Title: Machine Learning through Equilibrium Propagation: Extending the framework to quantum, thermal, and time dependent systems. - Quantum Foundations Seminar

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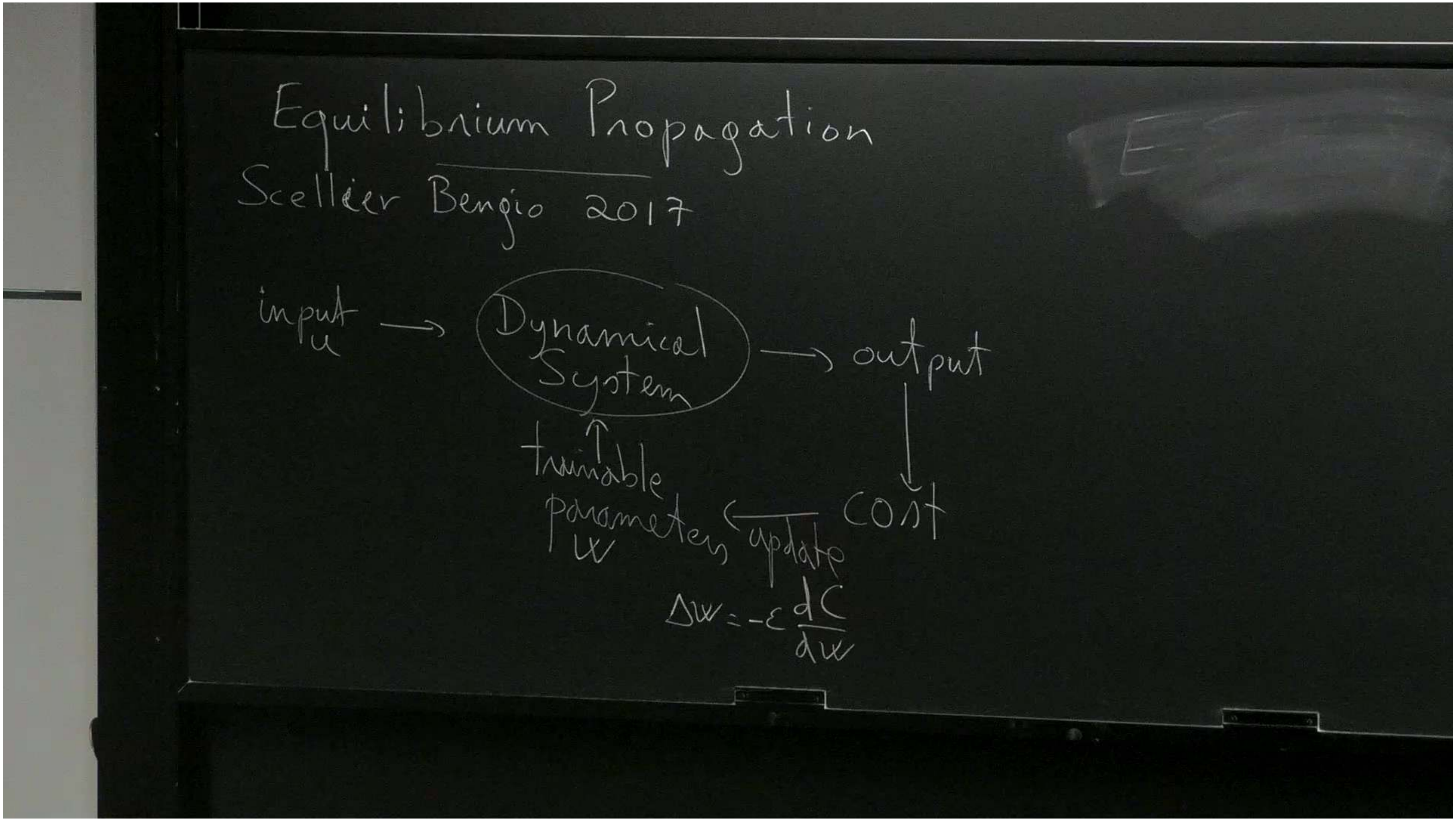
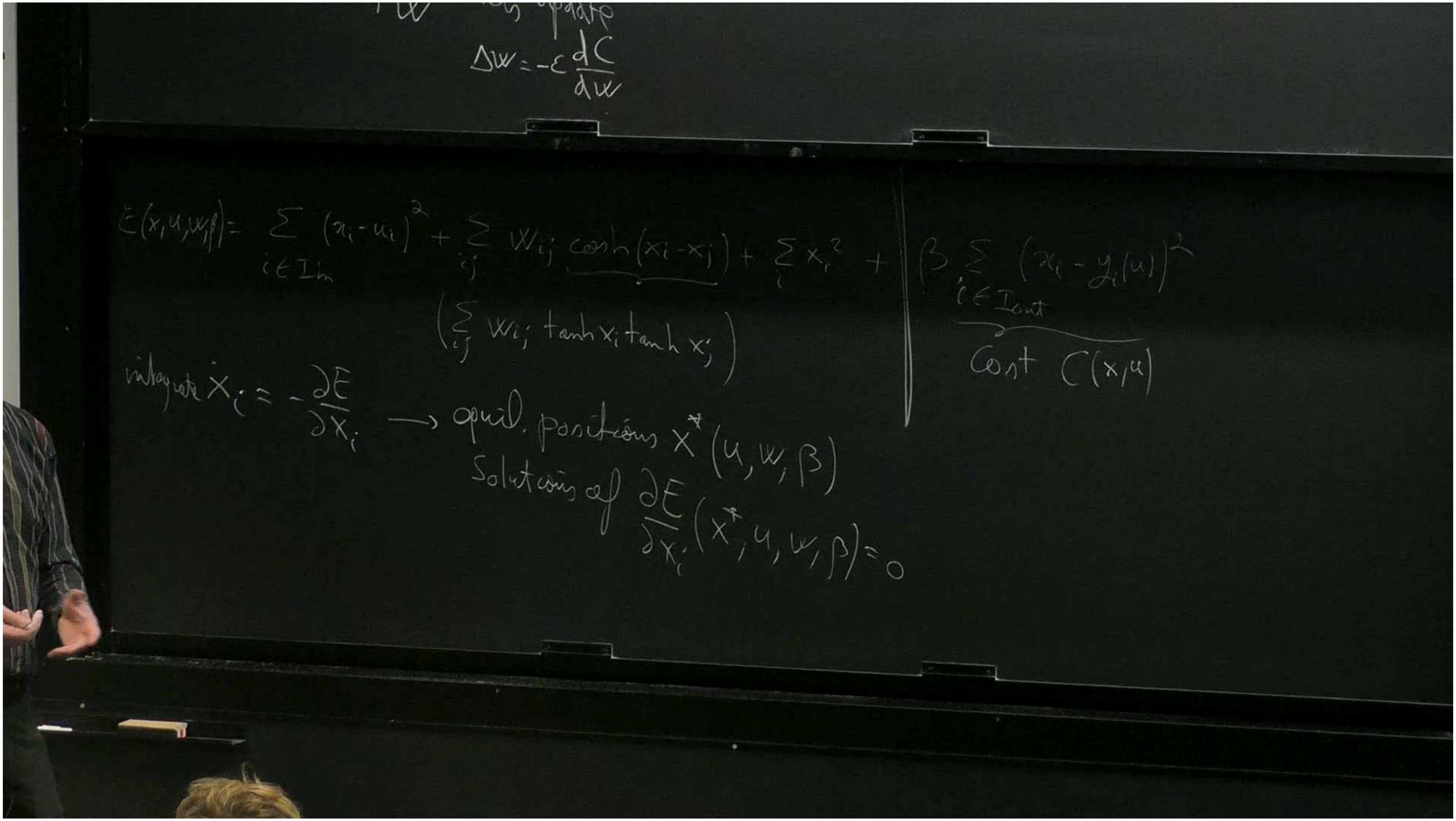
Abstract:

