Title: RG flows from perturbed 2d CFT - Quantum Fields & Strings Seminar

Speakers: Federico Ambrosino

Collection/Series: Quantum Fields and Strings

Subject: Quantum Fields and Strings

Date: November 28, 2025 - 4:00 PM

URL: https://pirsa.org/25110124

Abstract:

In this talk we characterize RG flows from UV fixed point described by a 2d CFT perturbed by one of its relevant operators.
The first part focuses on rational CFTs. By matching the anomalies associated to their non-invertible symmetries, one conjectures infinitely many new RG flows between Minimal models of Virasoro, Wn algebras, and more general coset models.
The second part presents a construction of a class of non-topological, but conserved operators in deformed CFT given by perturbed Verlinde lines. These provide infinite families of commuting non-local conserved charges associated to the new flows found by anomaly matching.
This is based on [2501.07511,2504.05277] and work in progress with T. Prochazka.

Perturbed 2d CFTs

Flows and conserved charges

Federico Ambrosino [Perimeter Institute]

Quantum Fields & Strings Seminar November 28th 2025

Based on [2501.07511] with Stefano Negro,
+ [2504.05277, 2511.02007] with Runkel & Watts,
+ to appear with Runkel, Watts & Konechny
+ to appear with Tomáš Procházka

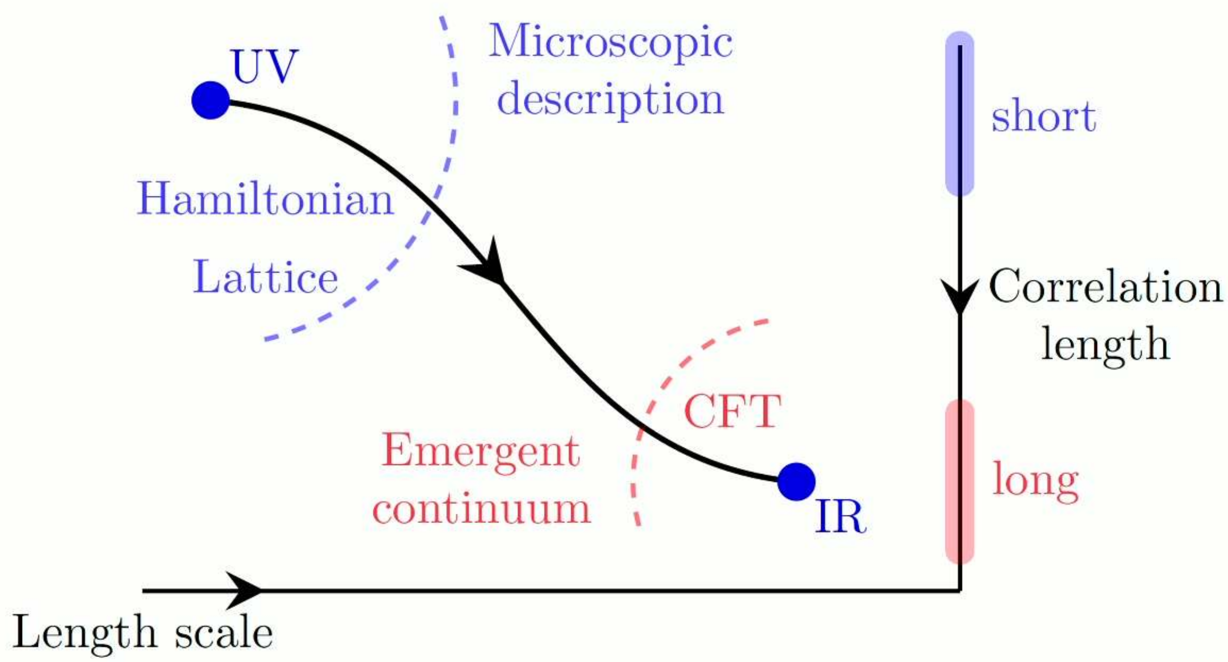
Deformed d dimensional CFT

$$\mathcal{T}_{\mathrm{UV}} = \mathrm{CFT}_{\mathrm{UV}} + g_\rho \int \phi_\rho \,, \qquad (\Delta_\rho \lesssim d)$$

