Title: Thermodynamics of gravity at finite cutoff

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Abstract:

Defining thermodynamic ensembles for gravitational systems is tied to specifying boundary conditions. In this talk I'll first briefly review how introducing a Dirichlet timelike boundary clarifies thermodynamics of de Sitter space. Then I'll discuss geometries subjected to conformal boundary conditions, where the conformal class of the boundary metric and the trace of the extrinsic curvature K are held fixed. In the high temperature limit the series of subextensive terms in the free energy are compared to predictions from thermal effective field theory, and we observe agreement in all considered cases. Ensembles with negative K include solutions with cosmic-type horizons, where the system boundary is smaller than the horizon. In some regions of parameter space, these solutions are the dominant phase and are necessary for consistency with thermal effective field theory.

Thermodynamics of gravity at finite cutoff

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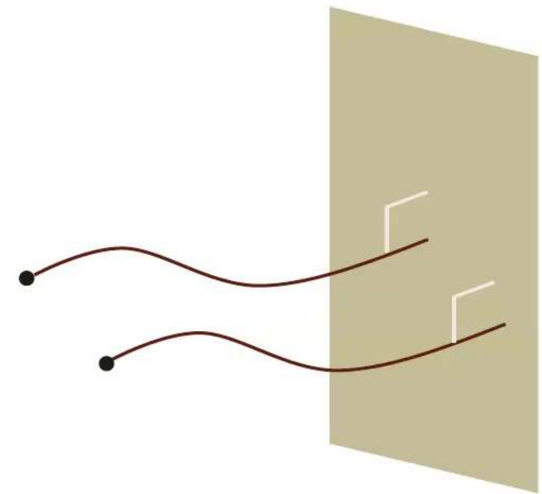
Boundary in gravitational systems

- Defining observables by setting anchor points;
  no local observable due to diffeomorphism invariance



Boundary in gravitational systems

- Defining observables by setting anchor points;
  no local observable due to diffeomorphism invariance

- Mapping to fundamental degrees of freedom: holography
  ingredients on the package

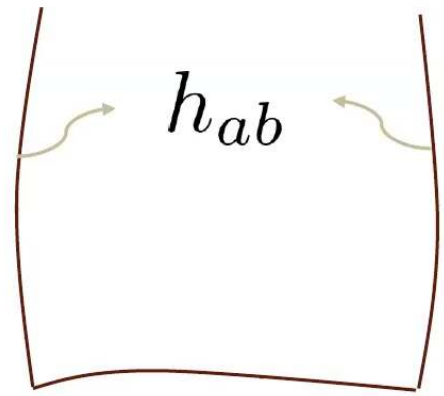
- Studying statistical ensembles and phase transitions;
  example: asymptotically AdS black holes [Hawking, Page '83]

Finite cutoff: which boundary conditions?

- Dirichlet:

  Fixing boundary metric

  $\rightarrow$ canonical ensemble with specified temperature and spatial geometry



- Initial boundary value problem *could be* ill-posed: existence and uniqueness problems

  [Anderson][An, Anderson][Witten][Anninos, Galante, Mühlmann]

Dirichlet problem for a spherical boundary does not suffer from non-uniqueness issues at the linearized level

[Anninos, Galante, Maneerat]

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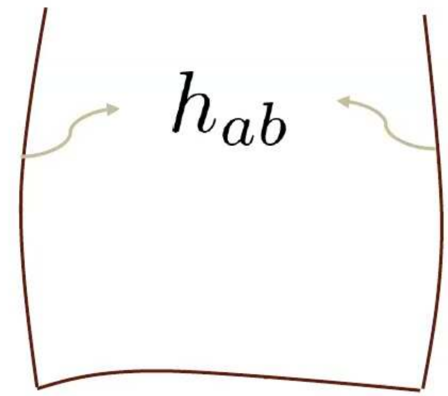

