Title: Superposing coherent states for fun & profit

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Abstract:

Coherent states conveniently represent the classically meaningful wavelike states of light, as opposed to the corpuscular number states, but superposing coherent states is mind-bogglingly unclassical, including as examples "cat states", "comb states" and "compass states". I present a history of superposing coherent states, especially for oscillators (including electromagnetic field states) and for spin, and extend to entangled coherent states. I then show that superposed coherent states are a viable path for photonic quantum computing and for reaching towards the ultimate limits of sensing. Finally, I discuss our experimental realisations (at University of New South Wales) of superposed coherent states in a spin-7/2 Antimony nucleus in a silicon substrate.

Superposing coherent states for fun and profit

DECEMBER 3

Barry C. Sanders UNIVERSITY OF CALGARY

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Robust macroscopic Schrödinger's cat on a nucleus

Pragati Gupta, Arjen Vaartjes, Xi Yu, Andrea Morello, and Barry C. Sanders

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Schrödinger cat states of a nuclear spin qudit in silicon

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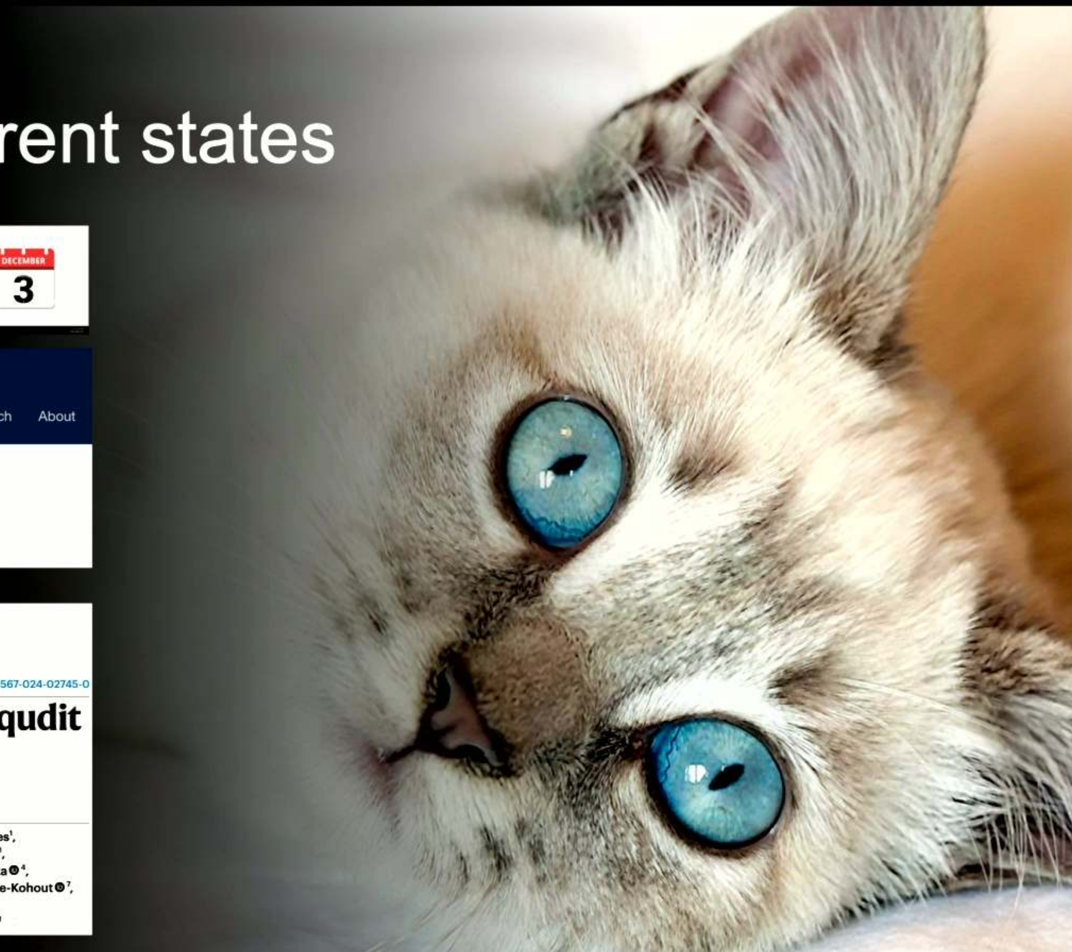

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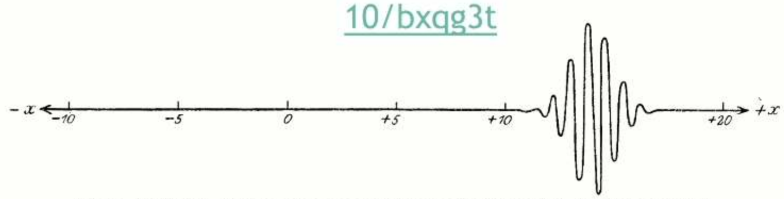

Fig. 2. Pendelnde Wellengruppe als undulationsmechanisches Bild des Massenpunktes.

VOLUME 10, NUMBER 3 PHYSICAL REVIEW LETTERS 1 FEBRUARY 1963

PHOTON CORRELATIONS*

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(Received 27 December 1962)

In 1956 Hanbury Brown and Twiss[1] reported that the photons of a light beam of narrow spectral width have a tendency to arrive in correlated pairs. We have developed general quantum mechanical methods for the investigation of such a method of describing the photon field which appears particularly well suited to the discussion of experiments performed with light beams, whether coherent or incoherent.

The correlations observed in the photoioniza-

together different set of basis states. We take these to be of the form

$$|\alpha_k\rangle = \exp(-\tfrac{1}{2}|\alpha_k|^2)\sum_n[\alpha_k{}^n/(n!)^{1/2}]|n\rangle, \qquad (1)$$

where α_k is an arbitrary complex amplitude. We shall call the $|\alpha_k\rangle$ coherent states; their use is well known in discussions of the harmonic oscillator in the classical limit. The expectation

10/c44wx8

EVEN AND ODD COHERENT STATES AND EXCITATIONS OF A SINGULAR OSCILLATOR

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Physica **72** (1974) 597–618

Synopsis

We introduce even and odd coherent states. The Green function, the transition amplitudes between the energy levels of a singular nonstationary oscillator in the case of constant frequency in the remote past and future and generating functions for these amplitudes are obtained, by a method similar to the usual coherent-states method. The transition amplitudes are expressed in terms of Jacobi polynomials. Various limit cases are considered of the exact formulas obtained. The results obtained for the one-dimensional problem are generalized to the N-dimensional case, and from there to the case of a charge moving in a singular electromagnetic field. The group-theoretical aspect of the problem is discussed and the groups $U(1,1)$, and $U(N,1)$ are shown to be the dynamical groups of the one-dimensional and N-dimensional cases, respectively.

One can construct two operators, each generating irreducible representations of the group consisting of two elements, the unit operator and the inversion operator $\hat{I}$. These are, first, the operator

$$\mathrm{ch}\,(\alpha\hat{a}^+ - \alpha^*\hat{a}) = \tfrac{1}{2}\,[\hat{\mathscr{D}}(\alpha) + \hat{\mathscr{D}}(-\alpha)] = \hat{\mathscr{D}}_+(\alpha), \tag{2.9}$$

generating the symmetric representation, and, second, the operator

$$\mathrm{sh}\,(\alpha\hat{a}^+ - \alpha^*\hat{a}) = \tfrac{1}{2}\,[\hat{\mathscr{D}}(\alpha) - \hat{\mathscr{D}}(-\alpha)] = \hat{\mathscr{D}}_-(\alpha), \tag{2.10}$$

generating the antisymmetric representation of this group.

It is obvious that the functions

$$|\alpha_+\rangle = \hat{\mathscr{D}}_+(\alpha)\,|0\rangle; \qquad |\alpha_-\rangle = \hat{\mathscr{D}}_-(\alpha)\,|0\rangle, \tag{2.11}$$

are even and odd, respectively, and the parity of these functions with respect to α is the same as with respect to x. So we call them even and odd coherent states.