$$\begin{tikzcd}
\mathcal{T}_{\mathrm{UV}} \arrow[d, squiggly] & & g_\rho \int \phi_\rho \arrow[dl, "\text{Gapped}"'] \arrow[dr, "\text{Gapless}"] & \\
\mathcal{T}_{\mathrm{IR}} = & \mathrm{TQFT} & & \mathrm{CFT}_{\mathrm{IR}}
\end{tikzcd}$$

Very hard to solve $\mathrm{CFT}_{\mathrm{d}}$

Condensed matter systems: gapless example



Deformed 2 dimensional CFT

$$\mathcal{T}_{\mathrm{UV}} = \mathrm{CFT}_{\mathrm{UV}} + g_\rho \int \phi_\sigma \,, \qquad (\Delta_\rho \lesssim 2)$$

Gapped

Gapless

$$\mathcal{T}_{\mathrm{IR}} = \qquad \mathrm{TQFT} \quad \mathrm{CFT}_{\mathrm{IR}}$$

Hopeful to solve $\mathrm{CFT}_{\mathrm{d=2}}$ (Virasoro algebra)

Deformed 2 dimensional Minimal Models

$$\mathcal{T}_{\mathrm{UV}} = \mathrm{RCFT} + g_\rho \int \phi_\rho \,, \qquad (\Delta_\rho \lesssim 2)$$

Gapped Gapless

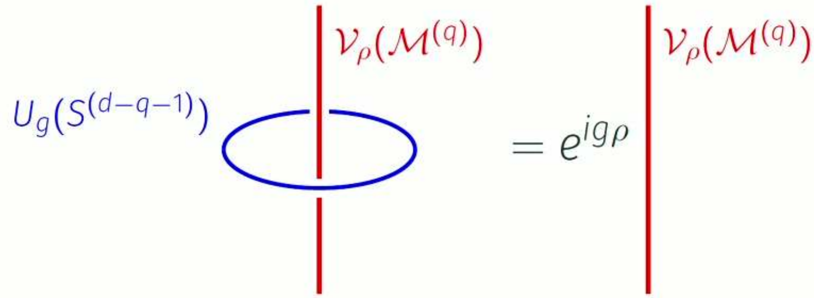
$$\mathcal{T}_{\mathrm{IR}} = \qquad \mathrm{TQFT} \quad \mathrm{CFT}_{\mathrm{IR}}$$

Solvable $\mathrm{CFT}_{\mathrm{UV}} = \mathrm{RCFT}$ (Virasoro algebra) !!

A defect fever

Topological operator = Symmetry

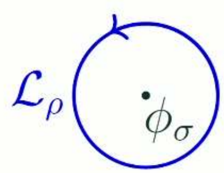
[Gaiotto, Kapustin, Seiberg, Willett]



<u>0-form</u>, higher-groups, <u>non-invertible</u>, emanent, mixed anomalies, gauging, SymTFT, higher-category ...

[Frolich, Fuchs, Runkel, Schwiegert, Gaiotto, Seiberg, Tachikawa, Bhardwaj, Chang, Lin, Shao, Wang, Yin, Copetti, Cordova, Komatsu, Schäfer-Nameki, Bottini, Tiwari, Cordova, Dumitrescu, Intriligator, Benini, Antinucci] +......∞

<u>Today:</u> QFT$_2$



Symmetries strike back

Symmetry: most important guiding principle in physics

More symmetries = More data

- Constraints on RG flows and S-Matrix:
  → Phases of 2d Adjoint QCD, Modified crossing...
  [Copetti, Cordova, Komatsu][Komargodski, Ohmori Seifnashri, Roumpedakis][Tanaka, Nakayama]
- Organizing principles
  → Particle-soliton degeneracy[Cordova, García-Sepúlveda, Holfester]
- Phenomenology?
  → Pion decay $\pi^0 \to \gamma\gamma$, axion coupling [Cordova, Shao. . .]

