Title: TBA - Quantum Information Seminar

Speakers: Zofia Adamska

Collection/Series: Quantum Information

Subject: Quantum Information

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Entangling logical qubits without physical operations

Koh, Gong, Diaconu, Tan, Geim, Gullans, Yao, Lukin, and **Majidy** *arXiv 2601.xxxx*

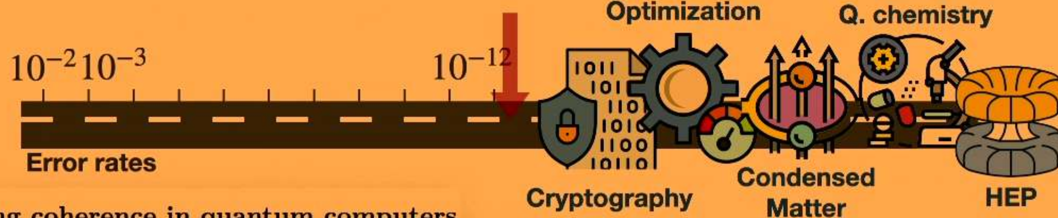
HARVARD UNIVERSITY

Banting Postdoctoral Fellowships

Quantum Error Correction: Intro

Motivation

- Error rates are too high
- QEC is the only known realistic route to sufficiently suppress errors

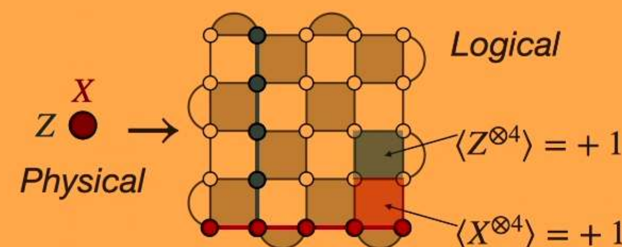


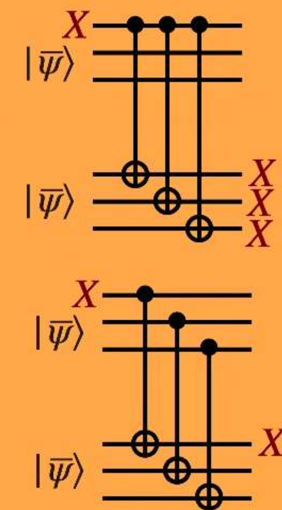
Maintaining coherence in quantum computers

Quantum computing: dream or nightmare?
S. Haroche, JM Raimond
(Published 1 August 1996)

What is QEC and how does it work?

- Store info. non-locally across qubits
- Partial (stabilizer) measurements
- Errors are discretized
- Below threshold
- Fault-tolerant processing





"Transversal"

$$R_n(\theta) \to \{I, X, Y, Z\} \quad e^{i\theta X_1}|\overline{\psi}\rangle = c\,|\overline{\psi}\rangle - isX_1\,|\overline{\psi}\rangle$$

$$p_{\mathrm{L}} \approx 0.1\left(\frac{p_{\mathrm{err}}}{p_{\mathrm{th}}}\right)^{\frac{d+1}{2}}$$

Is quantum mechanics useful?
R. Landauer
(Published 15 December 1995)

QEC: State of the field

2312.03982	2404.02280	2408.13687	2409.04628	2411.11822	2412.15165	2506.20661
Dec 23	Apr 24	Aug 24	Sep 24	Nov 24	Dec 24	Jun 25
			aws 2409.13025		2412.14256	2506.14688

Recent surge across various platforms

QEC: State of the field

2312.03982 — Dec 23

2404.02280 — Apr 24

2408.13687 — Aug 24

2409.04628 — Sep 24 — aws 2409.13025

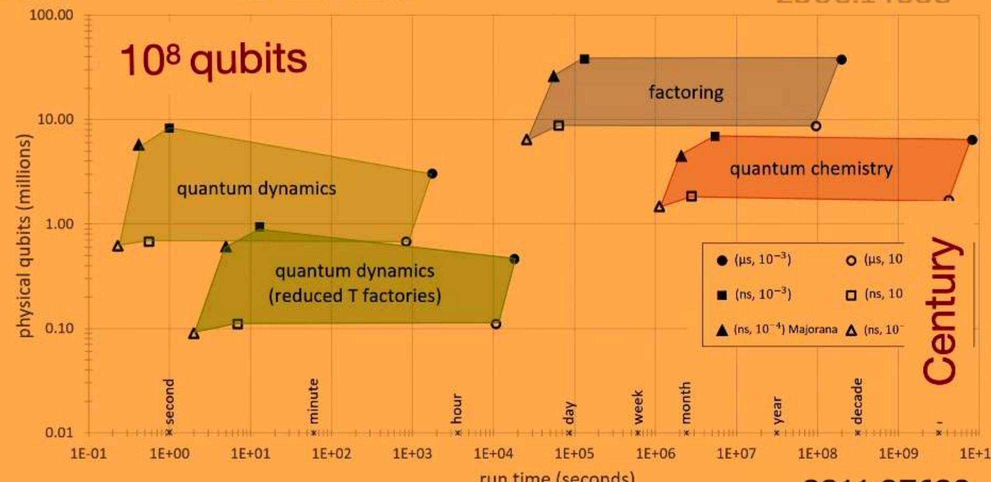
2411.11822 — Nov 24

2412.15165 — Dec 24 — 2412.14256

2506.20661 — Jun 25 — 2506.14688

Recent surge across various platforms

- Logical random circuit sampling (Atoms)
- Repeated error correction (Ions)
- Below threshold quantum memory (Transmon)
- Elements of universal + fault-tolerant (Atoms)