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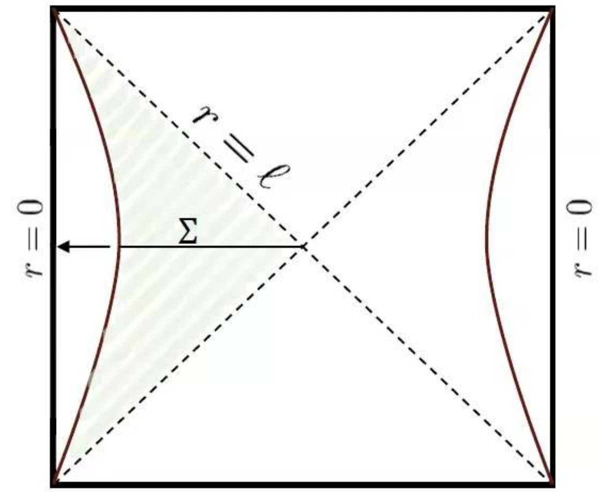
dS thermodynamics with Dirichlet wall

[Banihashemi, Jacobson]
[Banihashemi, Jacobson, Svesko, Visser]

- A boundary is needed to define the ensemble
- Clarifies thermodynamics of bounded static patch
- Standard first law of thermodynamics;



$$\boxed{\delta E_{\mathrm{BY}} = \delta E_{\mathrm{m}} + T\delta S_{\mathrm{BH}}}$$

positive temperature $T^{-1} = 2\pi\ell\sqrt{1 - R^2/\ell^2}$

Brown-York energy as internal energy $E_{\mathrm{BY}} = -\frac{1}{8\pi G}\oint_{\mathcal{S}} d^2x\sqrt{\sigma}\,k$

In the limit $R \to 0$: $0 = \delta E_{\mathrm{m}} + T\delta S_{\mathrm{BH}}$

$$\int \delta T_{ab} K^a d\Sigma^b = -\kappa_c \delta A_C (8\pi)^{-1}$$

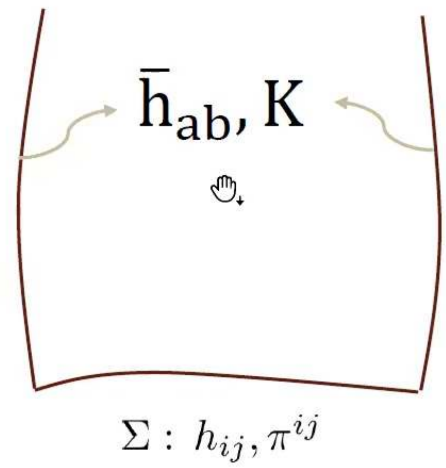
[Gibbons-Hawking]

➢ Reproduced by $\mathrm{T\overline{T}} + \Lambda_2$ deformation
[Batra-De Luca-Silverstein-Torroba-Yang]

Finite cutoff: which boundary conditions?

Conformal *(or York*)* *boundary conditions*

*York time, constant mean curvature slicing, and *cosmological action* [York '72][York '85]



Rest of the talk: work with **Edgar Shaghoulian** and **Sanjit Shashi**
[2409.07643][2503.17471]

Conformal boundary conditions

- Fixing conformal class of metric, or equivalently $\bar{h}_{ab} = h^{-1/d} h_{ab}$

  + trace of extrinsic curvature K ; "mixed" b.c.

- Better-posed initial boundary value problem? [An, Anderson] [Anninos, Galante, Maneerat]

  $\rightarrow$ Subject of current research [Liu, Reall, Santos, Wiseman]

- Extensive thermodynamic observables at high temperature: evidence for a local dual theory [Anninos, Galante, Maneerat]

Dirichlet: non-extensive behavior of observables at finite timelike boundaries → not a local QFT

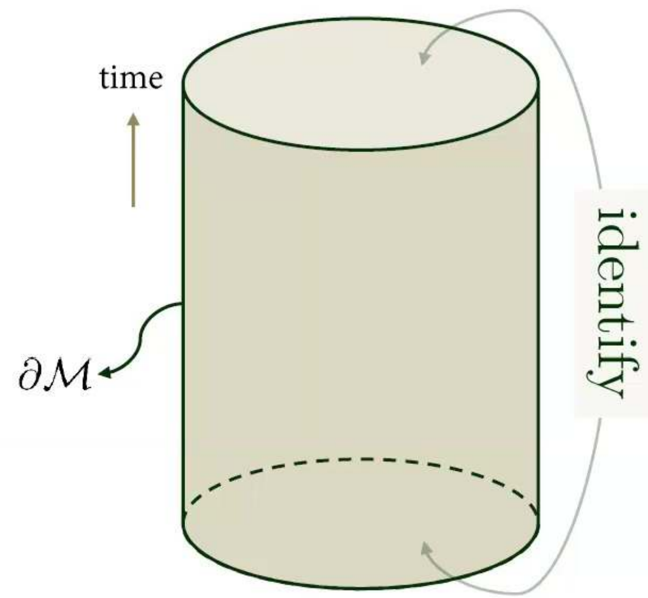
Conformal boundary conditions

- Fixing conformal class of metric, or equivalently $\overline{h}_{ab} = h^{-1/d} h_{ab}$