Today's talk

What can we learn by having the full symmetry data?

Usually hard in QFT: need all top. ops. , fusion, anomalies...

Rational 2d CFT: we know all topological defects

This Talk:

- Predict new RG flows
  - $\to$ Virasoro[FA, Negro][Tanaka, Nakayama] & $\mathcal{W}$ algebra [FA, Procházka]
  - $\to$ NLIE description [FA, S. Negro] (if interested...)
- Translational invariant charges $\leftrightarrow$ **Integrated** lines

[Runkel] [FA, Runkel, Watts][FA, Runkel, Watts, Konechny]

$\to$ Criterion for integrability?

$\to$ Explain "emergent" IR symmetries?

A primer on Virasoro Minimal models

$\mathcal{M}(p,q)$: Rational 2D CFT, A-series

- Central charge: $c = 1 - \frac{6(p-q)^2}{pq}$ Unitary iff $q = p+1$
- $\frac{(p-1)(q-1)}{2}$ primaries $\phi_{(r,s)} = \phi_{(p-r,q-s)}$
- weigths $h_{(r,s)}$ and C^t_{rs} all known
- Fusion Category: $\phi_\rho \otimes \phi_\sigma = \sum_\delta \mathcal{N}^\delta_{\rho\sigma}\, \phi_\delta$

Topological lines = Verlinde Lines $\{\mathcal{L}_\rho\} \overset{1\text{ to }1}{\Longleftrightarrow} \{\phi_\rho\}$

Non-invertible symmetry: $\mathcal{L}_\rho \times \mathcal{L}_\sigma = \sum_\delta \mathcal{N}^\delta_{\rho\sigma}\, \mathcal{L}_\delta$

Ward identity: $\mathcal{L}_\rho \; \phi_\sigma = \left[\frac{S_{\rho\sigma}}{S_{0\sigma}}\right] \; \phi_\sigma$

Example: the Ising model

2D Ising CFT $\mathcal{M}(3,4)$

$$c = \frac{1}{2}, \quad \text{Primaries}: \ 1_{0,0}, \epsilon_{\frac{1}{2},\frac{1}{2}}, \sigma_{\frac{1}{16},\frac{1}{16}}, \quad \text{TLD}: \ \mathbf{1}, \ \eta, \ \mathcal{N}$$

$$\text{Fusion Algebra TY}_2: \quad \eta^2 = \mathbf{1}, \quad \mathcal{N} \times \eta = \mathcal{N}, \quad \mathcal{N}^2 = \mathbf{1} + \eta$$

$\eta \longleftrightarrow \mathbb{Z}_2 \qquad \mathcal{N} \longleftrightarrow$ KW duality defect

Duality in QFT $\quad \mathrm{KW}: \mathrm{Ising}_T \longleftrightarrow \mathrm{Ising}_{T^{-1}}$

Symmetry in CFT $\quad \mathrm{KW}: \mathrm{Ising}_{T_c} \circlearrowleft$

TDL **honest** symmetries of CFT

Derfomation by primary field

Relevant deformation of minimal models

$$\mathcal{T}_{\mathrm{UV}} = \mathcal{M}(p,q) + g_\rho \int \phi_\sigma \,, \qquad (h_\rho < 1)$$

Gapped | Gapless

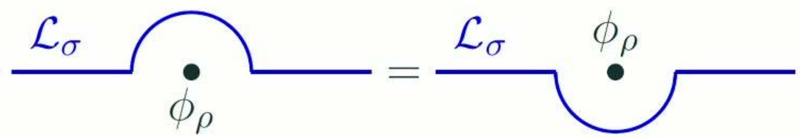
$$\mathcal{T}_{\mathrm{IR}} = \qquad \text{TQFT} \qquad \boxed{\text{CFT} \; = \mathcal{M}(p', q') \quad \text{TODAY}}$$

c-theorem with: $c_{\mathrm{eff}} = 1 - \frac{6}{pq}$ ($\mathcal{PT}$-sym) [Ravanini]