In 2017, Scellier and Bengio introduced the machine learning algorithm known as Equilibrium Propagation, in which the stationary state of a dynamical system is exploited for computation. A canonical example involves a network of nonlinear coupled elements -such as springs- described by a Hamiltonian H, where learning is carried out by tuning the parameters of H (for instance, the spring constants) so that its ground state implements the desired computation.

In this seminar, I will present the basic principles of the Equilibrium Propagation algorithm and discuss several recent extensions. These include the case of systems at finite temperature and of quantum systems, the extension to time-dependent systems governed by Lagrangian dynamics, and the case of nonreciprocal dynamical systems. These developments broaden the applicability of Equilibrium Propagation, making some new connections between machine learning, quantum mechanics, statistical physics, and dynamical systems theory.


Equilibrium Propagation
Scellier Bengio 2017
input
u
Dynamical System
output
trainable
parameters
w
update
cost
$\Delta w = -\varepsilon \frac{dC}{dw}$



update
$\Delta w = -\varepsilon \frac{dC}{dw}$
$E(x,u,w,\beta) = \sum_{i \in I_{in}} (x_i - u_i)^2 + \sum_{ij} w_{ij} \underbrace{\cosh(x_i - x_j)} + \sum_i x_i^2 + \beta \underbrace{\sum_{i \in I_{out}} (x_i - y_i(u))^2}$
$\left(\sum_{ij} w_{ij} \tanh x_i \tanh x_j \right)$
Cost $C(x,u)$
integrate $\dot{x}_i = -\frac{\partial E}{\partial x_i} \longrightarrow$ equil. positions $x^*(u,w,\beta)$
Solutions of $\frac{\partial E}{\partial x_i}(x^*,u,w,\beta) = 0$

$$\text{Cost} = C(x^*, u)$$

$$E^*(u, w, \beta) = E(x^*(u, w, \beta), u, w, \beta)$$

$$\frac{dE^*}{d\beta} = \underbrace{\cancel{\frac{\partial E}{\partial x_i}}}_{= 0} \frac{dx_i^*}{d\beta} + \frac{\partial E}{\partial \beta}^{*} = C^*(u, w, \beta)$$

$$\frac{dC^*}{dw_{ij}} = \frac{d^2 E^*}{dw_{ij}\, d\beta} = \frac{d}{d\beta}\left(\frac{dE^*}{dw_{ij}}\right) = \frac{d}{d\beta}\left(\underbrace{\cancel{\frac{\partial E^*}{\partial x_i}}}_{0} \frac{\partial x_i}{\partial w_{ij}} + \frac{\partial E}{\partial w_{ij}}\right) = \frac{d}{d\beta}\left(\underbrace{\frac{\partial E}{\partial w_{ij}}}_{\cosh(x_i^* - x_j^*)}\right)$$

$$\Delta w = -\varepsilon \frac{dC}{dw}$$

Quantum E.P.

eigenstates of H: $$\frac{\delta}{\delta|\psi\rangle} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = 0 \iff H|\psi\rangle = E|\psi\rangle$$

$$\hat{Z}_i \cdot y_i(\psi)$$

$$H = \sum_{i \in I_{in}} u_i \hat{Z}_i + \sum_{ij} W_{ij} X_i X_j + \sum_{ij} W'_{ij} Z_i Z_j + \beta \sum_{i \in I_{out}} \left(\hat{Z}_i - y_i(\psi)\right)^2$$

Pauli Operators

$|\Psi_0\rangle$: ground state

$$\text{Cost} = \langle \Psi_0 | \hat{C} | \Psi_0 \rangle$$

$$\Delta W_{ij} = -\varepsilon \frac{1}{2\beta} \left(\langle \Psi^0_{+\beta} | x_i x_j | \Psi^0_{+\beta} \rangle - \langle \Psi^0_{-\beta} | x_i x_j | \Psi^0_{-\beta} \rangle \right)$$

Thermal

$$\rho = \frac{1}{Z}\,\mathrm{Tr}\, e^{-H/T} \;\longrightarrow\; \text{minimizes the free energy}$$

$$\langle C \rangle_T = \frac{d}{d\beta} \ln Z$$

$$\Delta W_{ij} = -\varepsilon \, \frac{1}{2\beta} \left(\left\langle \frac{\partial H}{\partial W_{ij}} \right\rangle_{T+\beta} - \left\langle \frac{\partial H}{\partial W_{ij}} \right\rangle_{T-\beta} \right) = -\varepsilon \left(\left\langle \frac{\partial H}{\partial W_{ij}} \, C \right\rangle_T - \left\langle \frac{\partial H}{\partial W_{ij}} \right\rangle_T \langle C \rangle_T \right)$$

