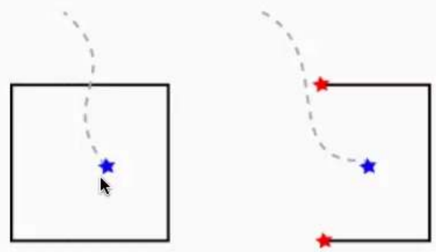
Remote detection versus braiding

2D TO — detector = anyon string operator



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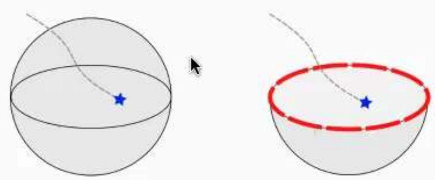
Remote detection versus braiding

2D TO — detector = anyon string operator

3D TO — point detector = loop creation/annihilation operator



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Remote detection versus braiding

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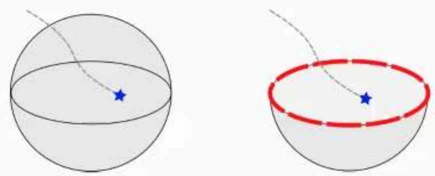

3D Fracton — No loop excitations ⇒ point-quasiparticles must have restricted mobility.

Q. Does fracton remote detection admit a "braiding"-like interpretation?

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Planon-Modular Fracton Orders

[EW–Qi–Dua–Hermele, '24]

Def. A fracton phase is planon-modular if every nontrivial excitation can be detected by braiding with a planon.

Planon = excitation mobile only within a plane

Leads to robust packaging of universal fusion data, and some statistical data.

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Phase invariants

The **detection-weight** of an excitation is the number of orientations of planon that detect it.

Let $S^{(i)}$ = excitations of detection-weight at most i.

Weighted quotient superselection sectors = $(S/S^{(1)}, S/S^{(2)}, \ldots)$.

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Phase invariants

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Weighted quotient superselection sectors = $(S/S^{(1)}, S/S^{(2)}, \ldots)$.

Model	WQSS
Toric code layers	0
X-cube	$(\mathbb{Z}_n^3, \mathbb{Z}_n, 0)$
4-planar X-cube	$(\mathbb{Z}_n^4, \mathbb{Z}_n, \mathbb{Z}_n, 0)$
FCC (n odd)	$(\mathbb{Z}_n^{40}, \mathbb{Z}_n^{10}, \mathbb{Z}_n^{10}, 0)$
FCC ($n = 2$)	$(\mathbb{Z}_2^{40}, \mathbb{Z}_2^{16}, \mathbb{Z}_2^{10}, \mathbb{Z}_2^{8}, \mathbb{Z}_2^{2}, \mathbb{Z}_2^{2}, 0)$

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RG

Theorem. If a p-modular phase A admits a bifurcating entanglement renormalization $A \to A \oplus B$, then B is a stack of *planon-only* theories.

Q. Does every p-modular phase admit a bifurcating RG?

Q. In the above theorem, is the B-theory always actually decoupled layers?

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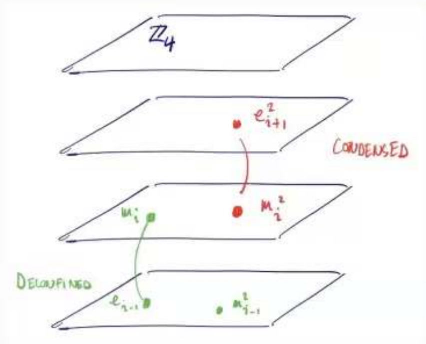
Planon-only fracton phases

[EW–Shirley–Beaudry–Hermele, '25]

Simplest subclass of fracton phases:

- Statistical processes match those of 2D topological order.
- Not all are decoupled layers [Shirley–Slagle–Chen '19, Ma *et al.* '20], e.g. *twisted 1-foliated model:*



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Algebraic packaging

We package the quasiparticle data as a *quadratic form* (S, q) over $R = \mathbb{Z}_n[t^{\pm}]$ [Knus '91].