Invariants under RG flows

$$\mathcal{M}(p,q) + g_\rho \int \phi_\sigma \rightsquigarrow ?$$

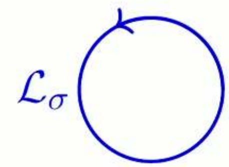
When: $[\mathcal{L}_\sigma, \phi_\rho]\, |\Phi\rangle = 0 \Leftrightarrow$



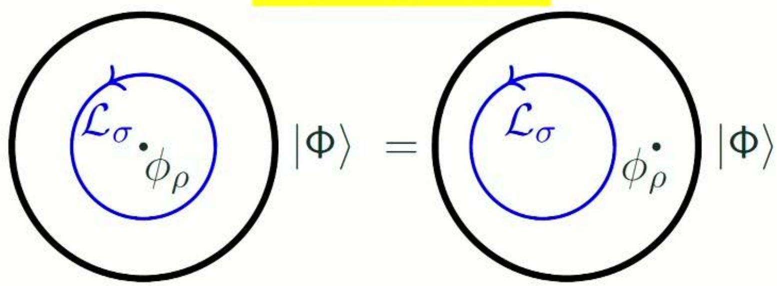
Then: preserved symmetry along flow &

[Chang, Lin, Shao, Wang, Yin] [Nakayama, Tanaka]

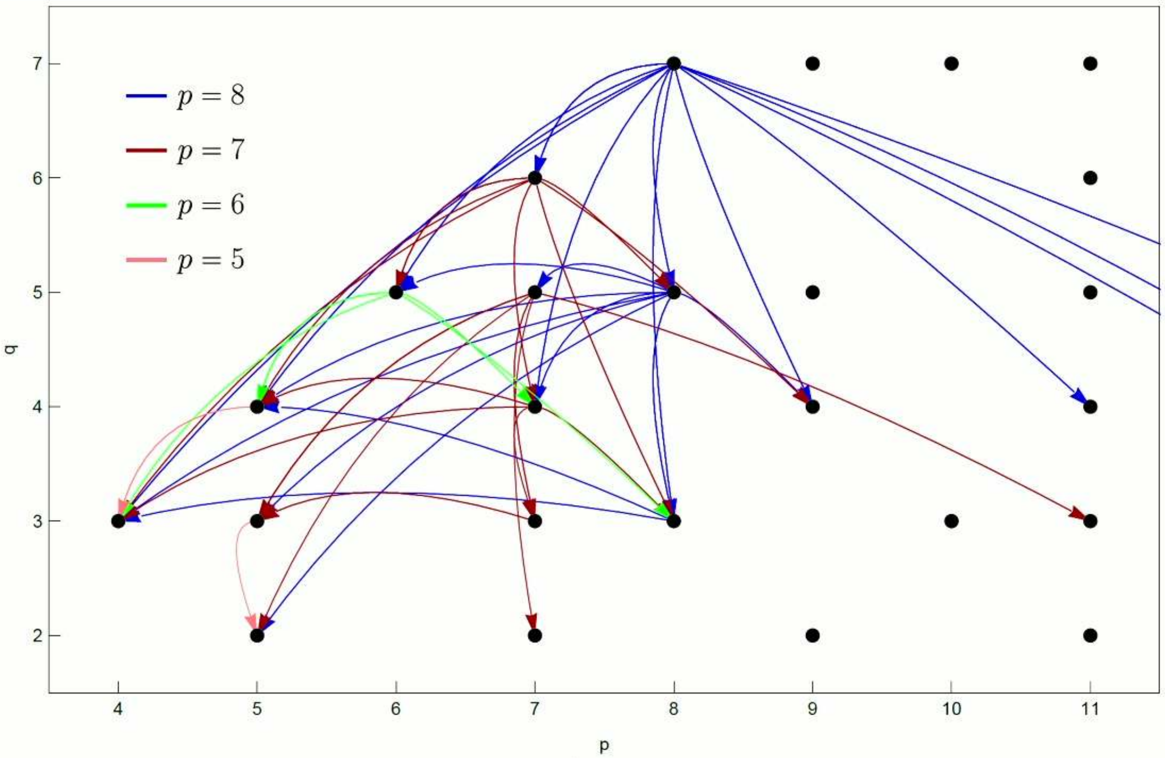
Quantum dimension $= \langle 0|\, \mathcal{L}_\sigma \,|0\rangle = d_\rho =$



RG invariant!