PHYSICAL REVIEW A VOLUME 33, NUMBER 1 JANUARY 1986

Quantum and classical Liouville dynamics of the anharmonic oscillator

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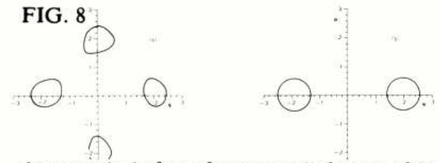
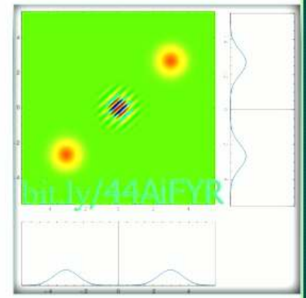
(Received 29 April 1985)

FIG. 8



al structure in the form of separate peaks form on a fairly flat background. These peaks smear, flatten, and reemerge as the cycle proceeds, eventually yielding two identical Gaussian peaks on opposite sides of the origin. The



VOLUME 57, NUMBER 1 PHYSICAL REVIEW LETTERS 7 JULY 1986

Generating Quantum Mechanical Superpositions of Macroscopically Distinguishable States via Amplitude Dispersion

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(Received 2 April 1986)

It is pointed out here that a coherent state propagating through an amplitude-dispersive medium will, under suitable conditions, evolve into a quantum superposition of two coherent states 180° out of phase with each other. The response of a homodyne detector to this superposition of macroscopically distinguishable states is calculated. Signatures which an experimentalist might look for in the homodyne detector's output in order to verify the generation of such states are described.

A "Schrödinger Cat" Superposition State of an Atom

C. Monroe,* D. M. Meekhof, B. E. King, D. J. Wineland

A "Schrödinger cat"–like state of matter was generated at the single atom level. A trapped $^9Be^+$ ion was laser-cooled to the zero-point energy and then prepared in a superposition of spatially separated coherent harmonic oscillator states. This state was created by application of a sequence of laser pulses, which entangles internal (electronic) and external (motional) states of the ion. The Schrödinger cat superposition was verified by detection of the quantum mechanical interference between the localized wave packets. This mesoscopic system may provide insight into the fuzzy boundary between the classical and quantum worlds by allowing controlled studies of quantum measurement and quantum decoherence.

Quantum mechanics allows the preparation of physical systems in superposition states, or states that are "smeared" between two or more distinct values. This curious principle of quantum mechanics (*1*) has been extremely successful at describing physical behavior in the microscopic world—from interactions of atoms with photons to interactions at the subnuclear level. But what happens when we extend the quantum superposition principle to macroscopic systems conventionally described by classical physics? Here, superpositions introduce a great amount of conceptual difficulty, as pointed out in 1935 by the celebrated Einstein-Podolsky-Rosen (*2*) and Schrödinger cat (*3*) paradoxes. For example, in Schrödinger's thought experiment (*3*), an unfortunate cat is placed in a quantum superposition of being dead and alive (correlated with a single radioactive atom that has and has not decayed). The state of the system can be represented by the entangled quantum mechanical wave function,

$$\psi = \frac{|\text{☺}\rangle|\uparrow\rangle + |\text{☹}\rangle|\downarrow\rangle}{\sqrt{2}} \qquad (1)$$

where $|\text{☺}\rangle$ and $|\text{☹}\rangle$ refer to the states of a live and dead cat, and $|\downarrow\rangle$ and $|\uparrow\rangle$ refer to the internal states of an atom that has and has not radioactively decayed. This situation defies our sense of reality because we only observe live or dead cats, and we expect that cats are either alive or dead independent of our observation (*4*). Schrödinger's cat paradox is a classic illustration of the conflict between the existence of quantum superpositions and our real-world experience of observation and measurement.

Although superposition states such as Schrödinger's cat do not appear in the macroscopic world, there is great interest in the realization of "Schrödinger cat"–like states in mesoscopic systems, or systems that have both macroscopic and microscopic features. In this context, the "cat" is generalized to

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Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement

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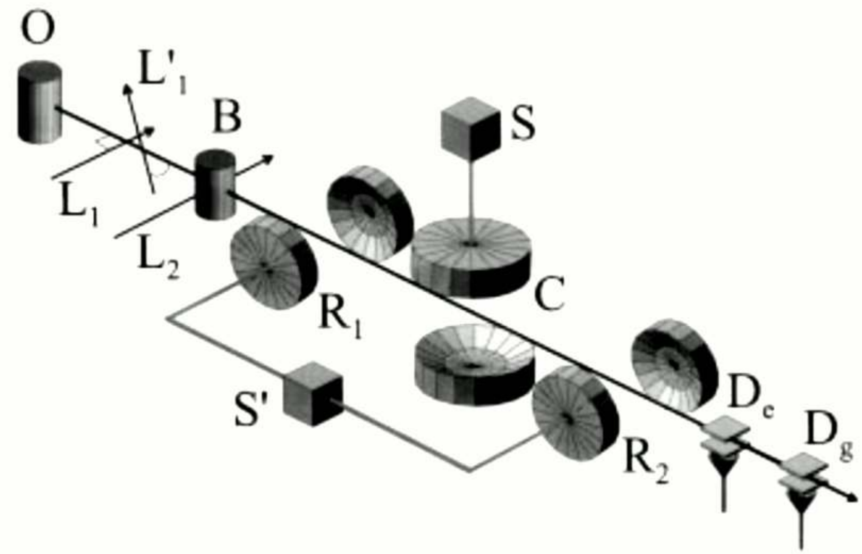
(Received 10 September 1996)

A mesoscopic superposition of quantum states involving radiation fields with classically distinct phases was created and its progressive decoherence observed. The experiment involved Rydberg atoms interacting one at a time with a few photon coherent field trapped in a high Q microwave cavity. The mesoscopic superposition was the equivalent of an "atom + measuring apparatus" system in which the "meter" was pointing simultaneously towards two different directions — a "Schrödinger cat." The decoherence phenomenon transforming this superposition into a statistical mixture was observed while it unfolded, providing a direct insight into a process at the heart of quantum measurement. [S0031-9007(96)01848-0]



Generating Optical Schrödinger Kittens for Quantum Information Processing

Alexei Ourjoumtsev, Rosa Tualle-Brouri, Julien Laurat, Philippe Grangier*

We present a detailed experimental analysis of a free-propagating light pulse prepared in a "Schrödinger kitten" state, which is defined as a quantum superposition of "classical" coherent states with small amplitudes. This kitten state is generated by subtracting one photon from a squeezed vacuum beam, and it clearly presents a negative Wigner function. The predicted influence of the experimental parameters is in excellent agreement with the experimental results. The amplitude of the coherent states can be amplified to transform our "Schrödinger kittens" into bigger Schrödinger cats, providing an essential tool for quantum information processing.

A key requirement for many quantum computation and communication protocols (*1*, *2*) is to use specific quantum states of light as a resource for information processing. We are interested in quantum states of propagating light beams, which can be analyzed either by photon counting or by homodyne detection, which measures the interference between the signal state and an intense reference beam with a relative phase θ. Homodyne detection measures a physical quantity called a "quadrature component" of the electric field, associated with the operator $\hat{x}_\theta = \hat{x}\cos\theta + \hat{p}\sin\theta$, where $\hat{x}$ and $\hat{p}$ are canonically conjugate field observables. The operators $\hat{x}$ and $\hat{p}$ are analogous to the position and the momentum of a particle, and they are often called quantum continuous variables (QCVs). From Heisenberg's inequalities, they cannot be determined simultaneously with an infinite precision, so one cannot generally define a proper phase-space density $\Pi(x, p)$ for the electric field. However, one can define a quasi-distribution $W(x, p)$ called the Wigner function, the marginals of which yield the probability distributions $P(x_\theta)$. By measuring the distributions $P(x_\theta)$ for several values of θ, one can reconstruct the Wigner function; this inverse process is known as quantum tomography (*3*).

