Title: Edge Modes: from classical to quantum and from discrete to continuum

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Abstract:

In this talk, I will describe recent developments in our understanding of edge modes associated to subregions in gauge theories, leveraging their realization as reference frames. I will begin with the picture in classical Maxwell theory, where I will introduce the concept of subregional Goldstone mode as a relational observable parametrizing the corner symmetry group. With this as a guiding principle, I will then move on to non-Abelian lattice gauge theory, where we can carry out the construction directly at the quantum level. I will characterize a novel hierarchy of relational subregional algebras, which encompasses the so-called electric and magnetic center algebras usually considered in the literature, for which we provide a new general definition. This leads to corresponding entropy hierarchies. Interestingly, some of the relational algebras can be factors, and so the physical Hilbert space factorizes. Except in the Abelian case, the subregional Goldstone mode is generically only defined on a subspace of the Hilbert space, stemming from the incompleteness of certain edge mode frames. I will conclude with on-going efforts to have a quantum description of edge modes in the continuum, employing algebraic QFT methods. An overarching theme will be the relation between the subregional Goldstone mode and the asymptotic soft sector of the theory.

Edge Modes

from classical to quantum and from discrete to continuum

Gonçalo Araújo Regado

Plan of the talk

1. Motivation
2. Classical edge modes in the continuum
    a. Subregional Goldstone mode (GM)
    b. Corner symmetries
3. Quantum edge modes on the lattice
    a. Physical Hilbert space factorisation
    b. Relational subregional algebras
4. Towards quantum edge modes in the continuum
5. Conclusion

Motivation

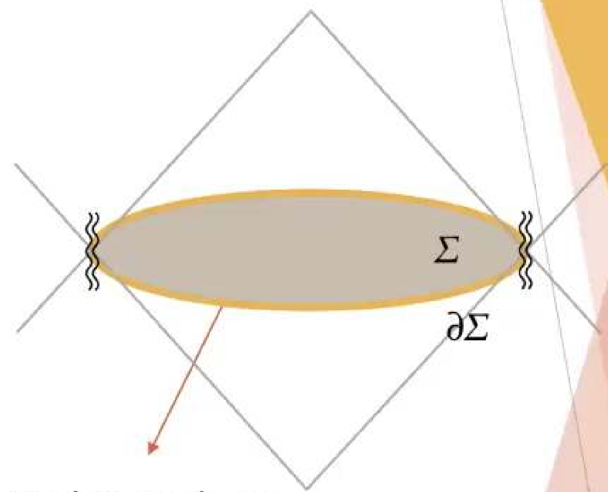
- Description of subregions in gauge theory
[Donnely, Freidel '16; Geiller, Jai-akson '19; Ball, Law, Wong '24]

- Meaning of entanglement entropy in gauge theory
[Donnely '12; Casini, Huerta, Rosabal '13; Donnely, Wall '14; Soni, Trivedi '15; Delcamp, Dittrich, Riello '16]

➢ Enter edge modes -- reference frames for corner gauge group

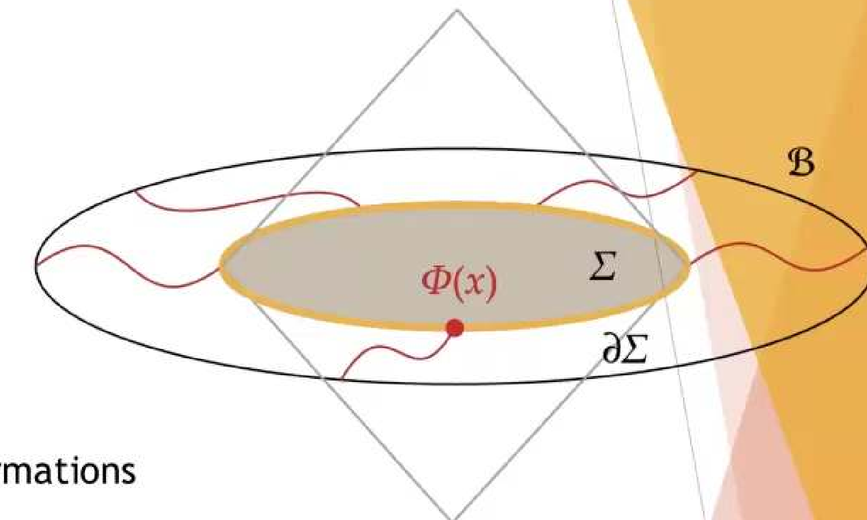
Are they new dofs?
[Carozza, Hoehn '21; Carozza, Eccles, Hoehn '22; Freidel, Kirklin '25]