Result



Main series: $\mathcal{M}(kp + l, q) \xrightarrow{\phi_{(1,2k+1)}} \mathcal{M}(kp - l, q)$ [Nakayama, Tanaka]

We have NLIE $\sim$ TBA (backup slides) [FA, S. Negro]

New flows

Main series: $\mathcal{M}(kp+l,q) \xrightarrow{\phi_{(1,2k+1)}} \mathcal{M}(kp-l,q)$ [Nakayama, Tanaka]

Integrable flows:

- $k, l = 1$: $\phi_{(1,3)}$ [Fendley, Saleur, Al. Zamolodchikov][Al. Zamolodchikov]
- $k = 1/2, k = 2$: $\phi_{(1,2)}, \phi_{(1,5)}$ [Dorey, Dunning, Tateo]
- $k = 3$: $\phi_{(1,7)}$ $\qquad \mathcal{M}(3,10) \to \mathcal{M}(3,8)$ [Narovlansky, Sun, Tarnopolsky]
- $\cdots$

Preserve $SU(2)_{q-2}$ fusion category:

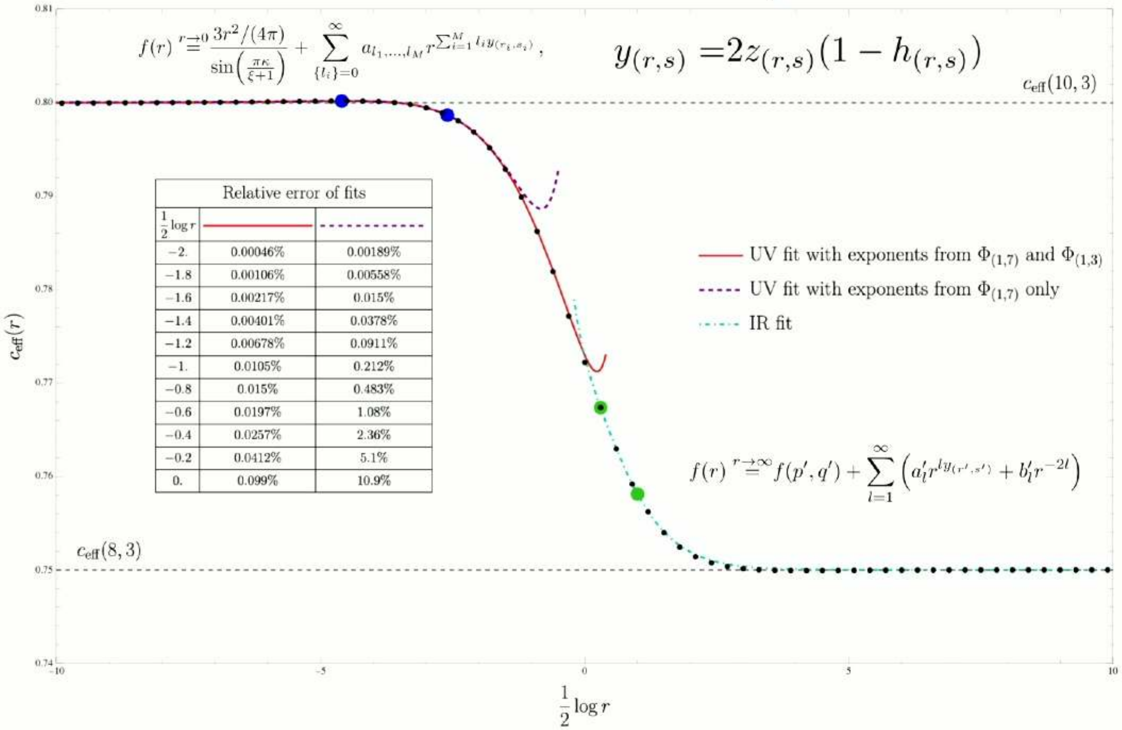
$$\{\mathcal{L}_{(1,1)} = \mathbf{1}, \mathcal{L}_{(2,1)}, \cdots, \mathcal{L}_{(q-1,1)}\}, \qquad \mathcal{L}_{(q-1,1)} \times \mathcal{L}_{(q-1,1)} = \mathbf{1}$$