2211.07629

Progress hinges on lowering overhead & error rates

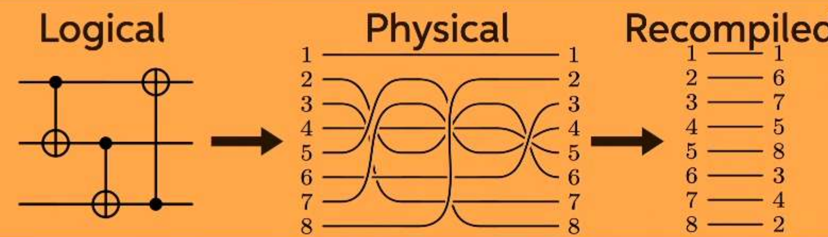
- Across all operations (Decoders, state-prep, etc.)
- Logical entangling gates
  - Repeated measurement rounds, dense two-qubit-gate layers, or ancillary codeblocks
  - Consume nearly half the total error budget in state-of-the-art experiments (2412.14256 & 2506.20661)

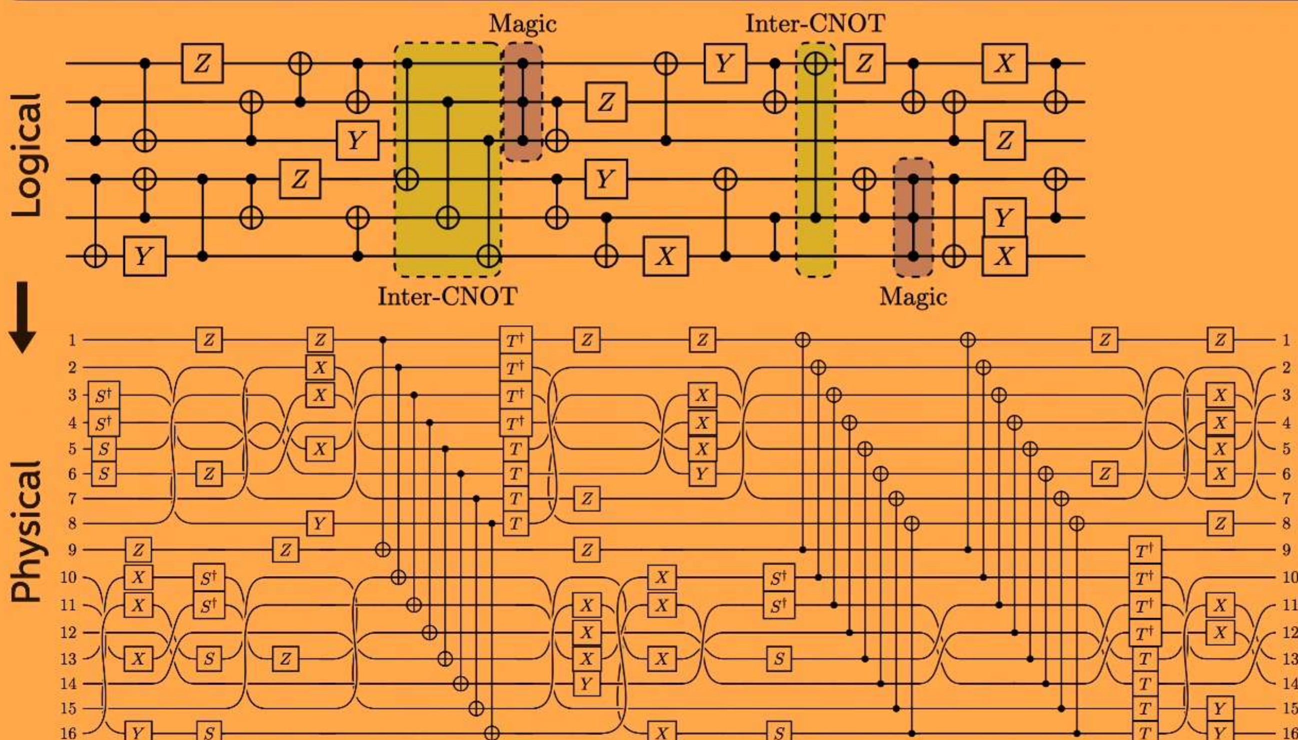
Minimum cost of a logical entangling gate