- New insights about entanglement, subregional algebras and connection with soft physics



New edge mode on the corner

What are edge modes?



➢ Start with free Maxwell theory

- Edge mode = non-local functional of A s.t.

$$\Phi(\mathrm{x}) \mapsto \Phi(x) + \alpha(x)$$

under corner-supported gauge transformations

- Prototypical example: Wilson line anchored on $\mathcal{B}$

$\Phi(x) := \int_{\gamma:\mathcal{B}\to\partial\Sigma} A$ -- extrinsic edge mode ⟶ complement-supported [Carozza, Hoehn '21]

- On-shell gauge-invariant subregion symplectic form:

$$\Omega^{ext} := \int_{\Sigma} \delta A \wedge \delta \star F - \int_{\partial\Sigma} \delta\Phi \wedge \delta \star F$$

[Donnely, Freidel '16]

Subregional Goldstone mode

- For any extrinsically-dressed subregion phase space we can find a gauge-invariant decomposition:

$$\Omega^{ext} \approx \Omega^{int} + \int_{\partial\Sigma} \delta\big(\widetilde{\Phi} - \Phi\big) \wedge \delta \star F$$

for some choice of $\widetilde{\Phi}$ -- intrinsic edge mode ⟶ built from dofs inside region

where Ω^{int} is entirely *bulk supported* on-shell.

> ➢ Gauge-invariant bulk/corner factorisation
>
> ➢ Physical conjugate to corner flux is always a cross-boundary observable

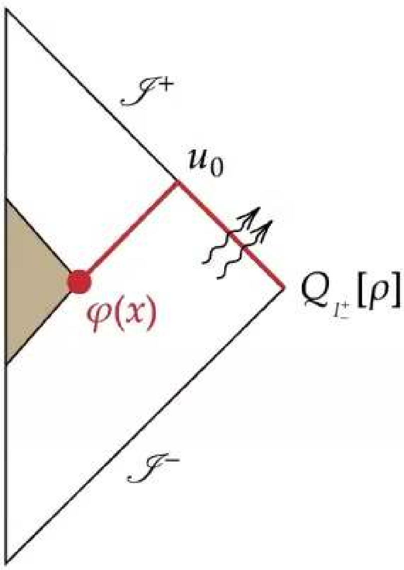
- Goldstone mode as a relational observable: $\varphi := \widetilde{\Phi} - \Phi$

[see Ball, Law, Wong '24 for gauge-fixed version;
Ball, Ciambelli '24 for non-abelian version]

Physical corner symmetries

- Non-degenerate flow: $\Phi(\mathrm{x}) \mapsto \Phi(\mathrm{x}) - \rho(x)$ while $A \mapsto A$
- Infinity of corner charges: $Q[\rho] := \int_{\partial\Sigma} \rho \star F$
- Flow parameterised by gauge-invariant GM: $\varphi(x) \mapsto \varphi(x) + \rho(x)$

} analogous to asymptotic soft sector [Strominger et al]



$$D^2\varphi(x) = D^2\phi(x) - \frac{1}{2}D^2N(x) + 2\lim_{r\to\infty} r^2 E_r(u_0, x)$$

asymptotic GM — asymptotic soft mode

$\varphi[\phi, N, \text{hard}]$

- Generators:
  - ➢ $Q_{\mathcal{I}^\pm}[\rho]$ -- large-gauge transfs.
  - ➢ Certain hard photon mode profiles