  + trace of extrinsic curvature K

- Euclidean action in pure gravity [York][Odak, Speziale]

$$I = -\frac{1}{16\pi G}\int_{\mathcal{M}} \sqrt{g}\, R - \frac{1}{8\pi G\, d}\int_{\partial\mathcal{M}} \sqrt{h}\, K$$



Thermal effective theory

- Effective action for a dimensionally reduced QFT over thermal circle at small β
  [Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom][Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma]

$$-\log Z = \int_{\Sigma} d^{d-1}x \sqrt{h_{\Sigma}} \left(-\frac{c_0}{\beta^{d-1}} + \frac{c_1}{\beta^{d-3}} R + \cdots \right)$$

- "Extensive": Free energy $F \sim -T \log Z$ scales with spatial volume and T^d

- Conjectured bound on c_1: positive for local QFTs
  motivated by results in free theories [Kutasov, Larsen][Melia, Pal][Benjamin, Lee, Ooguri, Simmons-Duffin]
  and CFTs dual to AdS gravity with asymptotic Dirichlet boundary [Allameh, Shaghoulian]

What is the fundamental theory?

- Insights from thermodynamics

- Compare thermodynamics of gravitational systems satisfying conformal boundary conditions with predictions from thermal effective field theory

- Explore behavior of subextensive corrections

Outline and summary

- Conformal b.c. on boundaries with topology $S^1 \times \Sigma^{d-1}$, $\Sigma = \{S, \mathbb{H}, \mathbb{T}\}$, $d > 2$
- Bulk "conformal" partition function = partition function of putative dual theory. Approximate partition function using Euclidean action of dominant bulk solution
- High temperature behavior of free energy and entropy
  Extensive; e.g., in 4-dimensional boundary: $F \sim \# V \tilde{T}^4 + \# \tilde{T}^2 R + \dots$
- Adding charge; Einstein-Maxwell theory
  solutions with cosmic-type horizon can accommodate negative K
- Cosmic solutions as dominant phase; e.g., AdS hyperbolic horizons

Conformal boundary conditions

- Spherical boundary $S^1 \times S^{d-1}$; $\quad ds^2 = \lambda^2(d\tilde{\tau}^2 + d\Omega_{d-1}^2)\,,\ \tilde{\tau} \sim \tilde{\tau} + \tilde{\beta}$

  instead of fixing the size of time circle and spatial sphere, **their ratio, $\tilde{\beta}$, is fixed**

- Ensemble data: conformal inverse temperature $\tilde{\beta}$ and constant K

- Partition function for the bulk canonical ensemble:

$$Z(\tilde{\beta}, K) \sim e^{-I_{\text{conf.}}}$$

Bulk solutions – Einstein gravity with zero Λ

- Boundary data

$$\tilde{\beta} = \frac{4\pi r_h}{d-2}\frac{\sqrt{f(r_c)}}{r_c}, \qquad K = \frac{1}{2r_c\sqrt{f(r_c)}}\left[2(d-1) - d\left(\frac{r_h}{r_c}\right)^{d-2}\right]$$

- Vacuum (flat spacetime)
- Two black holes at high enough conformal temperature
- Hawking-Page transition

High temperature behavior

- Expansion: $r_h = \frac{1}{\tilde{\beta}}\left(a_0 + a_1\tilde{\beta} + a_2\tilde{\beta}^2 + \cdots\right), \qquad r_c = \frac{1}{\tilde{\beta}}\left(w_0 + w_1\tilde{\beta} + w_2\tilde{\beta}^2 + \cdots\right)$
- On-shell action:

$$I = \mathrm{Vol}[S^{d-1}]\left[-\frac{(2\pi)^{d-1}}{4dGK^{d-1}}\frac{1}{\boxed{\tilde{\beta}^{d-1}}} - \frac{(2\pi)^{d-1}}{4GK^{d-1}}\frac{(d-1)(d-2)}{16\pi^2}\frac{1}{\boxed{\tilde{\beta}^{d-3}}} + \cdots\right]$$

Wilson coefficients: c_0 c_1

Matching thermal effective action

What is the matter with Dirichlet?