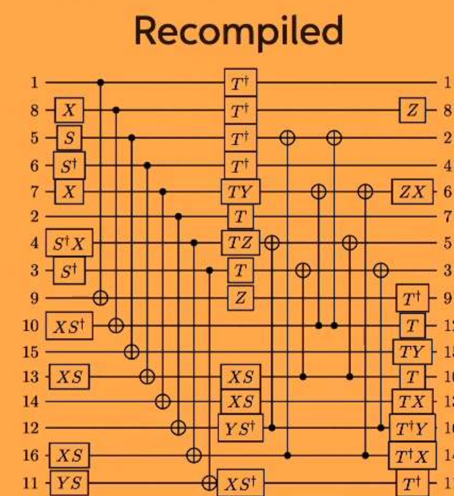
Entanglement via permutations $\text{SWAP}_{13} = \overline{\text{CNOT}}_{12}$

$$\overline{X}_1 = XXII \rightarrow IXXI = \overline{X}_1\overline{X}_2 \qquad \overline{Z}_1 = ZIZI \rightarrow ZIZI = \overline{Z}_1$$

$$\overline{X}_2 = XIXI \rightarrow XIXI = \overline{X}_2 \qquad \overline{Z}_2 = ZZII \rightarrow IZZI = \overline{Z}_1\overline{Z}_2$$



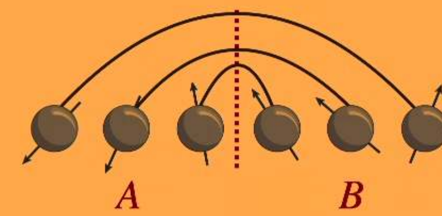




Zero-overhead
+ Perfect fidelity
= Minimum cost

Phantom codes

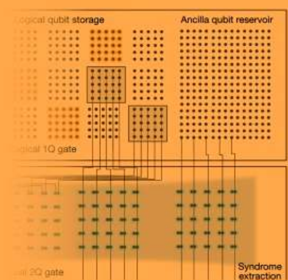
Phantom code: Stabilizer codes in which logical entangling gates between *every* ordered pair of logical qubits can be implemented *solely* via physical qubit permutations.



Two codes fall within this class

$[\![2^D, D, 2]\!]$ Hypercube[1,2]

- Harvard/MIT/QuEra's circuit sampling



$[\![12,2,4]\!]$ Carbon[3]

- Microsoft/Quantinuum's repeated rounds of QEC

Despite intriguing properties and experimental successes, phantom codes remain largely unexplored

Key open questions:

- Unknown whether additional classes of phantom codes exist
- What structural constraints they obey
- Whether their advantages persist for scalable applications under realistic noise conditions

[1]Vasmer and Kubica, *PRXQ* (2022), [2] Bluvstein, et al. *Nature* (2024), [3] Paetiznick, *et al. arXiv* (2024)

Our results

1. Substantially expand the phantom-code landscape (~2 to ~4)

- Numerically: Exhaustive enumeration ($n \leq 14$) and SAT search ($n \leq 21$)
- Analytically: Quantum Reed-Muller (higher k) and "binarization-and-concatenation" scheme (higher d).

2. Demonstrate the practical advantages of phantom codes for scalable applications in the presence of realistic noise

- Benchmark $[\![64,4,8]\!]$ phantom code against surface-code baselines
- End-to-end noisy simulations, including state preparation and full QEC cycles
- GHZ state preparation: ~140x fidelity improvement at 22% preselection (~ num. of qubits)
- Trotterized many-body simulation: ~207x fidelity improvement at 22% preselection (~ num of qubits)

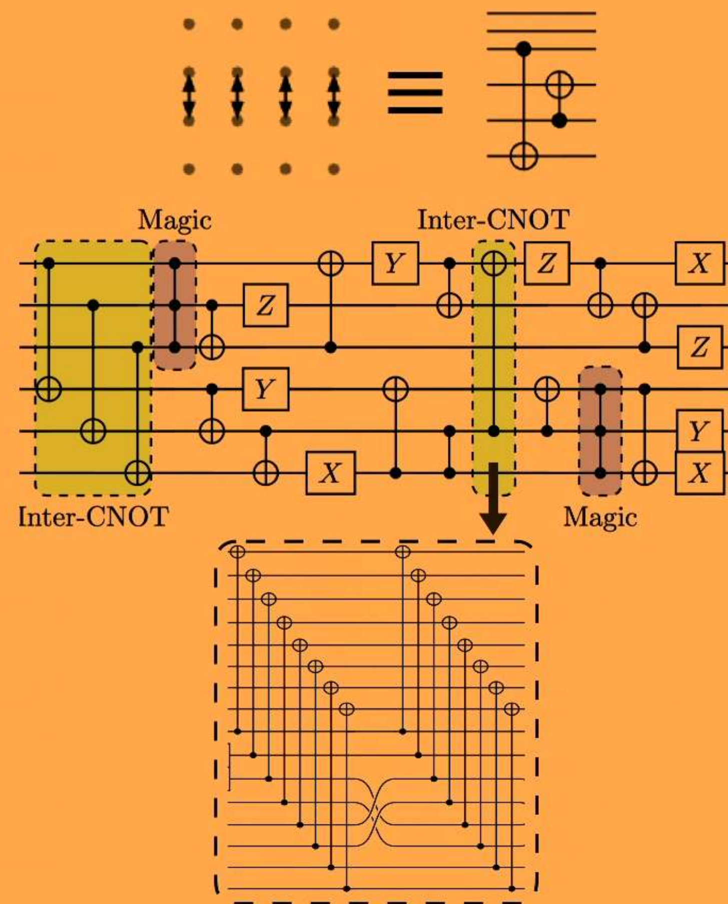
3. Furthermore…

- All codes discovered, we identify fault-tolerant logical Clifford and non-Clifford gates
- Develop a number of practical tools for implementing non-LDPC codes