For specific quantum states, the Wigner function can take negative values, thereby excluding any description by a classical phase-space den-

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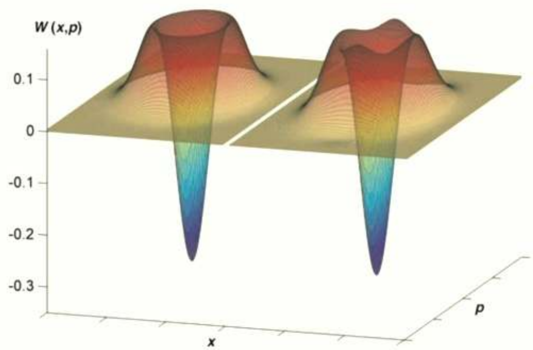

Fig. 1. Wigner function of an ideal photon-subtracted squeezed state (s = 0.6) (**left**), compared with the one of the closest Schrödinger kitten ($|\alpha_{opt}|^2$ = 0.8) (**right**). For any value of $|\alpha|^2 < 1$, the theoretical fidelity between the two matched states is larger than 0.997.

Reconstruction of non-classical cavity field states with snapshots of their decoherence

Samuel Deléglise, Igor Dotsenko, Clément Sayrin, Julien Bernu, Michel Brune, Jean-Michel Raimond & Serge Haroche

Nature 455, 510–514 (2008) | Cite this article

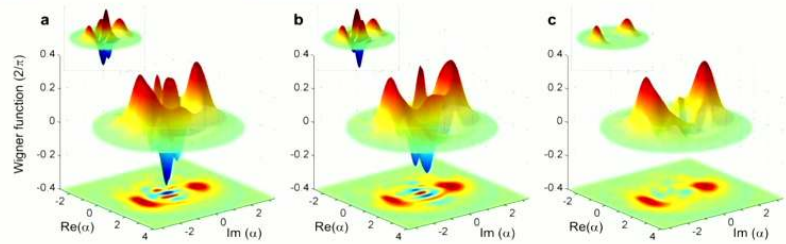

Figure 3 | Reconstructing Schrödinger cat states. The WFs of even (**a**) and odd (**b**) SC states (in units of $2/\pi$) with n_m=3.5 and χ=0.37π are reconstructed, following state preparation. The same detuning ($\delta/2\pi$ = 51 kHz) and interferometer phase ($\phi = -\Phi(0,\delta)+\pi$) are used for state preparation and reconstruction. The

the investigation of the quantum–classical boundary[5]. Although trapped-atom state reconstructions[6,7,8] have been achieved, it is challenging to perform similar experiments with trapped photons because cavities that can store light for very long times are required. Here we report the complete reconstruction and pictorial representation of a variety of radiation states trapped in a cavity in which several photons survive long enough to be repeatedly measured. Atoms crossing the cavity one by one are used to extract information about the field. We obtain images of coherent states[9], Fock states with a definite photon number and 'Schrödinger cat' states (superpositions of coherent states with different phases[10]). These states are equivalently represented by their density matrices or Wigner functions[11]. Quasi-classical coherent states have a Gaussian-shaped Wigner function, whereas the Wigner functions of Fock and Schrödinger cat states show oscillations and negativities revealing quantum interferences. Cavity damping induces decoherence that quickly washes out such

Sub-Planck structure in phase space and its relevance for quantum decoherence

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Heisenberg's principle[1] states that the product of uncertainties of position and momentum should be no less than the limit set by Planck's constant, $\hbar/2$. This is usually taken to imply that phase space structures associated with sub-Planck scales ($\ll \hbar$) do not exist, or at least that they do not matter. Here I show that this common assumption is false: non-local quantum superpositions (or 'Schrödinger's cat' states) that are confined to a phase space volume characterized by the classical action A, much larger than $\hbar$, develop spotty structure on the sub-Planck scale, $a = \hbar^2/A$.

NATURE | VOL 412 | 16 AUGUST 2001

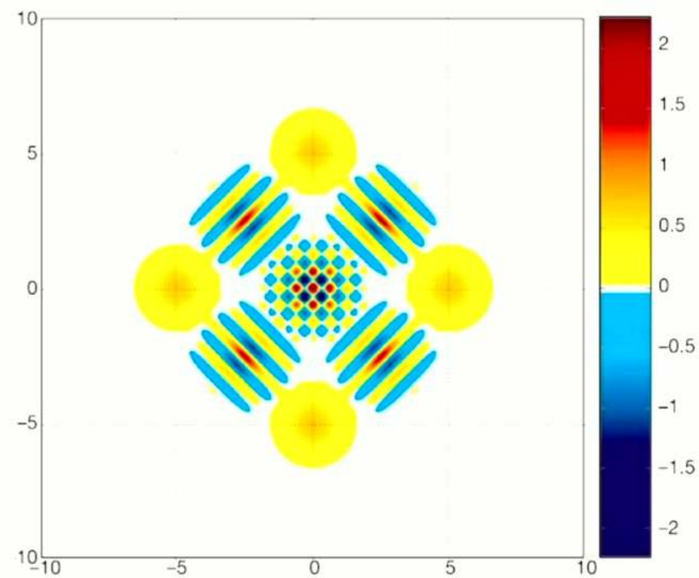

Figure 2 The compass state, equation (9). This is a superposition of minimum-uncertainty states ($|N\rangle, |W\rangle, |S\rangle, |E\rangle$) placed 'north',..., 'east', of the common centre. Units are selected so that the gaussians used are round and $\hbar = 1$. The form of the central interference pattern in W_{NWSE} can be inferred from the familiar structure of the super-

Deterministically Encoding Quantum Information Using 100-Photon Schrödinger Cat States

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In contrast to a single quantum bit, an oscillator can store multiple excitations and coherences provided one has the ability to generate and manipulate complex multiphoton states. We demonstrate multiphoton control by using a superconducting transmon qubit coupled to a waveguide cavity resonator with a highly ideal off-resonant coupling. This dispersive interaction is much greater than decoherence rates and higher-order nonlinearities to allow simultaneous manipulation of hundreds of photons. With a tool set of conditional qubit-photon logic, we mapped an arbitrary qubit state to a superposition of coherent states, known as a "cat state." We created cat states as large as 111 photons and extended this protocol to create superpositions of up to four coherent states. This control creates a powerful interface between discrete and continuous variable quantum computation and could enable applications in metrology and quantum information processing.

SCIENCE VOL 342 1 NOVEMBER 2013

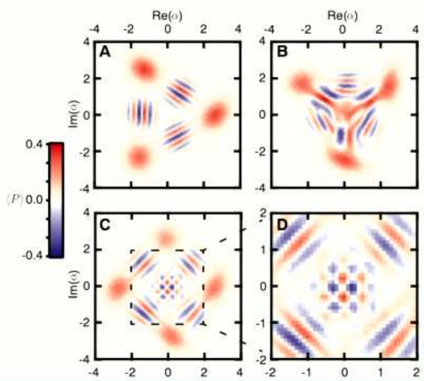

Fig. 4. Multicomponent cat states. By using conditional cavity phase shifts $C_{2\pi/3}$ and $C_{\pi/2}$, we created superpositions of three and four coherent states. Shown here is Wigner tomography of cavity states: (**A**) $|\beta\rangle + e^{i\lambda_1}|\beta e^{i2\pi/3}\rangle + e^{i\lambda_2}|\beta e^{i4\pi/3}\rangle$, where $|\beta| = \sqrt{7}$, $\lambda_1 = 0.6\pi$, and $\lambda_2 = -0.3\pi$; (**B**) $|0\rangle + e^{i\mu_1}|-i\beta\rangle + e^{i\mu_2}|\beta e^{i\pi/3}\rangle + e^{i\mu_3}|\beta e^{i2\pi/3}\rangle$, where $|\beta| = \sqrt{7}$, $\mu_1 = 0.5\pi$, $\mu_2 = -0.4\pi$, and $\mu_3 = -0.2\pi$; and (**C**) $|\beta\rangle + e^{i\nu_1}|i\beta\rangle + |-\beta\rangle + e^{i\nu_2}|-i\beta\rangle$, where $|\beta| = \sqrt{7}$, $\nu_1 = \pi$, and $\nu_2 = -0.2\pi$. (**D**) A closer inspection of the quantum interference in (C) reveals increased sensitivity to cavity displacements in both quadratures simultaneously.