$\{E_r(u_0), E_r(u_0')\} \neq 0$

Quantum edge modes on the lattice

Based on arXiv: 2506.23459

w/ Philipp Hoehn, Francesco Sartini

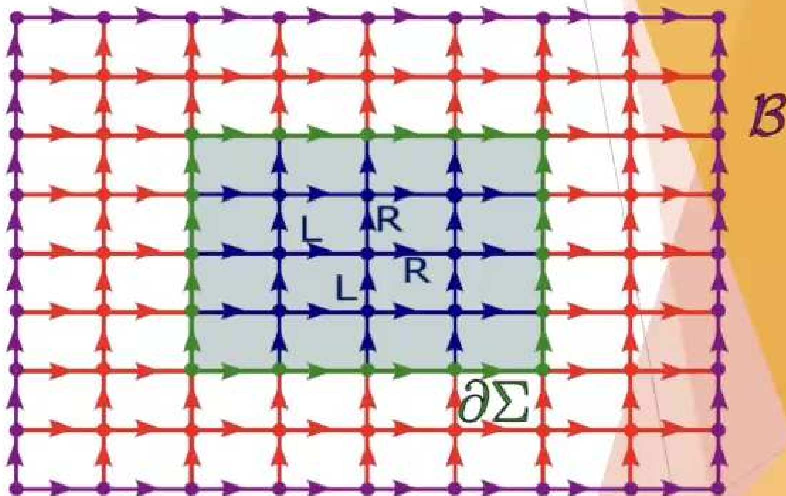
Lattice gauge theory

[Kogut, Susskind,...]

- Degrees of freedom on oriented edges: $L^2(G)$
- $\hat{g}_e$ -- holonomy operator on edge e
- Gauge transformations act on vertices v

$$\hat{g}_e \mapsto \rho\left(h_f^{-1}\right) \hat{g}_e \, \rho(h_i)$$

- $\hat{L}_e^a / \hat{R}_e^a$ -- left/right electric fields on edge e



General compact Lie group G
$\{a\}$ -- Lie algebra indices
ρ -- representation of G
$h = e^{i\theta^a T^a}$

$$U(h) := e^{-i\theta^a \hat{L}_e^a} \Rightarrow |g\rangle \mapsto |hg\rangle$$

$$V(h) := e^{+i\theta^a \hat{R}_e^a} \Rightarrow |g\rangle \mapsto |gh^{-1}\rangle$$

The Page-Wooters formalism

[Page, Wooters; de la Hamette, Galley, Hoehn, Loveridge, Muller '21,...]

- Setup: $\mathcal{H}_{kin} = \mathcal{H}_R \otimes \mathcal{H}_S$ ⟹ $\mathcal{H}_{phys} = (\mathcal{H}_R \otimes \mathcal{H}_S)^G = \Pi_{\text{phys}}(\mathcal{H}_{kin})$

  QRF system

  Gauge action: $U_R(g) \otimes U_S(g)$

- Reference frame ↔ $\{ |\tilde{g}\rangle_R : \tilde{g} \in {}^{G}/_{H}\}$ -- generalised coherent states $U_R(g)|\tilde{g}\rangle_R = |g\tilde{g}\rangle_R$

- R is ideal: $\langle \tilde{g}_1 | \tilde{g}_2 \rangle_R = \delta_G(\tilde{g}_1, \tilde{g}_2)$

$\Pi_{\text{phys}} := \int_G [dg]\, U_R(g) \otimes U_S(g)$ -- projector

H -- isotropy subgroup

The Page-Wooters formalism

$\mathcal{R}_R \mathcal{R}_R^{-1} = P_H = \mathbb{1}$

$\mathcal{R}_R^{-1} \mathcal{R}_R = \mathbb{1}$

- Page-Wooters conditioning map (jumping into the frame perspective):

$\mathfrak{R}_R(\tilde{g}) \coloneqq \langle \tilde{g}|_R \otimes 1_S$ -- quantum gauge-fixing