$$[\mathcal{L}_{(n,1)}, \phi_{(1,2m+1)}] = 0, \qquad m = 1, \cdots, k$$

Example: $\mathcal{M}(3,10) \to \mathcal{M}(3,8)$

Recent interest in this flow [Klebanov et al] [Miro' et al]

Solutions of our NLIE match UV and IR expansion



Relative error of fits

$\frac{1}{2}\log r$	UV fit ($\Phi_{(1,7)}$ and $\Phi_{(1,3)}$)	UV fit ($\Phi_{(1,7)}$ only)
−2.	0.00046%	0.00189%
−1.8	0.00106%	0.00558%
−1.6	0.00217%	0.015%
−1.4	0.00401%	0.0378%
−1.2	0.00678%	0.0911%
−1.	0.0105%	0.212%
−0.8	0.015%	0.483%
−0.6	0.0197%	1.08%
−0.4	0.0257%	2.36%
−0.2	0.0412%	5.1%
0.	0.099%	10.9%

$\mathcal{W}_N$ algebra

Virasoro algebra: generated by modes of spin 2 field $T(z)$

$\mathcal{W}_N$ algebra: additional currents of spin $s = 2, \cdots, N$

$\{T(z), W^{(3)}, \cdots, W^{(N)}\}$ [Zamolodchikov[2], Fateev, Lukyanov. . .]

$$T(z)W^{(s)}(u) \sim \frac{sW^{(s)}(u)}{(z-u)^2} + \frac{\partial W^{(s)}(u)}{z-u} + \cdots .$$

E.g. $\mathcal{W}^{(3)}$:

$$W(z)W(u) \sim \frac{c/3}{(z-u)^6} + \frac{2T(u)}{(z-u)^4} + \frac{\partial T(u)}{(z-u)^3} + \\ + \frac{1}{(z-u)^2}\left(\frac{3}{10}\partial^2 T(u) + \frac{32}{22+5c}\Lambda(u)\right) + \cdots$$

$\mathcal{W}_N$ Minimal models

Also $\mathcal{W}_N$ admit minimal truncations:

$\mathcal{W}_N$ minimal models: $\mathcal{W}_N(p, q)$

- Rational CFT: $c^{(N)}_{p,q} = (N-1)\left[1 - \frac{N(N+1)(p-q)^2}{pq}\right]$
- Unitary iff $p - q = \pm 1$
- Coset construction $\frac{\widehat{\mathfrak{su}}(N)_{\kappa-N} \times \widehat{\mathfrak{su}}(N)_{p=1}}{\widehat{\mathfrak{su}}(N)_{\kappa+p-N}}$ $\kappa = p/(q-p)$

Can we find RG flows?