A Schrödinger cat living in two boxes

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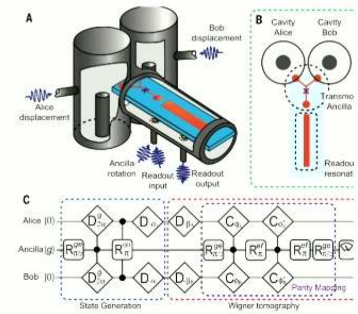
Quantum superpositions of distinct coherent states in a single-mode harmonic oscillator, known as "cat states," have been an elegant demonstration of Schrödinger's famous cat paradox. Here, we realize a two-mode cat state of electromagnetic fields in two microwave cavities bridged by a superconducting artificial atom, which can also be viewed as an entangled pair of single-cavity cat states. We present full quantum state tomography of this complex cat state over a Hilbert space exceeding 100 dimensions via quantum nondemolition measurements of the joint photon number parity. The ability to manipulate such multicavity quantum states paves the way for logical operations between redundantly encoded qubits for fault-tolerant quantum computation and communication.

(i.e., $|\alpha|^2 \gtrsim 2$), $|\psi_\pm\rangle$ can be considered a single cat state living in two boxes, whose superposed components are coherent states in a linearly hybridized mode of Alice and Bob (*14*). Alternatively, in the more natural eigenmode basis, $|\psi_\pm\rangle$ has been known as the entangled coherent states in theoretical studies (*15*) and may also be understood as two single-cavity cat states that are en-



. Cartoon schematic of device architecture and experimental protocol. (A) A 3D ... consisting of two coaxial cavities (Alice and Bob), a Y-shaped transmon with a single ... (marked by ×), and a stripline readout resonator. All components are housed inside a ... high-purity aluminum, with artificial windows drawn for illustration purposes. (B) A top

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IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

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REVIEW

Review of entangled coherent states

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Abstract
We review entangled coherent state research since its first implicit use in 1967 to the present. Entangled coherent states are important to quantum superselection principles, quantum information processing, quantum optics and mathematical physics. Despite their inherent fragility, entangled coherent states have been produced in a conditional propagating-wave quantum optics realization. Fundamentally the states are intriguing because they entangle the coherent states, which are in a sense the most classical of all states of a dynamical system.

PHYSICAL REVIEW VOLUME 155, NUMBER 5 25 MARCH 1967

Charge Superselection Rule*

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(Received 25 August 1966)

Questions concerning superselection rules are considered. Two experiments are discussed. In the first, coherent superpositions of different angular-momentum states are constructed. In the second, coherent superpositions of states with different charge are constructed in complete analogy with the angular-momentum case. We suggest that, contrary to a widespread belief, interference may be possible between states with different charges.

A selection rule for a quantity A asserts the conservation of A. Selection rules exist for energy, momentum, angular momentum, electric charge, lepton number, and baryon number, and the number of fermions modulo 2. According to Wick, Wightman, and Wigner, some of these selection rules can be elevated to the status of superselection rules. A superselection rule asserts the impossibility of preparing a coherent superposition of two eigenstates of A with different eigenvalues. More generally, no physically realizable density matrix contains off-diagonal elements connecting states of different A.

In the above experiment, the two states $|Q\theta\rangle$ and $|Q'\theta'\rangle$ for C_1 and C_2 are, of course, not eigenstates of charge. Actually they are minimum uncertainty packets in charge and phase. Hence we have not yet been able to set up the required relative reference frames with a system of definite charge. However, if we superpose states $|Q\theta_1\rangle$ and $|Q'\theta_2\rangle$, keeping the phase difference $(\theta_1-\theta_2)$ fixed at $(\theta-\theta')$ according to

$$|i\rangle=\int |Q\theta_1\rangle|Q'\theta_2\rangle\delta(\theta_1-\theta_2-\theta+\theta')e^{i(Q+Q')\theta_1}, \quad (10)$$

then we obtain a state of definite charge $Q+Q'$. Furthermore, all probabilities for proton or neutron states are precisely the same as for the initial state $|Q\theta\rangle\ |Q'\theta'\rangle$. Hence the apparatus described by Eq. (10) constructs a state which, when referred to the phase frame of C_2, is a coherent superposition of proton and neutron with phase $(\theta-\theta')$.

Bosonic quantum codes for amplitude damping

PDF

Isaac L. Chuang[1,2], Debbie W. Leung[1], and Yoshihisa Yamamoto[1]

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Phys. Rev. A **56**, 1114 – **Published 1 August, 1997**

DOI: https://doi.org/10.1103/PhysRevA.56.1114

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Abstract

Traditional quantum error correction involves the redundant encoding of k quantum bits using n quantum bits to allow the detection and correction of any t bit error. The smallest general $t = 1$ code requires $n = 5$ for $k = 1$. However, the dominant error process in a physical system is often well known, thus inviting the following question: Given a specific error model, can more efficient codes be devised? We demonstrate alternative codes that correct just amplitude damping errors that allow, for example, a $t = 1$, $k = 1$ code using effectively $n = 4.6$. Our scheme is based on using bosonic states of photons in a finite number of optical modes. We present necessary and sufficient conditions for the codes and describe construction algorithms, physical implementation, and performance bounds.

PHYSICAL REVIEW A VOLUME 59, NUMBER 4 APRIL 1999

Macroscopically distinct quantum-superposition states as a bosonic code for amplitude damping

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Department of Physics, The University of Queensland, Queensland 4072, Australia
(Received 14 September 1998)

We show how macroscopically distinct quantum superposition states (Schrödinger cat states) may be used as logical qubit encodings for the correction of spontaneous emission errors. Spontaneous emission causes a bit flip error, which is easily corrected by a standard error correction circuit. The method works arbitrarily well as the distance between the amplitudes of the superposed coherent states increases. [S1050-2947(99)06503-8]

Cat codes

PHYSICAL REVIEW A, VOLUME 62, 052108

Entangled coherent-state qubits in an ion trap

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(Received 13 October 1999; published 16 October 2000)

We show how entangled qubits can be encoded as entangled coherent states of two-dimensional center-of-mass vibrational motion for two ions in an ion trap. The entangled qubit state is equivalent to the canonical Bell state, and we introduce a proposal for entanglement transfer from the two vibrational modes to the electronic states of the two ions in order for the Bell state to be detected by resonance fluorescence shelving methods.

PHYSICAL REVIEW A, VOLUME 64, 012310

Encoding a qubit in an oscillator

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(Received 9 August 2000; published 11 June 2001)

Quantum error-correcting codes are constructed that embed a finite-dimensional code space in the infinite-dimensional Hilbert space of a system described by continuous quantum variables. These codes exploit the noncommutative geometry of phase space to protect against errors that shift the values of the canonical variables q and p. In the setting of quantum optics, fault-tolerant universal quantum computation can be

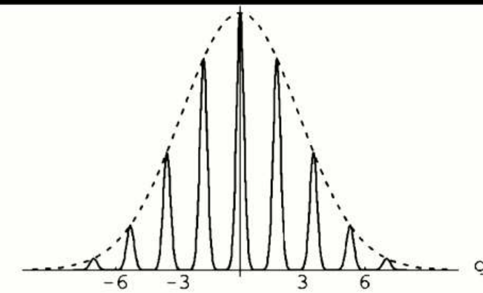

FIG. 2. Probability distribution in position space $P(q) = \frac{1}{2}|\langle q|(|\bar{0}\rangle + |\bar{1}\rangle)|^2$ for an approximate code word with $\Delta = \kappa = 0.25$. The dashed line is the distribution's Gaussian envelope.
$= 0.25$. The dashed line is the distribution's Gaussian envelope.