Phantom codes: Important Properties

Three different, but related ideas

- Codes with permutation symmetries ($[\![16,6,4]\!]$ Tesseract[1])
  - Can't do arbitrary CNOTs
- Logical Pauli frame tracking
  - Can't handle magic
- Automorphism gates (permutations + local Cliffords)
  - Can't achieve zero-cost nor perfect fidelity



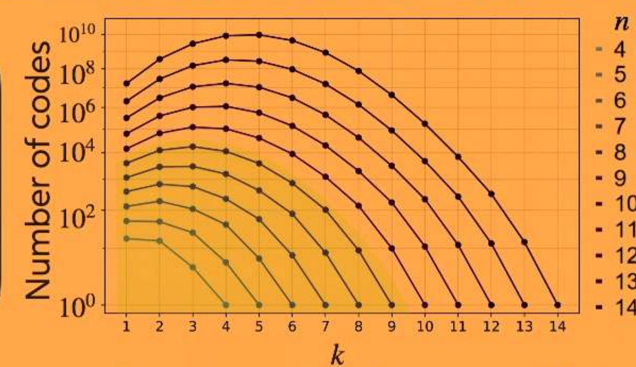
Features

- CSS codes: Efficient arbitrary CNOT circuits across codeblocks.
- Phantom property is independent of logical basis.

[1] Delfosse and Reichardt *arXiv* (2020)

Exhaustive code enumeration

1.39×10^4 out of 2.71×10^{10} are phantom

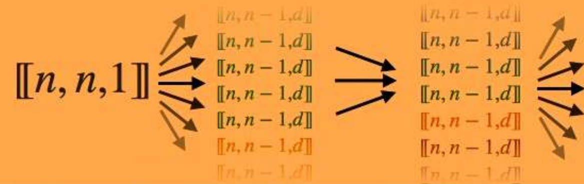


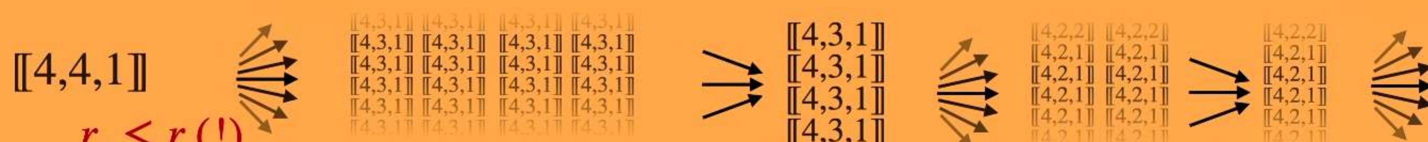
~1/1,000,000 of codes are $n \leq 9$

Method

- Enumerating all stabilizer codes
- Group isomorphic codes (GI-hard)
- Cross, Vandeth 2501.17447 ($n = 9$)

Add stabilizer   Group   Repeat





$r_x \leq r_z (!)$

$H_x = []$ $H_z = []$

$L_x = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$ $L_z = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$

$H_x = [1000]$

$L_x = \begin{bmatrix} 0100 \\ 0010 \\ 0001 \end{bmatrix}$ $L_z = \begin{bmatrix} 0100 \\ 0010 \\ 0001 \end{bmatrix}$

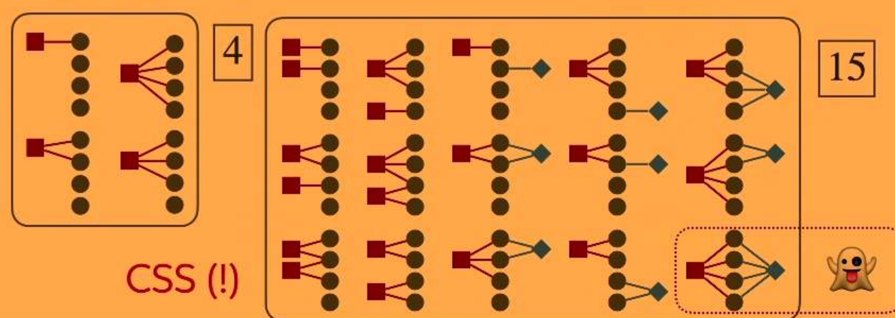
Or…

$H_x = [0100]$

$H_x = [1100]$

$\vdots$

$H_x = [1111]$



CSS (!)

SAT-based code discovery

What's a SAT solver?