YES! Using non-invertible sym [WIP with Prochazka]

A uniform description: truncation of $\mathcal{W}_\infty$

No canonical embedding $\mathcal{W}_n[c] \hookrightarrow \mathcal{W}_{n+1}[c]$

- No natural inclusion of DS reductions from $\widehat{\mathfrak{sl}}_n$
- OPE & currents don't restrict nicely

$\Rightarrow$ Nice inclusions into bigger algebra $\mathcal{W}_\infty[\lambda, c]$

- Contains currents $s = 2, 3, \cdots, \infty$
- Interpolating family: $\mathcal{W}_\infty[\lambda = N, c] \simeq \mathcal{W}_N(c)$
- $\mathcal{S}_3$ symmetry: $\mathcal{W}_\infty[\lambda_1, c] \simeq \mathcal{W}_\infty[\lambda_2, c] \simeq \mathcal{W}_\infty[\lambda_3, c]$

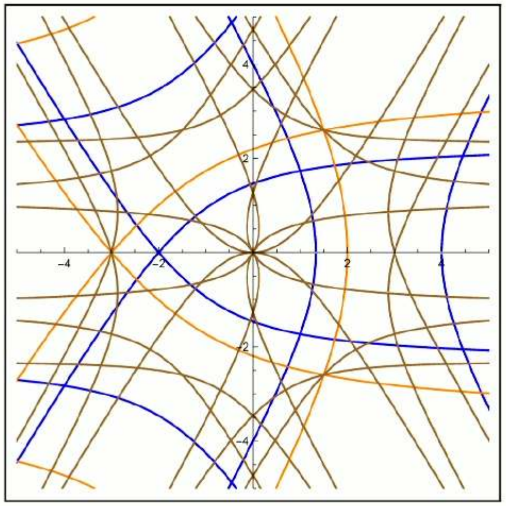
$$\mathcal{W}_\infty[\lambda_1, \lambda_2, \lambda_3], \quad \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3} = 0, \quad c = (\lambda_1-1)(\lambda_2-1)(\lambda_3-1)$$

[Gaberdiel, Gopakumar,Gaiotto, Rapcak, Prochazka, Eberhardt, Linshaw . . .]

A uniform description: truncation of $\mathcal{W}_\infty$

$\mathcal{W}_{1+\infty}$ has null vectors: $\frac{N_1}{\lambda_1} + \frac{N_2}{\lambda_2} + \frac{N_3}{\lambda_3} = 0$

$$\mathcal{W}_\infty[\lambda_1, \lambda_2, \lambda_3] \simeq Y_{N_1,N_2,N_3}[c]$$



Double truncations are RCFTs:

[Prochazka, Gaiotto, Rapcak]

$$\boxed{W_N(p,q) = Y_{p-N,q-N,0} \cap Y_{0,0,N}}$$

Uniform useful description for representation theory etc...

Many generalizations: Grassmanian VOA [Prochazka, Eberhardt]

$$\frac{SU(N)_\kappa \times SU(N)_p}{SU(N)_{\kappa+p}} \qquad \frac{\mathfrak{g}_\kappa \times \mathfrak{g}_p}{\mathfrak{g}_{\kappa+p}} \qquad \dots$$

Primaries

[Isomorphism $Y[\widehat{\mathfrak{gl}_1}]$ or box counting]
Primaries in $\mathcal{W}_N(p, q)$ representation $(A_N, A_N)/$Relations:

$$\left(\ydiagram{6,5,2}\, , \ydiagram{5,2}\right)$$

Primaries

[Isomorphism $Y[\widehat{\mathfrak{gl}_1}]$ or box counting]

Primaries in $\mathcal{W}_N(p,q)$ representation (A_N, A_N)/Relations:

$$\left(\overbrace{\ydiagram{6,5,2}}^{\leq q-N}, \overbrace{\ydiagram{5,2}}^{\leq p-N}\right\} < N\Big)$$