$\mathcal{R}_R : \mathcal{H}_{phys} \to P_H(\mathcal{H}_S)$

$\mathcal{R}_R^{-1} = \Pi_{phys} \, |\tilde{g}\rangle_R \otimes \mathbb{1}_S$

- It induces isomorphism: $\mathfrak{R}_R(\mathcal{H}_{phys}) \cong \mathbb{P}_H(\mathcal{H}_S)$

relational observable

- Relational observables on $\mathcal{H}_{phys}$: $\mathfrak{R}_R^{-1}\, \mathfrak{H}(f_S)\, \mathfrak{R}_R\, \Pi_{phys} = \mathcal{G}(|\tilde{g}\rangle\langle\tilde{g}|_R \otimes \mathfrak{H}(f_S))\, \Pi_{phys}$

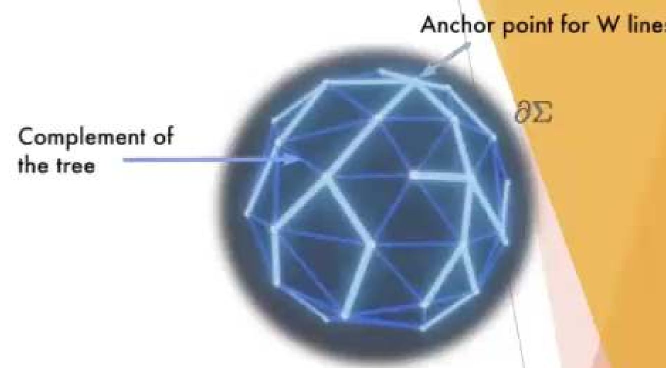
QRF can only "see" H-coarse-grained system

$\mathbb{P}_H \coloneqq \int_H [dh]\ U_S(h)$ -- projector

$\mathfrak{H}(\cdot) \coloneqq \int_H [dh] U_S(h)(\cdot)U_S^\dagger(h)$ -- isotropy average

$\mathcal{G}(\cdot) \coloneqq \int_G [dg]\, U_R(g) \otimes U_S(g)\ (\cdot)\ U_R^\dagger(g) \otimes U_S^\dagger(g)$ - G-twirl

Page-Wooters on the lattice



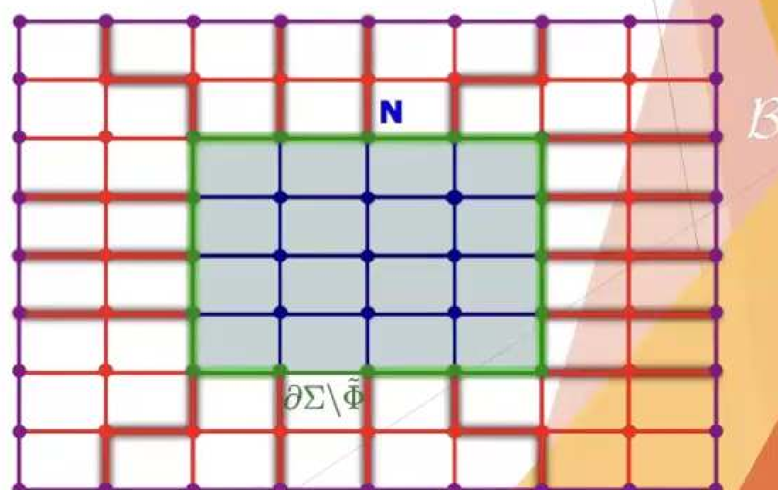
- Construct frame at each vertex on $\partial\Sigma$:

➢ Extrinsic $\{\Phi_v \mid v \in \partial\Sigma\}$ -- anchored on $\mathcal{B}$

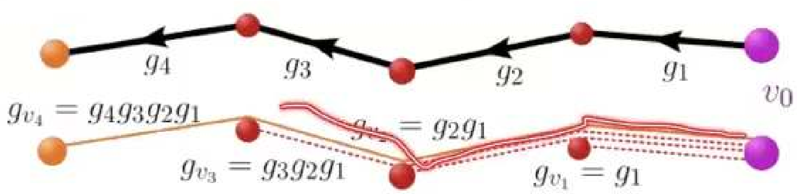
$H = \emptyset$ -- complete frame

➢ Intrinsic $\{\widetilde{\Phi}_v \mid v \in \partial\Sigma \setminus N\}$ -- anchored on some fixed vertex $N \in \partial\Sigma$