$\Delta w = -\varepsilon \frac{\quad}{dw}$

Lagrangian Dynamics

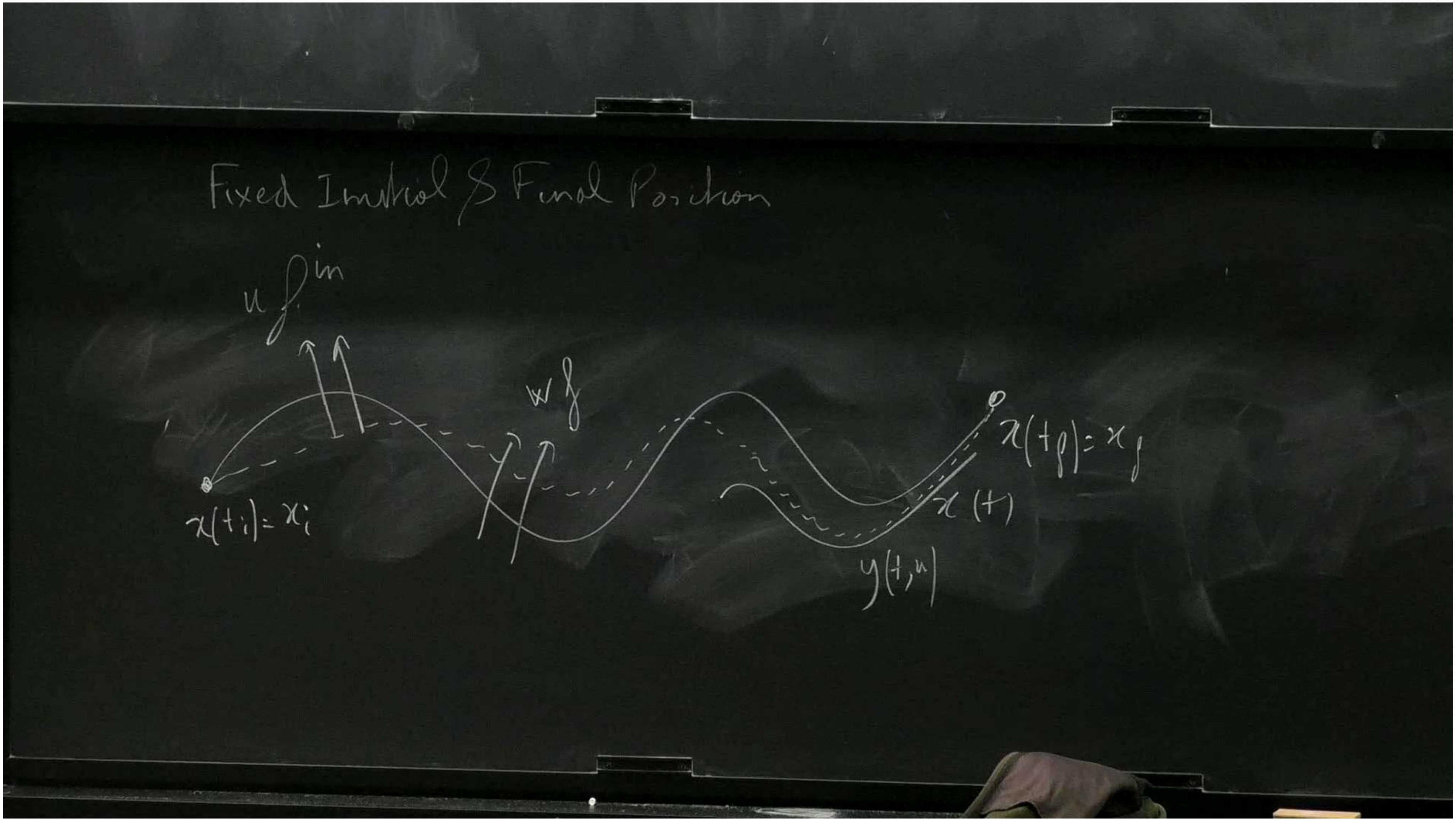