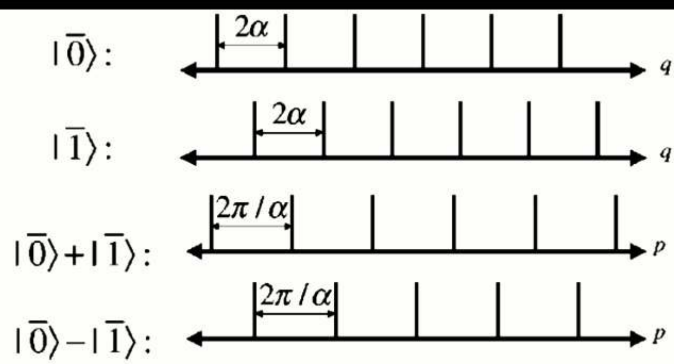

FIG. 1. Code words of the $n=2$ code. The states $|\bar{0}\rangle$, $|\bar{1}\rangle$ are

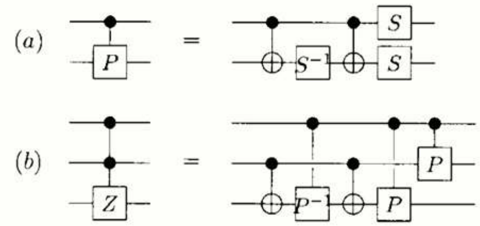

FIG. 7. Construction of the three-qudit gate $\Lambda^2(Z)$. (a) A $\Lambda(P)$ gate can be constructed (up to an overall phase) from two S gates, an S^{-1} gate, and two CNOT'S. The circuit is executed from left to right. (b) A $\Lambda^2(Z)$ gates can be constructed from two $\Lambda(P)$ gates, a $\Lambda(P^{-1})$ gate, and two CNOT'S.

Encoding a qubit in a trapped-ion mechanical oscillator

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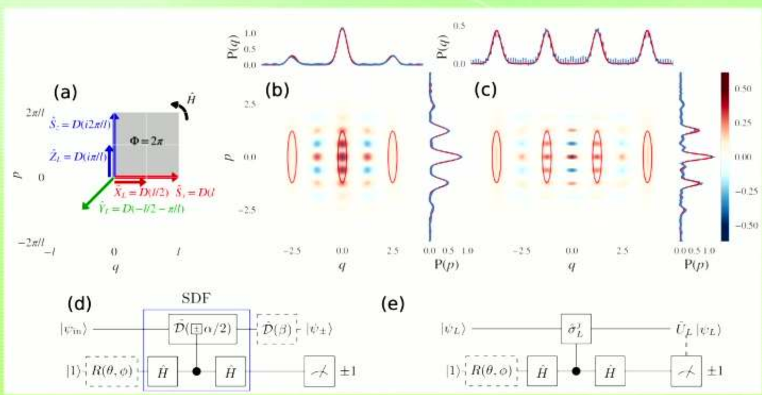
Nature **566**, 513–517 (2019) | Cite this article

approximations to such states, but these have thus far remained out of reach[7,8,9,10,11]. Here we demonstrate such an encoded qubit using a superposition of displaced squeezed states of the harmonic motion of a single trapped $^{40}Ca^{+}$ ion, controlling and measuring the mechanical oscillator through coupling to an ancillary internal-state qubit[12]. We prepare and reconstruct logical states with an average squared fidelity of 87.3 ± 0.7 per cent. Also, we demonstrate a universal logical single-qubit gate set, which we analyse using process tomography. For Pauli gates we reach process fidelities of about 97 per cent, whereas for



Experimentally, we create the code states in the axial motional mode of a single $^{40}\mathrm{Ca}^{+}$ ion at close to $\omega_{\mathrm{m}} \approx 2\pi \times 1.85$ MHz by starting from a squeezed vacuum state produced by reservoir engineering [22], followed by repeated application of a modular variable measurement sequence [6, 11, 23, 24]. A modular measurement is implemented by a two stage process shown in the circuit in figure 1 (d) excluding operations inside the dashed boxes. First i) the ancillary internal-state qubit

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Coherent States for Arbitrary Lie Group

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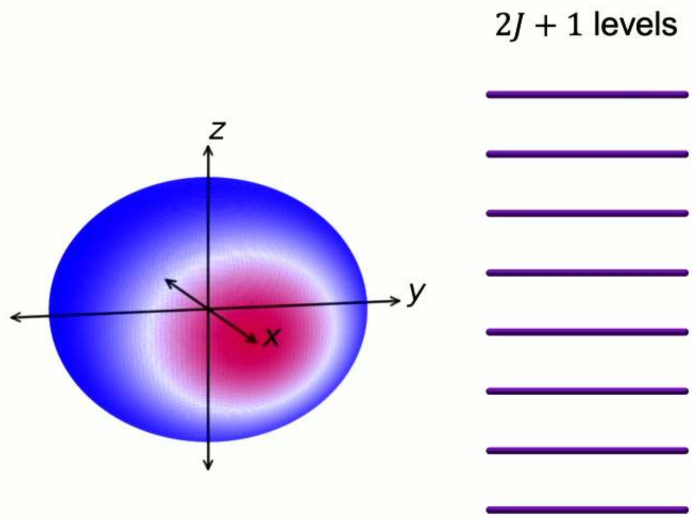
Received November 11, 1971

Abstract. The concept of coherent states originally closely related to the nilpotent group of Weyl is generalized to arbitrary Lie group. For the simplest Lie groups the system of coherent states is constructed and its features are investigated.

4. Case of the Simplest Compact Group-$SU(2)$ Group

The $SU(2)$ group is the unitary matrix group of the second order with unity determinant. It is locally isomorphic to the $SO(3)$ group – the rotation group of three-dimensional space and it is the most investigated group among all the non-Abelian Lie groups[10]. Nevertheless, the coherent states for this group as a special system do not appear to have been considered so far.



PHYSICAL REVIEW A VOLUME 40, NUMBER 5 SEPTEMBER 1, 1989

Quantum dynamics of the nonlinear rotator and the effects of continual spin measurement

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The quantum and classical dynamics of the nonlinear oscillator are contrasted by comparing the evolution of the quantum Q function with that of a similar classical probability distribution. The quantum nonlinear rotator is shown to generate a superposition of two distinct coherent states from a coherent-state input. Measurements of the angular-momentum components and the signature of a superposition state are discussed. The effects of a continual measurement of one angular-momentum component are introduced into the model, and its effects on quantum coherences are shown to degrade the quantum coherence effects. We present a quantum optical model that obeys the nonlinear rotator dynamics and can generate superpositions of SU(2) coherent states of the two-mode electromagnetic field.