- Boolean Formulas in Conjunctive Normal Form (CNF), $x_i \in \{0,1\}$: $(x_1 \vee \neg x_3 \vee x_5) \wedge (x_2 \vee x_4) \wedge \cdots$ where $\vee$, $\neg$, $\wedge$ are logical OR, NOT, and AND
- Find whether the Boolean formula can be made TRUE
- Outcome: Solution or UNSAT with proof
- 3-SAT is NP-complete (as hard as any NP problem)

k	2					3			4
d	2	3	4	5	6	2	3	4	2
Phantom	4	11	12	18	20	7	14	16	15
General	4	10	12	18	20	6	11	14	6

$$F(\text{CNOT}_{12}) = \left(\begin{array}{ccc|ccc} 1 & \boxed{1} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} X_1 \to X_1 X_2 \\ \\ \\ \\ Z_2 \to Z_1 Z_2 \\ \\ \end{array}$$

Logical operators $L := \begin{pmatrix} L_x \\ L_z \end{pmatrix}$

Parity check matrix $H := \begin{pmatrix} H_x \\ H_z \end{pmatrix}$,

Symplectic inner product $\Omega = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

Permutation for CNOT P_{ab}

$$L(P_{ab} \oplus P_{ab})\Omega L^{\mathrm{T}} = F(\text{CNOT}_{ab})\Omega,$$
$$H(P_{ab} \oplus P_{ab})\Omega H^{\mathrm{T}} = 0,$$
$$H(P_{ab} \oplus P_{ab})\Omega L^{\mathrm{T}} = 0,$$
$$L(P_{ab} \oplus P_{ab})\Omega H^{\mathrm{T}} = 0,$$

Gate checker

- Variable: P_{ab}. Input: H, L.

Code discovery

- Variables: H, L, P_{ab}

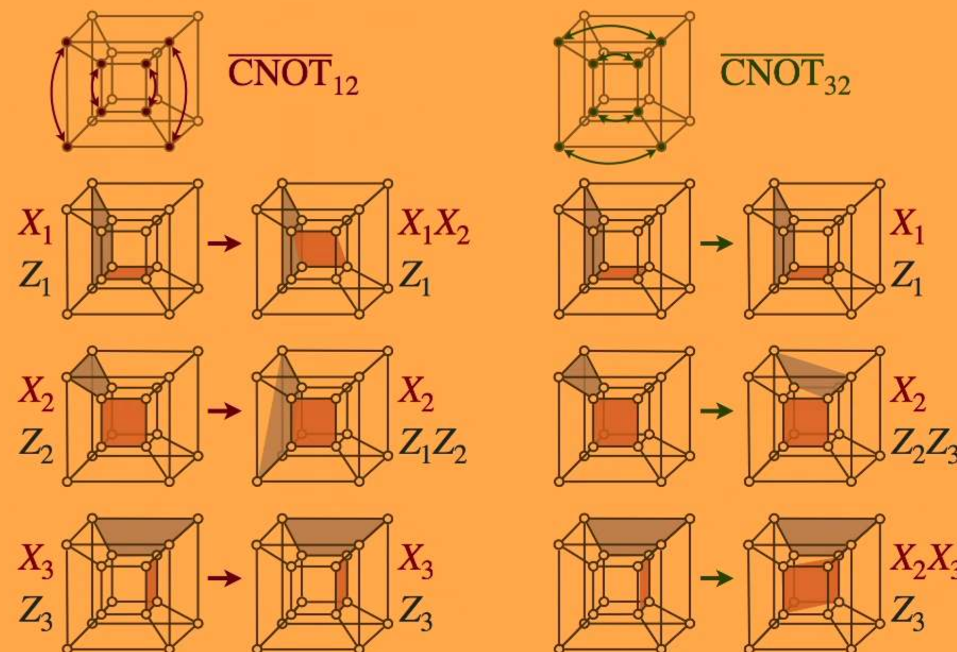
Phantom qRM codes

Method

- Start from $[\![2^m, \binom{m}{\ell}, \min(2^{m-l}, 2^l)]\!]$ qRM code
- Permutations perserve qRM structure
- List all monomials of degree ≤ m

$[\![16,6,4]\!]$ example from RM(1,4) and RM(1,4)

R(0,4)	**I**	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	x1	0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
	x2	0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1
	x3	0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1
R(1,4)	**x4**	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
	x1x2	0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1
	x1x3	0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1
	x1x4	0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 1
	~~**x2x3**~~	~~0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1~~
Fix	~~**x2x4**~~	~~0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1~~
	~~**x3x4**~~	~~0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1~~
R(2,4)		



Binarize & concatenate

Method

- Start from GF(4) qudit code, e.g. $[\![3,1,2]\!]_4$

$$H_X = \begin{pmatrix}\omega & \omega^2 & 1\end{pmatrix}, \quad H_Z = \begin{pmatrix}1 & \omega & \omega^2\end{pmatrix}$$