$S = \int dt\, L(x, x, t)$

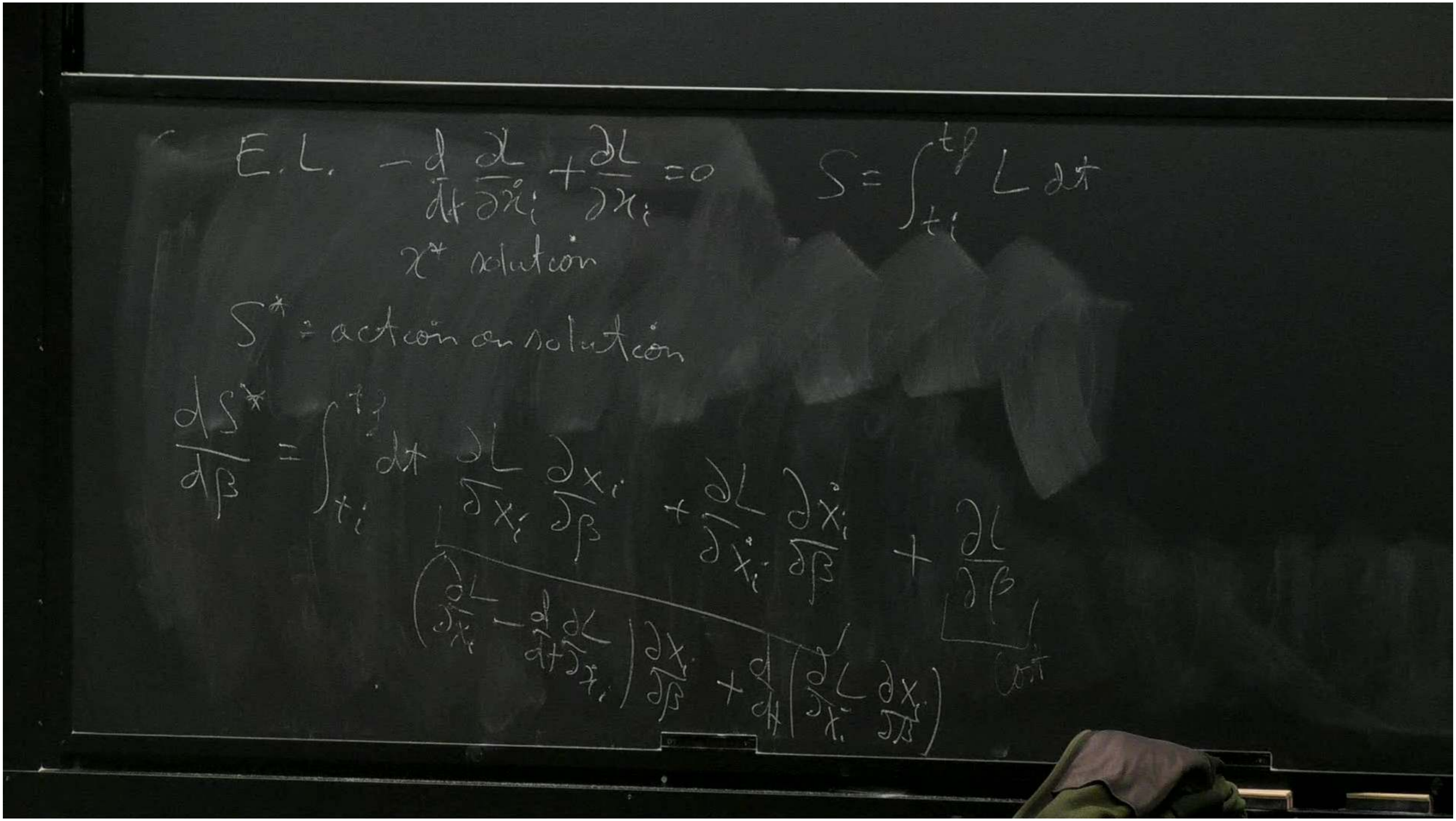
$\frac{\delta S}{\delta x} = 0 \longrightarrow$ E.L. eqts