$H \cong G$ -- incomplete frame



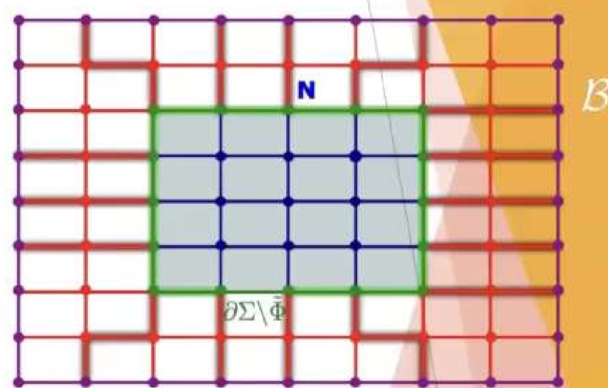
- Requires kinematical re-factorisation:
(composite Wilson line dof lives on final vertex)



The physical Hilbert space factorises

- Take the kinematical Hilbert space wrt corner gauge transformations:

$$\mathcal{H}_{kin}^{\partial\Sigma} = \mathcal{H}_{\Phi} \otimes \mathcal{H}_{\tilde{\Phi}} \otimes \mathcal{H}_{\Sigma\backslash\tilde{\Phi}}^{phys} \otimes \mathcal{H}_{\bar{\Sigma}\backslash\Phi}^{phys}$$



- Physical Hilbert space: $\mathcal{H}_{phys} = \Pi_{phys}^{\partial\Sigma}(\mathcal{H}_{kin}^{\partial\Sigma})$

$$\mathcal{R}_{\tilde{\Phi}}(\mathcal{H}_{phys}) = \mathbb{P}_{H}\left(\mathcal{H}_{\tilde{\Phi}} \otimes \mathcal{H}^{phys}_{\Sigma\backslash\tilde{\Phi}} \otimes \mathcal{H}^{phys}_{\bar{\Sigma}\backslash\tilde{\Phi}}\right)$$

- Extrinsic perspective yields a factorisation:

$$\mathfrak{R}_{\Phi}(\mathcal{H}_{phys}) \cong \mathcal{H}_{\tilde{\Phi}} \otimes \mathcal{H}_{\Sigma\backslash\tilde{\Phi}}^{phys} \otimes \mathcal{H}_{\bar{\Sigma}\backslash\Phi}^{phys}$$

→ captures physical entanglement

inherited from kinematical TPS (after non-local refactorisation)

- ➢ contrast with:

$$\mathcal{H}_{phys} \ncong \mathcal{H}_{\Sigma}^{phys} \otimes \mathcal{H}_{\bar{\Sigma}}^{phys}$$

[extended Hilbert space: Donnely '12, Soni, Trivedi '15,...]

Corner symmetries

- Frame reorientations = physical transformations:

$$V_R(g)|\tilde{g}\rangle_R = |\tilde{g}g^{-1}\rangle_R \qquad \mathcal{H}_{kin} = \mathcal{H}_R \otimes \mathcal{H}_S$$

(while leaving S invariant)

- On the lattice, these are physical complement-supported unitaries:

$$V_\Phi(g_{\partial\Sigma}) \otimes 1_{\tilde{\Phi}} \otimes 1_{\Sigma\backslash\tilde{\Phi}} \otimes \mathcal{U}_{\bar{\Sigma}\backslash\Phi} \qquad \mathcal{H}_{kin}^{\partial\Sigma} = \mathcal{H}_\Phi \otimes \mathcal{H}_{\tilde{\Phi}} \otimes \mathcal{H}_{\Sigma\backslash\tilde{\Phi}}^{phys} \otimes \mathcal{H}_{\bar{\Sigma}\backslash\Phi}^{phys}$$