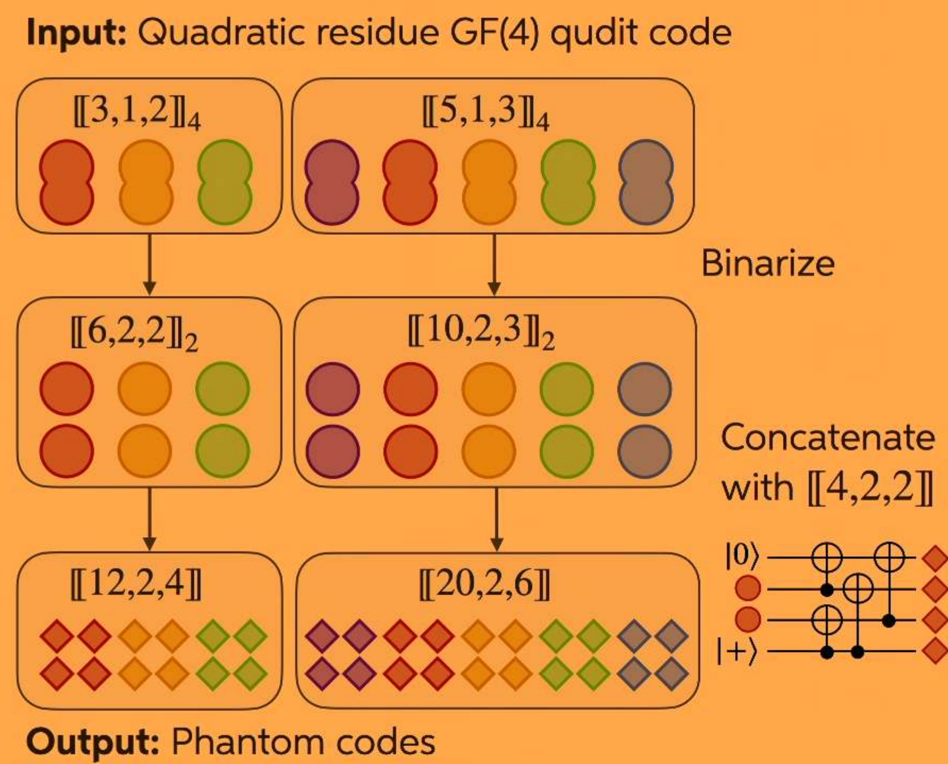
- Binarize the code, e.g. to get $[\![6,2,2]\!]_2$:

$$H_X = (10 \quad 01 \quad 11), \quad H_Z = (01 \quad 11 \quad 10)$$

- Concatenate with $[\![4,2,2]\!]_2$, e.g. to get $[\![12,2,4]\!]_2$

Properties

- Self-dual GF(4) code —> Hadamard on qubit code
- Hermitician GF(4) code —> CZ on qubit code



Logical gates beyond phantom

Clifford gate

- Automorphism of extended binary symplectic rep.

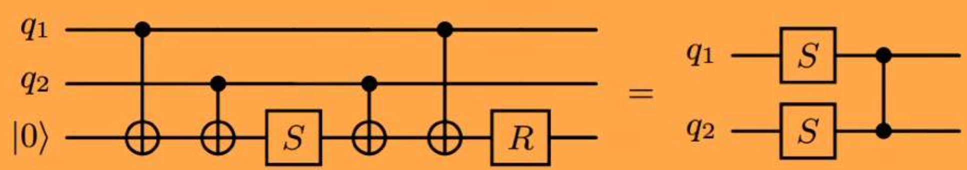
$$\begin{array}{ccc} H_x & H_z & H_x \oplus H_z \end{array}$$
$$\begin{bmatrix} 1111 & 0000 & 1111 \\ 0000 & 1111 & 1111 \end{bmatrix} \qquad H^{\otimes 4} = \overline{H}^{\otimes 2}\overline{\mathrm{SWAP}}$$

Magic

- Fix Clifford hierarchy level $(T, T^{1/2}, \ldots)$ and use solver
- QRM: Split code into smaller hypercubes.

Fold-S

- Circuit to embed code into larger code, use same method as for magic to find S, reverse the embedding



Code	Clifford	Fold-S	$d=2$ Magic	Found
$[\![4,2,2]\!]$	CZ, $H^{\otimes 2}$	Yes	–	[24]
$[\![8,3,2]\!]$	CZ_{ij}	Yes	$\mathrm{C^2Z}$	[24]
$[\![16,4,2]\!]$	CZ_{ij}	Yes	$\mathrm{C^3Z}$	[24]
$[\![12,2,4]\!]$	$H^{\otimes 2}$	Yes	–	[6]
$[\![7,3,2]\!]$	CZ_{ij}	Yes	–	Enumeration
$[\![12,2,2]\!]$	CZ, S_i	NA	CS	Enumeration
$[\![14,3,3]\!]$	–	Yes	–	Enumeration
$[\![18,2,5]\!]$	$H^{\otimes 2}$	Yes	–	SAT search
$[\![20,2,6]\!]$	CZ, $H^{\otimes 2}$	Yes	–	SAT search
$[\![21,2,3]\!]$	CZ, S_i	NA	–	SAT search
$[\![16,3,4]\!]$	–	Yes	$\mathrm{C^2Z}$	qRM
$[\![32,4,4]\!]$	–	Yes	$\mathrm{C^3Z}$	qRM
$[\![64,5,4]\!]$	–	Yes	$\mathrm{C^4Z}$	qRM
$[\![64,4,8]\!]$	–	Yes	$\mathrm{C^3Z}$	qRM
$[\![44,2,10]\!]$	$H^{\otimes 2}$	Yes	–	B&C
$[\![52,2,10]\!]$	CZ, $H^{\otimes 2}$	?	–	B&C
$[\![68,2,14]\!]$	CZ, $H^{\otimes 2}$	?	–	B&C
$[\![116,2,22]\!]$	CZ, $H^{\otimes 2}$	?	–	B&C