- Braiding is a *Hermitian form*

$$S \times S \longrightarrow R.$$

- Remote detection $\iff$ planon-modularity $\iff$ injectivity of

$$S \longrightarrow \mathrm{Hom}_R(S, R).$$

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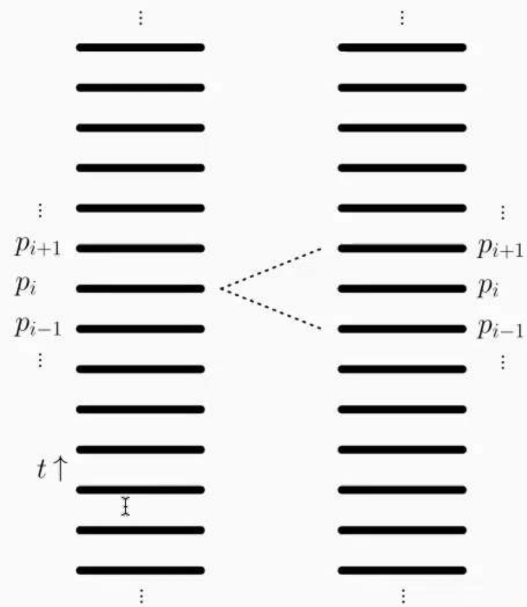
Remote detection is not enough

Model

- One $\mathbb{Z}_2$ generator per layer.
- p_i braids nontrivially only with $p_{i\pm1}$.



15

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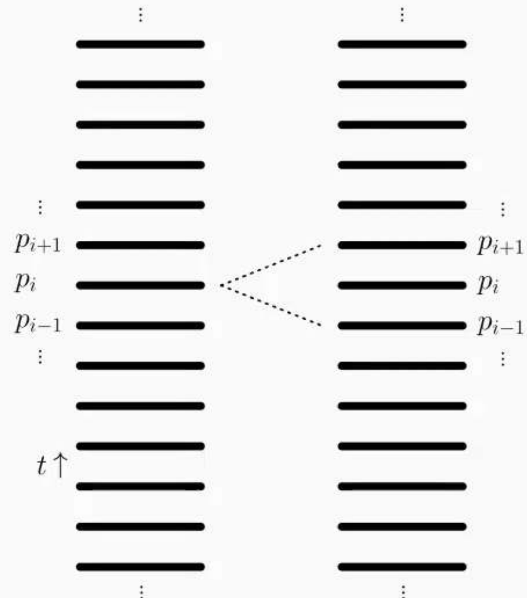

Unphysical!

Upon compactification, the composite excitation

$$\sum_{i \in \mathbb{Z}_{2N}} p_{2i}$$

becomes a *transparent anyon*.

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Remote detection is not enough

Model

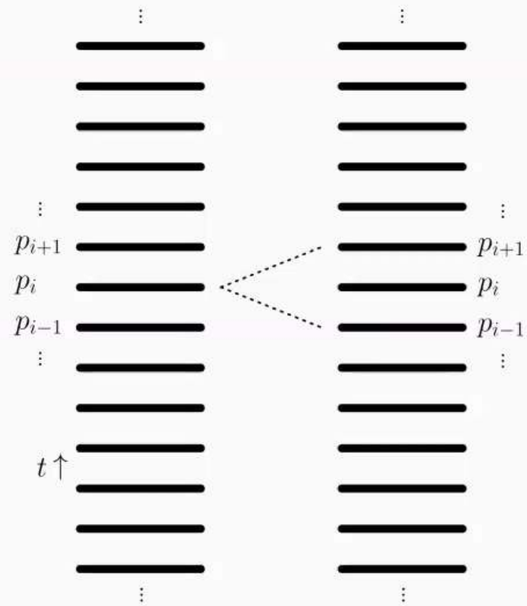
- One $\mathbb{Z}_2$ generator per layer.
- p_i braids nontrivially only with $p_{i\pm 1}$.

Unphysical!

Without compactification, the detector corresponding to the infinite planon

$$\sum_{i \in \mathbb{Z}} p_{2i}$$

is ineffective.