subregion left invariant

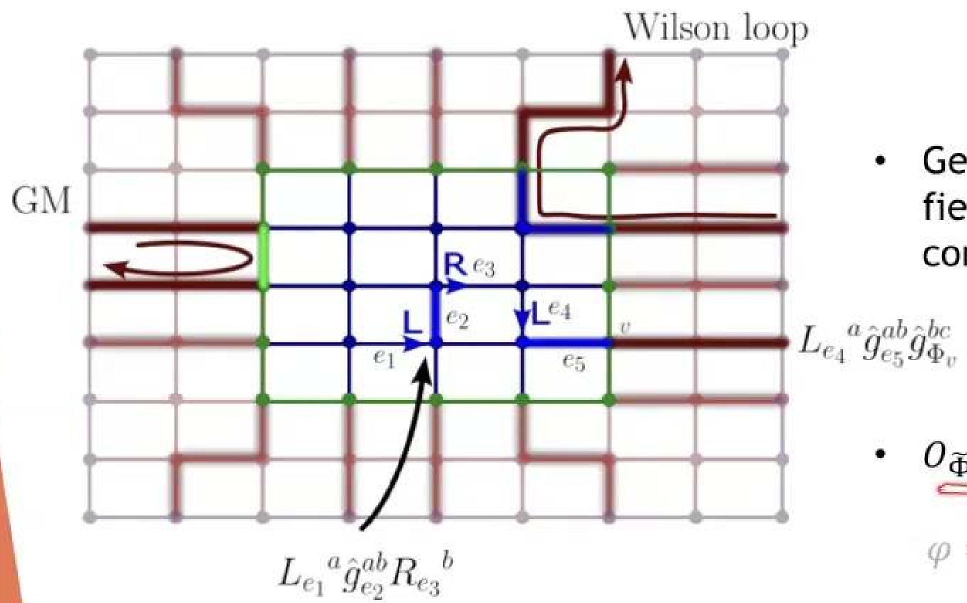
- On $\mathfrak{R}_\Phi(\mathcal{H}_{phys})$ they look like corner-supported gauge transformations.

Relational algebras

- Relational observables = dressed observables:

G-twirl

$$O_{f_S|R}(g) \coloneqq \mathcal{G}(|g\rangle\langle g|_R \otimes f_S) \quad \text{where} \quad \mathcal{G}(\cdot) \coloneqq \int_G [dg]\, U_R(g) \otimes U_S(g)\, (\cdot)\, U_R^\dagger(g) \otimes U_S^\dagger(g)$$



- Get: Wilson loops, $\mathcal{B}$-anchored Wilson lines, electric fields parallel transported to $\mathcal{B}$, quadratic combinations of electric fields…

- $O_{\widetilde{\Phi}|\Phi}$ -- GM observables

$\varphi = \widetilde{\Phi} - \Phi$

Electric centre algebra

- The corner symmetry transformations act as automorphisms on $\mathcal{A}^{phys}_{\Sigma|\Phi}$.
- If we are ignorant about the GM parameterising this orbit then we have access to a *coarser algebra:*

$$\mathcal{A}_E := \underbrace{\mathbb{G}_{\partial\Sigma}(\mathcal{A}^{phys}_{\Sigma|\Phi})}_{\text{twirl over physical symmetry}}$$

→ Non-trivial centre = Casimir of perpendicular electric flux at each corner vertex

$$\rho^{phys}_{\Sigma|\Phi} = \begin{pmatrix} \blacksquare & \cdots & \blacksquare \\ \vdots & \ddots & \vdots \\ \blacksquare & \cdots & \blacksquare \end{pmatrix} \xrightarrow{\mathbb{G}_{\partial\Sigma}} . \quad \rho_E = \begin{pmatrix} \blacksquare & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \blacksquare \end{pmatrix}$$

in the basis diagonalising the corner Casimir operators, the twirl decoheres the density matrix

CPTP map

[Casini, Huerta, Rosabal '13; Soni, Trivedi '15]

Intersections of relational algebras

- Subsystem relativity = different frames associate different algebras to the same kinematical subsystems