Entangled coherent states for systems with $SU(2)$ and $SU(1,1)$ symmetries J. Phys. A: Math. Gen. **33** (2000) 7451–7467

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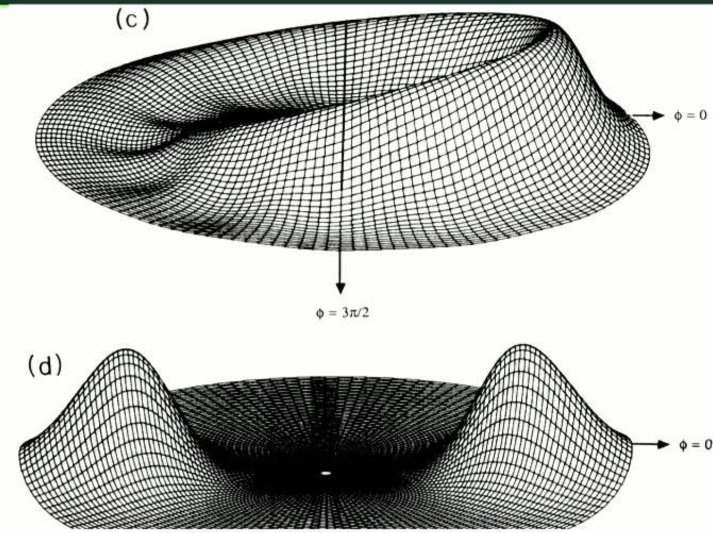
Abstract. Entangled $SU(2)$ and $SU(1,1)$ coherent states are developed as superpositions of multiparticle $SU(2)$ and $SU(1,1)$ coherent states. As a special example of entangled $SU(2)$ coherent states, entangled binomial states are introduced and these entangled binomial states enable the contraction from entangled $SU(2)$ coherent states to entangled harmonic oscillator coherent



PHYSICAL REVIEW A **103**, 053711 (2021)

Sub-Planck structures: Analogies between the Heisenberg-Weyl and SU(2) groups

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PHYSICAL REVIEW A VOLUME 54, NUMBER 6 DECEMBER 1996

Optimal frequency measurements with maximally correlated states

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(Received 16 August 1996)

We show how maximally correlated states of N two-level particles can be used in spectroscopy to yield a frequency uncertainty equal to $(NT)^{-1}$, where T is the time of a single measurement. From the time-energy uncertainty relation we show that this is the best precision possible. We rephrase these results in the language of particle interferometry and obtain a state and detection operator which can be used to achieve a phase uncertainty exactly equal to the $1/N$ Heisenberg limit, where N is the number of particles used in the measurement. [S1050-2947(96)50712-2]

are in the ground (spin-down) state $|g\rangle$ (with energy E_g) and $|N/2,N/2\rangle$ denote the state where all the particles are in the excited (spin-up) state $|e\rangle$ (with energy E_e). We show how the state

$$|\Psi_M\rangle \equiv \{|N/2,N/2\rangle + |N/2,-N/2\rangle\}/\sqrt{2} \qquad (1)$$

can be used to measure $\omega_0 = (E_e - E_g)/\hbar$ with a frequency uncertainty equal to $(NT)^{-1}$, where T is the time of a single measurement. This state is ''maximally correlated'' in the

Superpositions of $SU(3)$ coherent states via a nonlinear evolution

K Nemoto[1] and B C Sanders[2]

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PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

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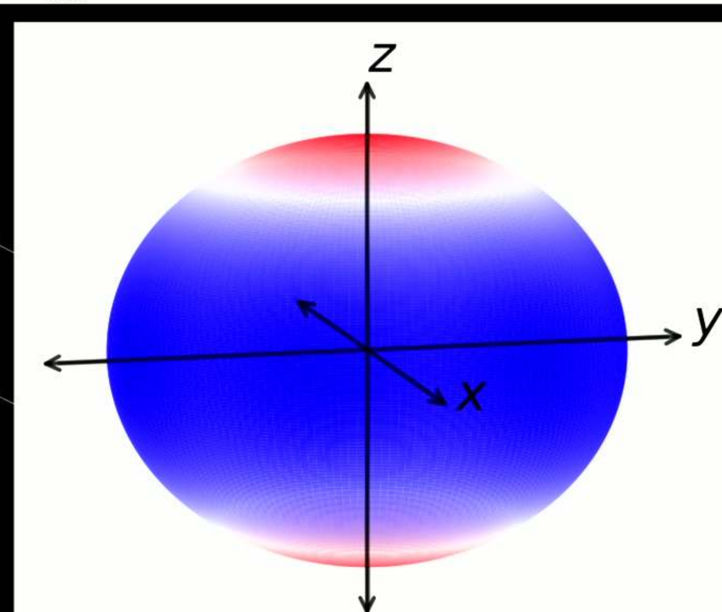
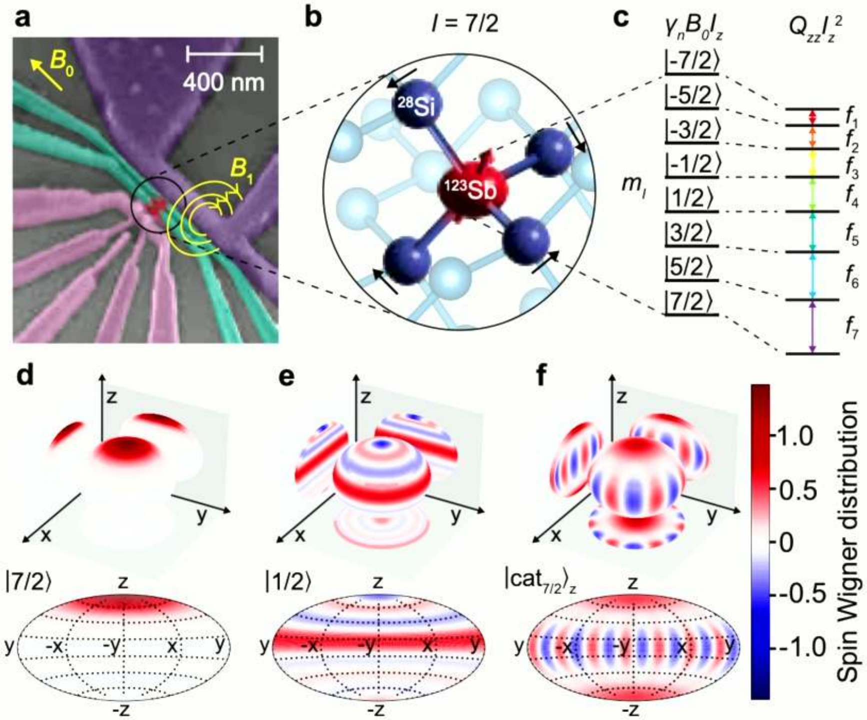
(Received August 25, 1953)

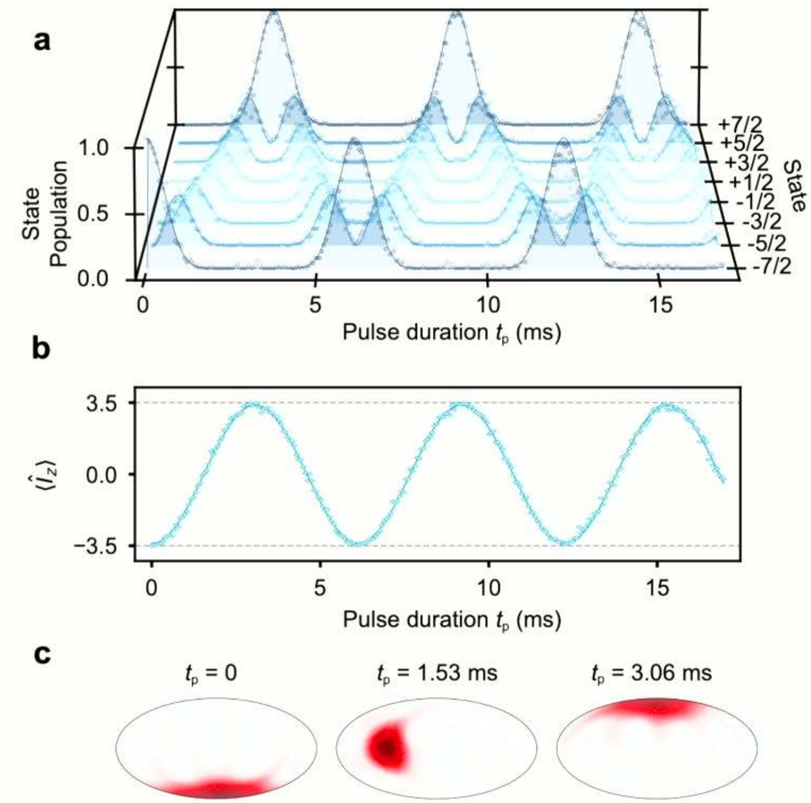
By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line breadths are calculated. For a gas of large extent the effect of photon recoil momentum on coherence is calculated. The effect of a radiation pulse in exciting "super-radiant" states is discussed. The angular correlation between successive photons spontaneously emitted by a gas initially in thermal equilibrium is calculated.