Numerical benchmarking

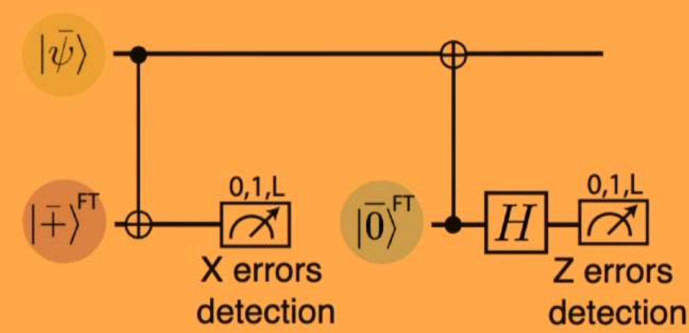
Numerical benchmarking

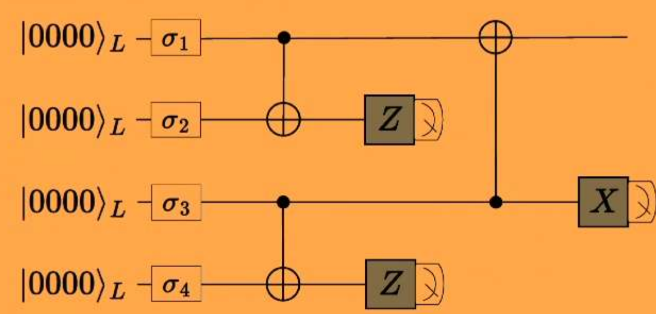
Surface-code baseline (25 yrs of optimization)
- LPDC (low-weight checks)
- Low-overhead state preparation
- Efficient hook-free bare ancilla syndrome measurement
- Near-optimal matching decoder
- Correlated decoding + algorithmic fault-tolerance (don't need d rounds of SE)

How we operate on the $[\![64,4,8]\!]$ (non-LDPC)
- Bare-ancilla SE is not possible, so we use Steane-style QEC
- Developed a pre-selection based FT state-prep protocol
  - Strictness post-selection, 1e-3 > ~22% acceptance
- Developed a spatiotemporal sliding window MLE decoder

Common points
- Circuit-level depolarizing noise with error rates based on atoms
- Average over all 0 and all + initial states

Steane QEC



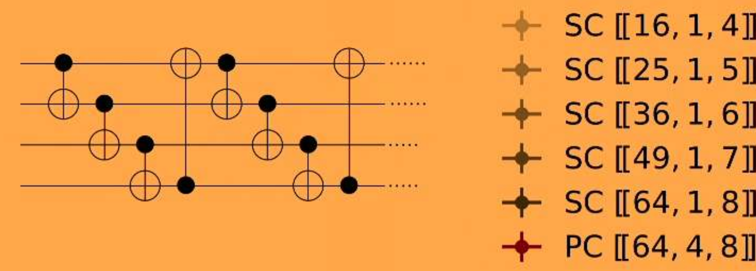


σ_i are permutations

Numerical benchmarking

Benchmark 1: Repeated in-block CNOTs

- In-block CNOTs are free, state prep is more costly—When does reduction in gate cost outweighs the state prep cost?

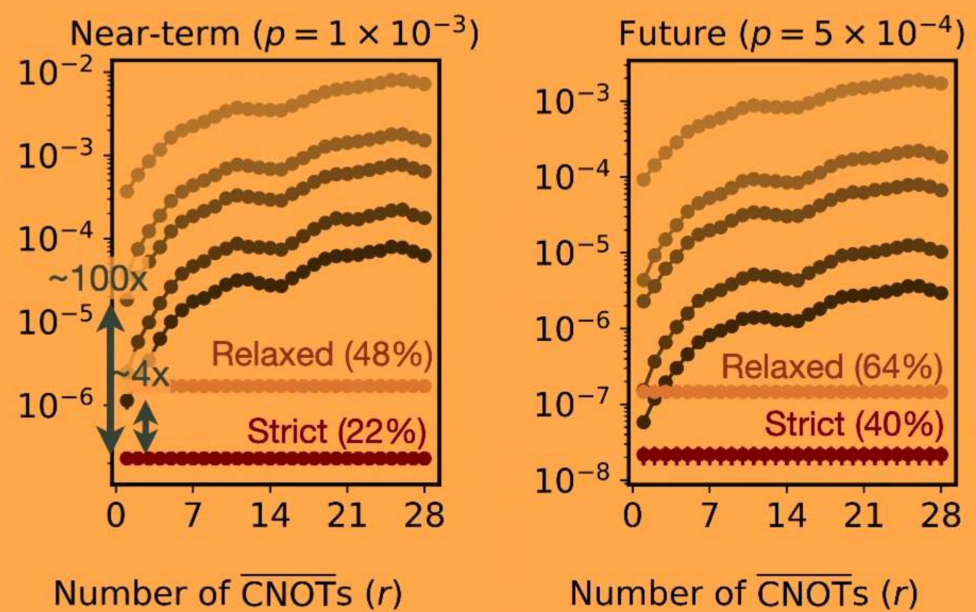


Total qubit counts (data + ancilla + factory)