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Enhanced principle

Excitation-detector principle: *every nontrivial quasiparticle is detectable, and every nontrivial detector is effective.*

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Algebraic packaging

We package the quasiparticle data as a *quadratic form* (S, q) over $R = \mathbb{Z}_n[t^{\pm}]$ [Knus '91].

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14

Enhanced principle

Excitation-detector principle: *every nontrivial quasiparticle is detectable, and every nontrivial detector is effective.*

Theorem. All compactifications of an (abelian) planon-only theory result in modular 2D anyon theories if and only if the excitation-detector principle is satisfied.

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Realizability

Hence, it is *necessary* for a quadratic form (S, q) to satisfy the excitation-detector principle for it to be the data of a physical planon-only fracton phase.

Conjecture. This is also a sufficient condition.

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Prime fusion order classification

Theorem. All nondegenerate planon-only theories (S, q) of prime fusion order (i.e., $pS = 0$), are *decoupled layers*.

This resolves the realizability conjecture in the prime fusion order case.

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Planon-only classification

[EW–Shirley–Beaudry–Hermele, in preparation]

Using techniques similar to those in [Haah-Fidkowski-Hastings, '18], we (coarsely) classify planon-only abelian phases of arbitrary fusion order.

- At prime-squared fusion order, up to topological spins, there is only the twisted 1-foliated model.
- At higher fusion order, a finite list of possibilities.

Corollary. Every such phase is a fixed point under foliated RG.

In progress: prove realizability conjecture.

Homological Fracton Braiding

[Shirley-EW-Yang-Beaudry-Hermele, in preparation]

See also [Ruba-Yang '23, Ruba-Yang '25].

Let $R = \mathbf{Z}_n[\mathbf{Z}^3]$, $R_\infty = \mathbf{Z}_n[[\mathbf{Z}^3]]$.

Space of detection operators of a fracton phase:

$$D = \mathrm{Tor}_2^R(S, R_\infty)$$

Remote detection pairing:

$$D \times S \to R_\infty$$

Equivalent to symmetric form [Schlichting '10, Schlichting '21]:

$$S \to S^\sharp = \mathrm{Ext}_R^2(S, R)$$

Excitation-detector principle $\Leftrightarrow$ *braiding form is isomorphism.*

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Homological Braiding ctd.

This formula also applies to 2D topological phases:

$$S \to S^\sharp = \mathrm{Ext}^2_{R_{2D}}(S, R_{2D}) \xrightarrow{\text{can}} \mathrm{Hom}_{\mathbf{Z}}(S, \mathbf{Q}/\mathbf{Z}) \tag{1}$$

is equivalent to

$$S \times S \to \mathbf{Q}/\mathbf{Z}.$$

21

Homological Braiding ctd.

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is equivalent to

$$S \times S \to \mathbf{Q}/\mathbf{Z}.$$

In planon-only fracton case,

$$S \to S^\sharp = \mathrm{Ext}^2_{R_{3D}}(S, R_{3D}) \xrightarrow{\text{can}} \mathrm{Hom}_{R_{1D}}(S, R_{1D})$$

is equivalent to

$$S \times S \to R_{1D}.$$

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Key points

- Remote detection useful organizing principle, e.g. in planon-modular fracton phases.
- Strengthen principle of remote detection to excitation-detector principle: *every quasiparticle is detectable, and every detector is effective*.
- Classification of planon-only abelian fracton phases $\Rightarrow$ all are foliated RG fixed points.
- Braiding as symmetric form equivalent to remote detection pairing.
- Braided fracton fusion theory = nondegenerate symmetric form $S \to S^\sharp$.

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Future directions and open questions

- Fracton topological spins = quadratic refinement of braiding form?
- Show that all nondegenerate planon-modular fracton phases are fixed points of foliated RG.
- Classify low planarity planon-modular phases.
- Braiding in general abelian gapped phases = symmetric form in derived category.
- Apply derived category perspective to QEC problems — e.g. analyzing BB codes.
- Nonabelian generalization?

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Thank you!