High temperature free energy (and entropy) not extensive

$$I \approx \mathrm{Vol}[S^{d-1}] \left[-\frac{(2\pi)^{d-1}}{4G} \left(\underbrace{\frac{1}{K(r_c,\beta)}}_{\text{ruins extensive scaling}} \right)^{d-1} \frac{r_c^{d-1}}{\beta^{d-1}} \right]$$

Turning on Λ and adding charge

- Partition function

$$Z(\tilde{\beta}, \tilde{\Phi}) = \mathrm{Tr} \exp\left[-\tilde{\beta}(\tilde{H} - \tilde{\Phi} Q)\right]$$

- Geometry: (A)dS-Reissner–Nordström

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_k^2 \quad , \quad f(r) = k - \frac{\mu}{r^{d-2}} + \frac{q^2}{r^{2(d-2)}} \pm \frac{r^2}{\ell^2}$$

$k = 1, 0, -1$ indicating horizon topology

Boundary data

$$\tilde{\beta} = \pm \frac{4\pi}{f'(r_h)} \frac{\sqrt{f(r_c)}}{r_c}, \qquad K = \pm \frac{d-1}{r_c \sqrt{f(r_c)}} \left[f(r_c) + \frac{r_c f'(r_c)}{2(d-1)} \right]$$

$$\tilde{\Phi} = \left(\frac{q}{r_h^{d-2}} - \frac{q}{r_c^{d-2}} \right) \frac{r_c}{\eta \sqrt{f(r_c)}}, \qquad \eta = \sqrt{\frac{2(d-2)}{d-1}}$$

Solve for r_h , r_c and q ; not doable analytically

Comparing to thermal effective action

$$-\log Z(\tilde{\beta}, \tilde{\Omega}, \tilde{\Phi}) = \int_\Sigma d^{d-1}x \sqrt{\tilde{h}_\Sigma} \left[-\frac{\boxed{c_0}}{\tilde{\beta}^{d-1}} + \frac{1}{\tilde{\beta}^{d-3}} \left(\boxed{c_1} \tilde{R} + c_2 \tilde{F}^2 + \boxed{c_3} \tilde{\Phi}^2 \right) + \cdots \right]$$

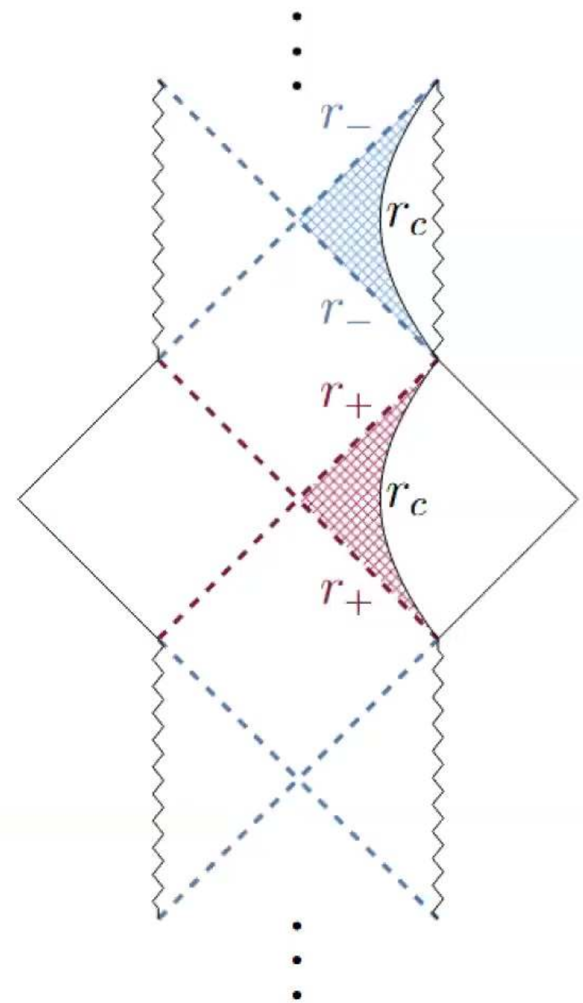
$$r_h = \frac{1}{\tilde{\beta}} \left(a_0 + a_1 \tilde{\beta} + a_2 \tilde{\beta}^2 + \cdots \right), \qquad r_c = \frac{1}{\tilde{\beta}} \left(w_0 + w_1 \tilde{\beta} + w_2 \tilde{\beta}^2 + \cdots \right)$$