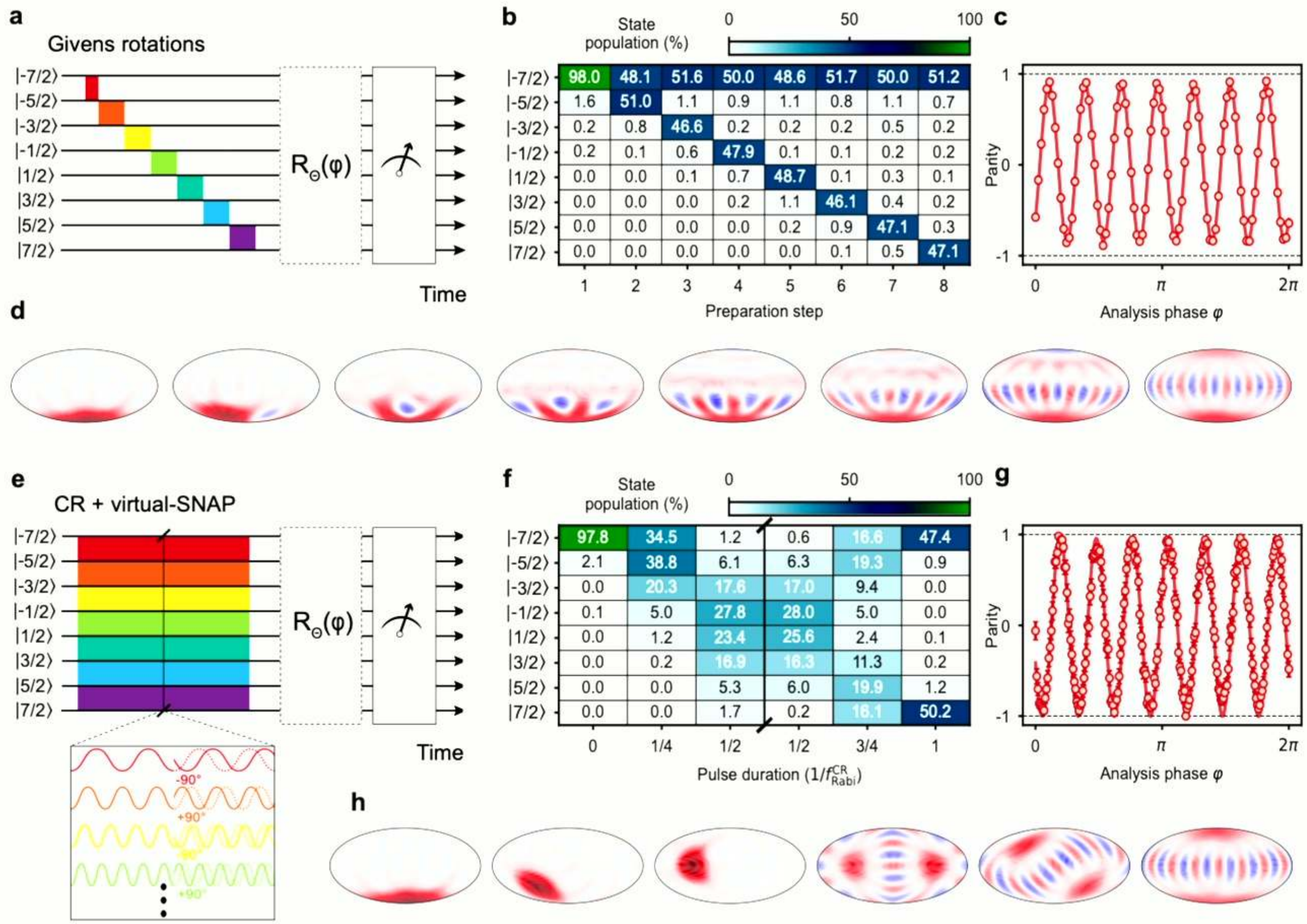


Antimony donor in a nanoscale device



Covariant rotations driven by a multi-tone pulse





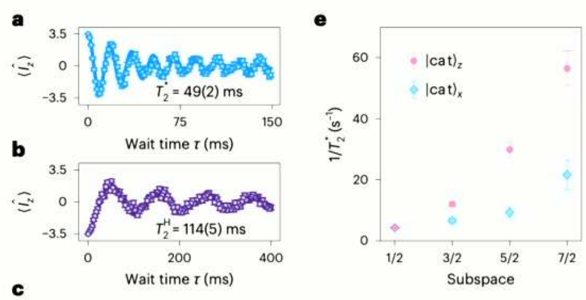
a
Givens rotations
|-7/2⟩
|-5/2⟩
|-3/2⟩
|-1/2⟩
|1/2⟩
|3/2⟩
|5/2⟩
|7/2⟩
$R_\Theta(\varphi)$
Time
b
State population (%)
0
50
100

	1	2	3	4	5	6	7	8
|-7/2⟩	98.0	48.1	51.6	50.0	48.6	51.7	50.0	51.2
|-5/2⟩	1.6	51.0	1.1	0.9	1.1	0.8	1.1	0.7
|-3/2⟩	0.2	0.8	46.6	0.2	0.2	0.2	0.5	0.2
|-1/2⟩	0.2	0.1	0.6	47.9	0.1	0.1	0.2	0.2
|1/2⟩	0.0	0.0	0.1	0.7	48.7	0.1	0.3	0.1
|3/2⟩	0.0	0.0	0.0	0.2	1.1	46.1	0.4	0.2
|5/2⟩	0.0	0.0	0.0	0.0	0.2	0.9	47.1	0.3
|7/2⟩	0.0	0.0	0.0	0.0	0.0	0.1	0.5	47.1

Preparation step
c
Parity
1
0
-1
0
π
2π
Analysis phase φ
d
e
CR + virtual-SNAP
|-7/2⟩
|-5/2⟩
|-3/2⟩
|-1/2⟩
|1/2⟩
|3/2⟩
|5/2⟩
|7/2⟩
$R_\Theta(\varphi)$
Time
-90°
+90°
+90°
+90°
f
State population (%)
0
50
100

	0	1/4	1/2	1/2	3/4	1
|-7/2⟩	97.8	34.5	1.2	0.6	16.6	47.4
|-5/2⟩	2.1	38.8	6.1	6.3	19.3	0.9
|-3/2⟩	0.0	20.3	17.6	17.0	9.4	0.0
|-1/2⟩	0.1	5.0	27.8	28.0	5.0	0.0
|1/2⟩	0.0	1.2	23.4	25.6	2.4	0.1
|3/2⟩	0.0	0.2	16.9	16.3	11.3	0.2
|5/2⟩	0.0	0.0	5.3	6.0	19.9	1.2
|7/2⟩	0.0	0.0	1.7	0.2	16.1	50.2