- Phantom: 256 qubits (22% pre.)
- $d = 5$ SC: 287 qubits
- $d = 8$ SC: 511 qubits

Comparison

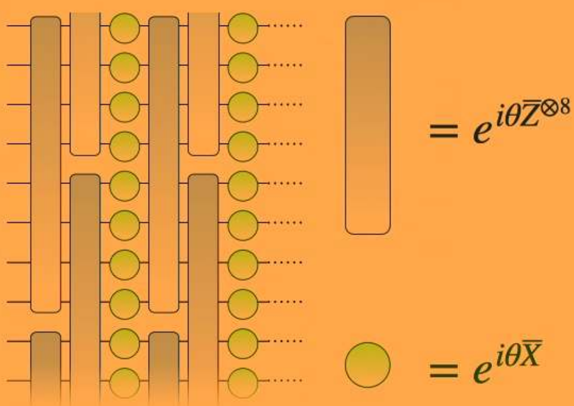
- 100x improvement and comparable qubits
- 4x improvement and 2x less qubits

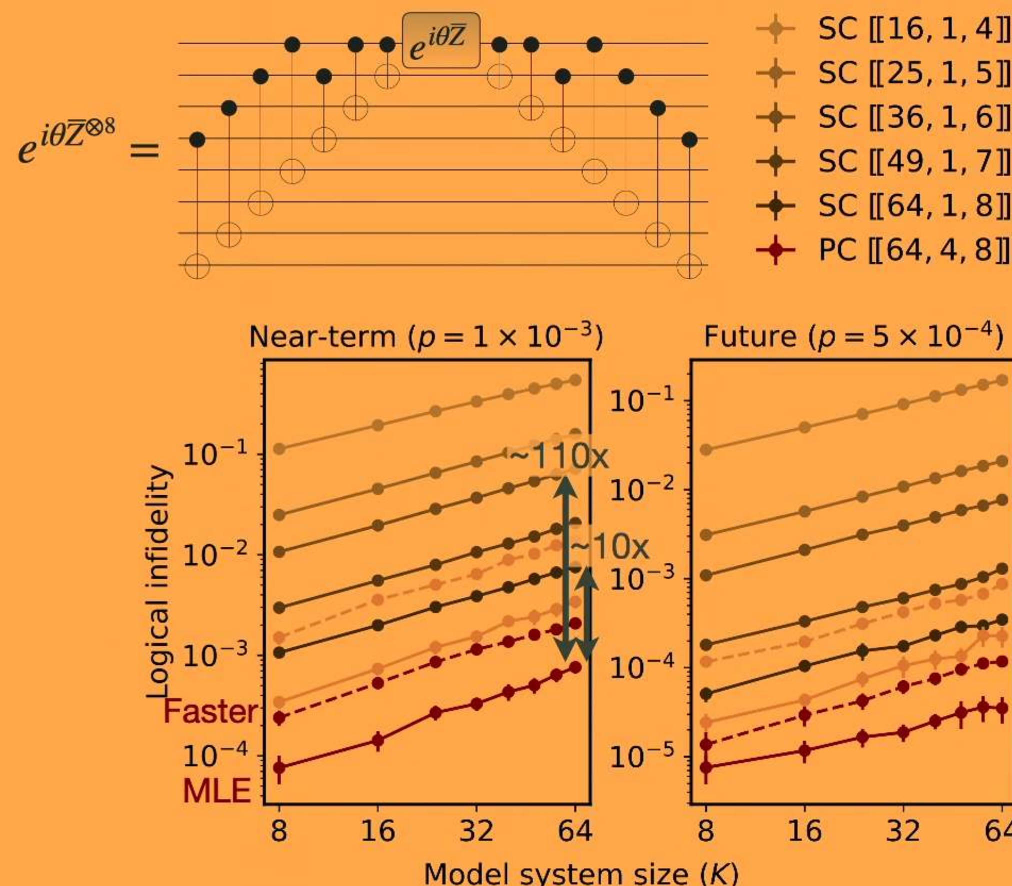


Numerical benchmarking

Benchmark 3: Trotterized many-body sim.

- Sustained advantage for scalable application





Comparison (same qubits as GHZ prep)

- 100x improvement and comparable qubits
- 10x improvement and 1.8x less qubits

Summary & Outlook

Summary

- Substantially expand the phantom-code landscape
- Demonstrate the practical advantages of phantom codes

Outlook

- Exploring limits
  - All phantom codes discovered are “non-LDPC”: motivates new constructions or formal no-go’s
  - Empirical bound $k = \mathcal{O}(\log n)$: Can large automorphism groups coexist with high encoding rates?
  - Magic was $d = 2$—reflect a deeper structural obstruction?
- Phantom minimizes temporal cost, high code size. LDPC do the opposite. Is there a continuum?
- Software and decoding—automorphism-aware compilers that treat permutations as free?
- Experimental implementations

Thanks for listening

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Sasha
Geim

Michael
Gullans

Norman
Yao

Misha
Lukin