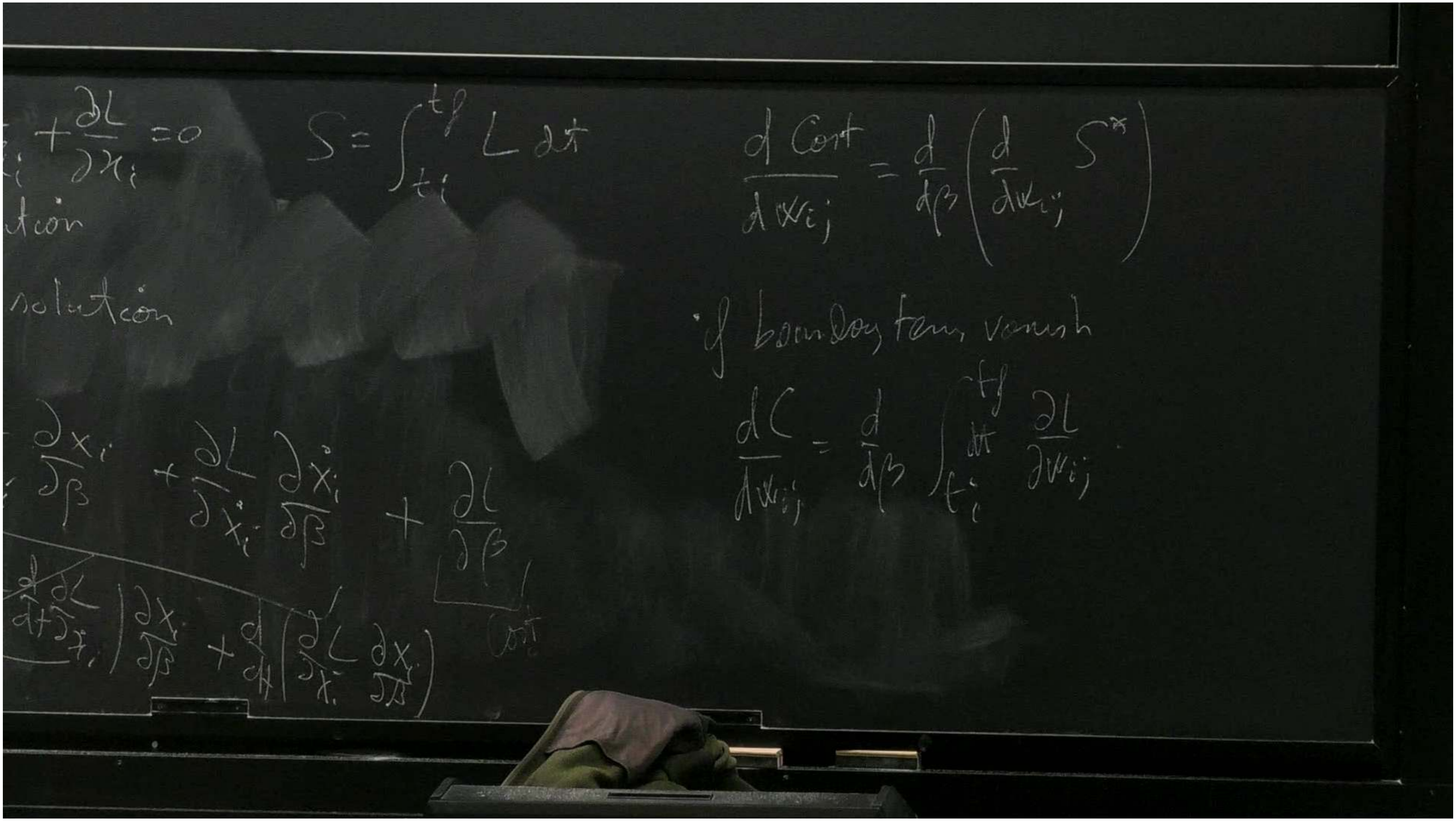
$x \in \mathbb{R}^n$

$L = \frac{1}{2} m \dot{x}^2 - V(x,t) - \sum_i u_i V_i^{in}(x,t) - \sum_i \psi_i V_i(x,t) + \beta C(x,u,t)$

$\text{Cost} = \int dt\, \mu(t) \left(x - y(t,u)\right)^2$


Fixed Initial & Final Position
u f^in
w f
x(t_i) = x_i
x(t_f) = x_f
x(t)
y(t,u)


E.L. $-\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} + \frac{\partial L}{\partial x_i} = 0$
$S = \int_{t_i}^{t_f} L \, dt$
x^* solution
S^* = action on solution
$\frac{dS^*}{d\beta} = \int_{t_i}^{t_f} dt \, \frac{\partial L}{\partial x_i}\frac{\partial x_i}{\partial \beta} + \frac{\partial L}{\partial \dot{x}_i}\frac{\partial \dot{x}_i}{\partial \beta} + \boxed{\frac{\partial L}{\partial \beta}}$
$\left(\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}\right)\frac{\partial x_i}{\partial \beta} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\frac{\partial x_i}{\partial \beta}\right)$
const


$S = \int_{t_i}^{t_f} L \, dt$
$\frac{d \, \mathrm{Cost}}{d w_{ij}} = \frac{d}{d\beta}\left(\frac{d}{d w_{ij}} S^*\right)$
if boundary terms vanish
$\frac{dC}{d w_{ij}} = \frac{d}{d\beta} \int_{t_i}^{t_f} dt \, \frac{\partial L}{\partial w_{ij}}$
solution
$\frac{\partial L}{\partial \dot{x}_i} \frac{\partial \dot{x}_i}{\partial \beta} + \frac{\partial L}{\partial \beta}$
Cost