Pulse duration ($1/f^{CR}_{Rabi}$)
g
Parity
1
0
-1
0
π
2π
Analysis phase φ
h



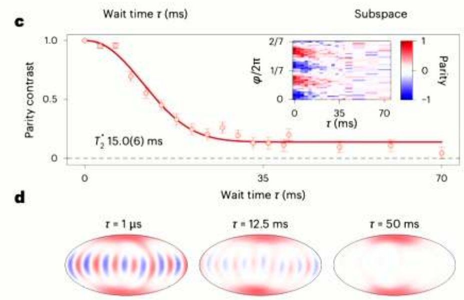
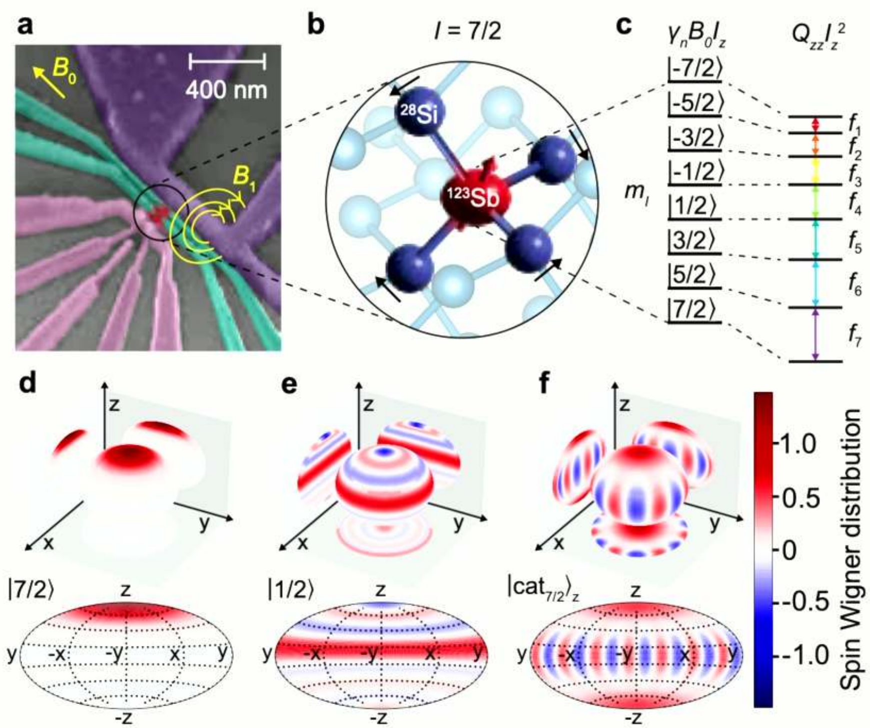


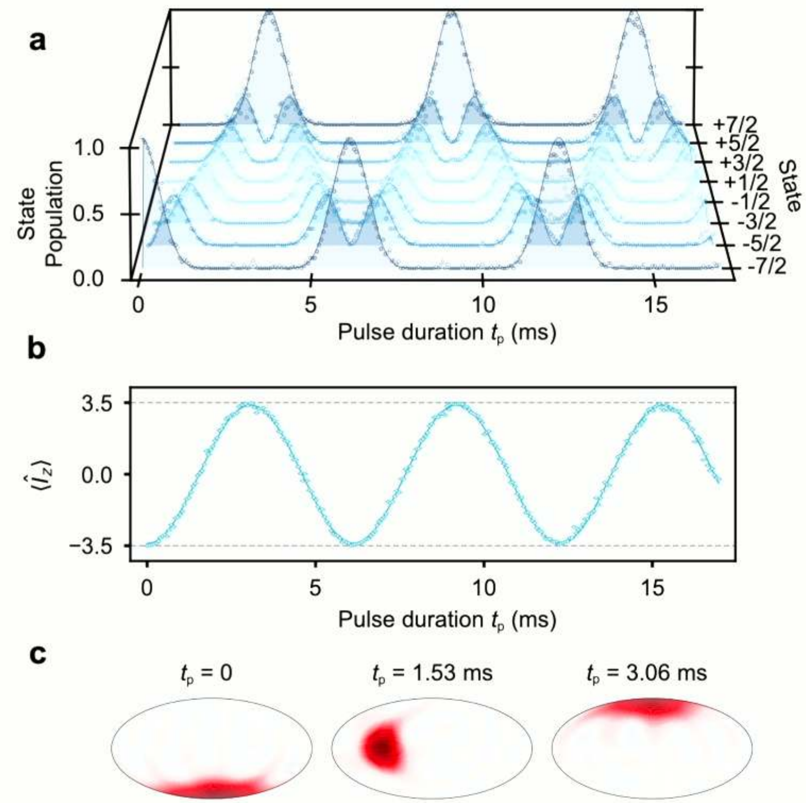
Fig. 4 | Dephasing of spin coherent states and Schrödinger cat states. **a**, Decay of Ramsey fringes for a spin coherent state prepared with a $SU(2)$ π/2 pulse starting from $|-7/2\rangle$, left to freely evolve for time τ and then rotated with a second $SU(2)$ π/2 pulse around an angle φ increasing with τ. The dephasing time was $T_2^* = 49(2)$ ms. **b**, Hahn echo experiment, which had another $SU(2)$ π pulse at the halfway point, yielding coherence time $T_2^{\rm H} = 114(5)$ ms. Each $\hat{I}_z$ expectation in **a** and **b** was calculated from the probabilities of all eight nuclear spin states, each measured over 100 repetitions. **c**, Decay of the parity oscillations of a $|\mathrm{cat}_{7/2}\rangle_z$ state with wait time, extracted from the data in the inset. Each parity measurement was an average over 50 repetitions. The cat state was prepared by the virtual-SNAP method and had coherence time $T_2^* = 15.0(6)$ ms.

d, Reconstructed Wigner functions of an initial $|\mathrm{cat}_{7/2}\rangle_z$ after the indicated wait times. Note the disappearance of interference fringes around the equator at long times. **e**, Dephasing rates for z cats (circles) and x cats (diamonds) as a function of subspace size. The dephasing rate was extracted from fitting the decay of the return probability, each averaged over 80 repetitions of spin projection measurements. The subspace cats $|\mathrm{cat}_l\rangle_x$ were prepared by the virtual-SNAP method and $|\mathrm{cat}_l\rangle_z$ by Givens rotations (Methods). Note that the x and z states in the $m_l = 1/2$ subspace coincide.

Antimony donor in a nanoscale device



Covariant rotations driven by a multi-tone pulse



PHYSICAL REVIEW VOLUME 155, NUMBER 5 25 MARCH 1967

Charge Superselection Rule*

YAKIR AHARONOV AND LEONARD SUSSKIND
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(Received 25 August 1966)

Questions concerning superselection rules are considered. Two experiments are discussed. In the first, coherent superpositions of different angular-momentum states are constructed. In the second, coherent superpositions of states with different charge are constructed in complete analogy with the angular-momentum case. We suggest that, contrary to a widespread belief, interference may be possible between states with different charges.

A selection rule for a quantity A asserts the conservation of A. Selection rules exist for energy, momentum, angular momentum, electric charge, lepton number, and baryon number, and the number of fermions modulo 2. According to Wick, Wightman, and Wigner, some of these selection rules can be elevated to the status of superselection rules. A superselection rule asserts the impossibility of preparing a coherent superposition of two eigenstates of A with different eigenvalues. More generally, no physically realizable density matrix contains off-diagonal elements connecting states of different A.

In the above experiment, the two states $|Q\theta\rangle$ and $|Q'\theta'\rangle$ for C_1 and C_2 are, of course, not eigenstates of charge. Actually they are minimum uncertainty packets in charge and phase. Hence we have not yet been able to set up the required relative reference frames with a system of definite charge. However, if we superpose states $|Q\theta_1\rangle$ and $|Q'\theta_2\rangle$, keeping the phase difference $(\theta_1-\theta_2)$ fixed at $(\theta-\theta')$ according to

$$|i\rangle=\int|Q\theta_1\rangle|Q'\theta_2\rangle\delta(\theta_1-\theta_2-\theta+\theta')e^{i(Q+Q')\theta_1}, \quad (10)$$

then we obtain a state of definite charge $Q+Q'$. Furthermore, all probabilities for proton or neutron states are precisely the same as for the initial state $|Q\theta\rangle\ |Q'\theta'\rangle$. Hence the apparatus described by Eq. (10) constructs a state which, when referred to the phase frame of C_2, is a coherent superposition of proton and neutron with phase $(\theta-\theta')$.