[Hoehn, Kotecha, Mele '23; De Vuyst, Eccles, Hoehn, Kirklin '24,…]

$$\bigcap_{\text{extrinsic}\,\Phi} \mathcal{A}^{phys}_{\Sigma|\Phi} =: \mathcal{A}_E$$ ⟶ Electric centre algebra is what all extrinsic edge modes agree on

$$\bigcap_{\text{intrinsic}\,\tilde{\Phi}} \mathcal{A}^{phys}_{\Sigma\setminus\tilde{\Phi}|\tilde{\Phi}} =: \mathcal{A}_M$$ ⟶ Magnetic centre algebra is what all intrinsic edge modes agree on

Non-trivial centre $= \{\partial\Sigma$ Wilson loops $\vee$ electric part$\}$

(magnetic centre: $\partial\Sigma$ Wilson loops)

[generalisation of Delcamp, Dittrich, Riello '16]

Algebra and entropy hierarchy

$$\mathcal{A}^{phys}_{\Sigma|\Phi} \supset \mathcal{A}_E \supset \mathcal{A}^{phys}_{\Sigma\backslash\tilde{\Phi}|\tilde{\Phi}} \supset \mathcal{A}_M$$

Relational algebras

QRF independent

algebra	entanglement entropy	distillable
$\mathcal{A}^{\text{phys}}_{\Sigma\|\Phi}$	$S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\|\Phi})$	✓
$\mathcal{A}_E$	$H(\{p_{\mathfrak{e}}\}) + \sum_{\mathfrak{e}} p_{\mathfrak{e}} (\log d_{\mathfrak{e}} + S_{\text{vN}}(\rho_{\mathfrak{e}}))$	×
$\mathcal{A}^{\text{phys}}_{\Sigma\backslash\bar{\Phi}\|\bar{\Phi}}$	A: $S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\backslash\bar{\Phi}\|\bar{\Phi}})$	✓
	nA: $H(\{p_{\bar{\mathfrak{e}}}\}) + \sum_{\bar{\mathfrak{e}}} p_{\bar{\mathfrak{e}}} (\log d_{\bar{\mathfrak{e}}} + S_{\text{vN}}(\rho_{\bar{\mathfrak{e}}}))$	×
$\mathcal{A}_M$	$H(\{p^m_{\bar{\mathfrak{e}}}\}) + \sum_{m,\bar{\mathfrak{e}}} p^m_{\bar{\mathfrak{e}}} (\log d^m_{\bar{\mathfrak{e}}} + S_{\text{vN}}(\rho^m_{\bar{\mathfrak{e}}}))$	×

- Relative entropy hierarchy:

$$S(\rho_{|\Phi}||\sigma_{|\Phi}) \geq S(\rho_{\text{E}}||\sigma_{\text{E}}) \geq S(\rho_{|\tilde{\Phi}}||\sigma_{|\tilde{\Phi}}) \geq^* S(\rho_{\text{M}}||\sigma_{\text{M}})$$

- vN entropy hierarchy:

$$S_{vN}(\rho_{|\Phi}) \leq S_{vN}(\rho_{\text{E}}) \qquad S_{vN}(\rho_{|\tilde{\Phi}}) \leq^* S_{vN}(\rho_{\text{M}})$$

Towards quantum edge modes in the continuum

Work in progress

w/ Philipp Hoehn, Alok Laddha, Bilyana Tomova

Conclusion and Outlook

- Summary:
  - Defined a subregional GM + relationship to asymptotic soft sector
  - Physical Hilbert space factorisation on the lattice
  - Relational algebra hierarchy
  - Attempts to go to continuum
- Future directions:
  - Continuum QED + implications for scattering theory (infrared problem) [Kulish, Fadeev, Buchholz, Wald,...]
  - Gravity – corner program [Freidel, Ciambelli, Geiller, Wieland, ...]
  - Phenomenological implications of GM sector $\langle Q(\rho)\, Q(\rho')\rangle$

    [Zurek, Verlinde, He, Raclariu, Ciambelli, Mitra,...]

Thank you!