Example 1— $\Lambda = 0\,, \widetilde{\Phi} \neq 0$

Reissner-Nordström geometry
with spherical horizons

"Cosmic horizon": $r_c < r < r_-$

Can accommodate negative K



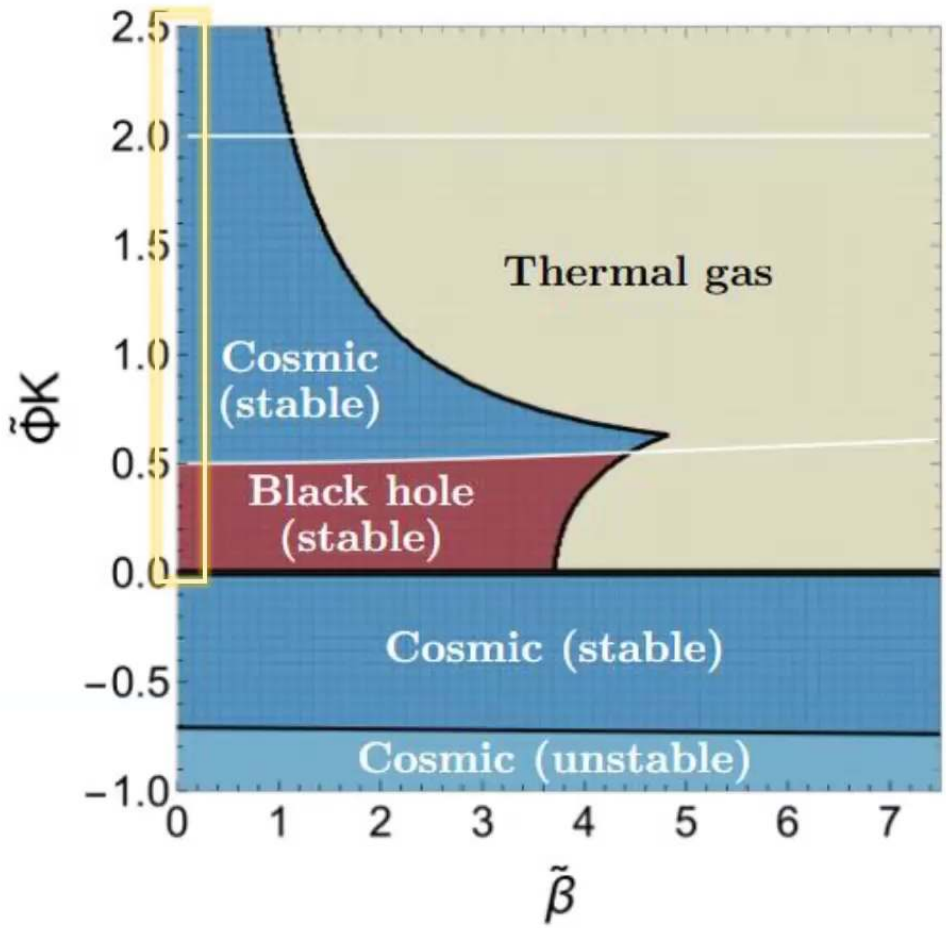
High temperature expansion

Extensive cosmic solution for

$$(\tilde{\Phi}K)^2 > \frac{1}{8}(d-1)(d-2)$$

Needed for consistency with EFT

Wilson coefficient: $c_3 = -\frac{(2\pi)^{d-3}d^2}{2GK^{d-1}}$

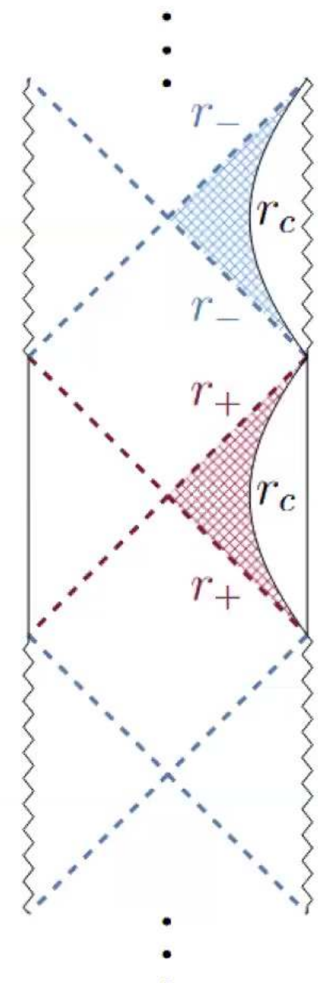


Example 3— $\Lambda < 0\,, \widetilde{\Phi} = 0$

Hyperbolic AdS black hole

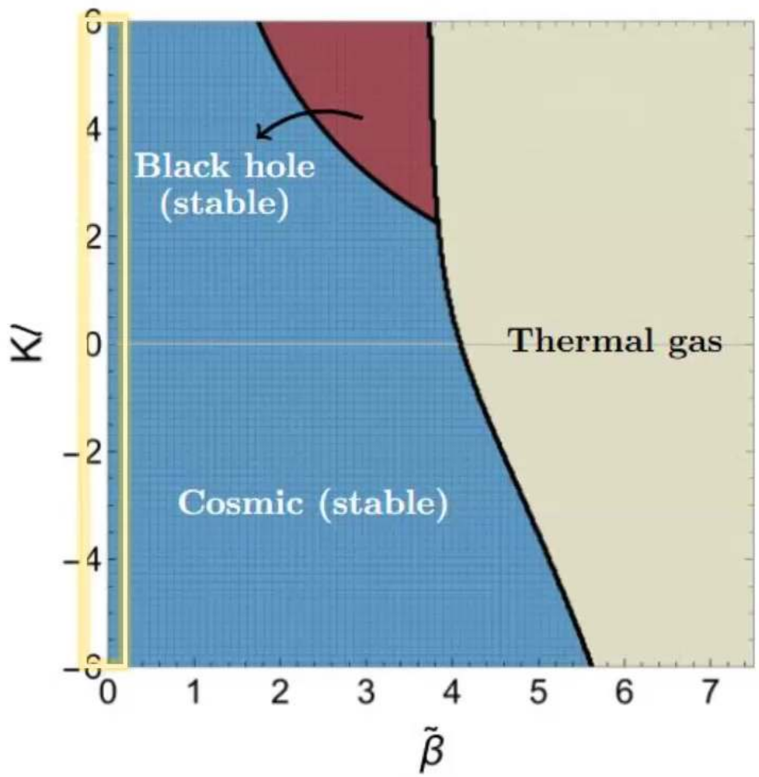
$$f(r) = -1 - \frac{\mu}{r^{d-2}} + \frac{r^2}{\ell^2}$$

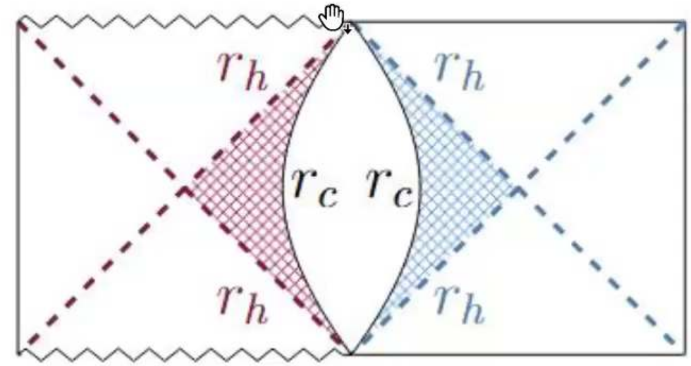
Cosmic solutions when $\mu < 0$



Example 4— $\Lambda > 0\,, \widetilde{\Phi} = 0$

Spherical horizons in Schwarzschild-dS





$$f(r) = 1 - \frac{\mu}{r^{d-2}} - \frac{r^2}{\ell^2}\,, \mu > 0$$

also found by [Anninos, Galante, Maneerat]

Recap

- Gravitational systems with conformal boundary conditions show agreement with structure of thermal effective theory
  possible because of solutions with various horizon topology

- Solutions with cosmic horizons are important

- Conjectured QFT bound $c_1 \geq 0$ is violated
  Gravity not fully decoupled..?
  coupling of QFT degrees of freedom with Weyl factor

$$c_1 = -\frac{(2\pi)^{d-3}}{16GK^{d-1}} < 0$$

- Understanding K in the putative boundary dual..?

Thank you!