For Virasoro: $(N=2)\ \phi \sim (\ydiagram{4}, \ydiagram{3})$

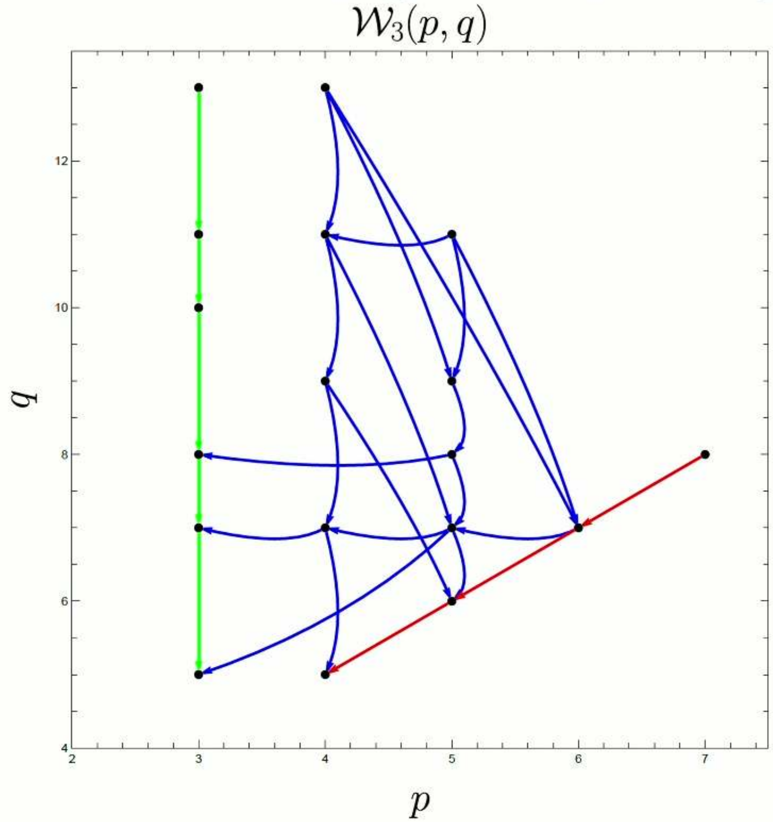
$$\phi_{(1,2k+1)} \sim (\cdot, \overbrace{\ydiagram{2}}^{2k}), \qquad \mathcal{M}(kq+l, q) \xrightarrow{\phi_{(1,2k+1)}} \mathcal{M}(kq-l, q)$$

Commutes with

$$SU(2)_{q-2} : \left\{ (\cdot,\cdot), (\ydiagram{1}, \cdot), \cdots, \underbrace{(\overbrace{\ydiagram{5}}^{q-2}, \cdot)}_{\mathbb{Z}_2} \right\}$$

A quick preview: example for $\mathcal{W}_3$

Using non-invertible-symmetry-matching:



- Unitary series: [Poghossian, Poghosyan]
  $\mathcal{W}_3(q+1, q) \to \mathcal{W}_3(q, q-1)$
  Triggered by $(\cdot, \boxplus) \sim \phi_{(1,3)}$

- Reproduces all the known integrable flows (3 classes)

- Extend to new ∞ families!

A quick preview: example for $\mathcal{W}_N$

New families: <u>simplest</u> $\mathcal{W}_N(kq+i,q) \xrightarrow{\phi_{(0,k)}} \mathcal{W}_N(kq-i,q)$

$$\varphi_{(0,k)} = \Big(\cdot, \overbrace{\underbrace{\ydiagram{6,3,3}}_{k}}^{(N-1)k}\Big), \qquad k = 1, \cdots, \left\lfloor \frac{p-N}{N-1} \right\rfloor$$

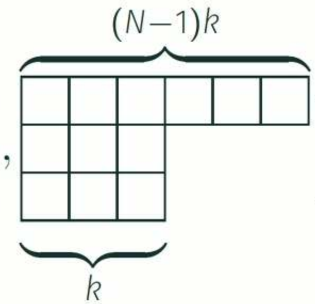
- $\mathcal{W}_j[\varphi_{(0,k)}] = 0 \Longrightarrow$ Preserved $\mathcal{W}_N$
- Commutes with $SU(N)_{q-N} \supset \mathbb{Z}_N$ symmetry:

$$\left\{ (\cdot,\cdot), (\ydiagram{1},\cdot), (\ydiagram{2},\cdot), (\ydiagram{1,1},\cdot), (\ydiagram{2,1},\cdot), (\ydiagram{2,2},\cdot) \right\}$$

Generalizes nicely to $\mathcal{W}_\infty$ (**?**)

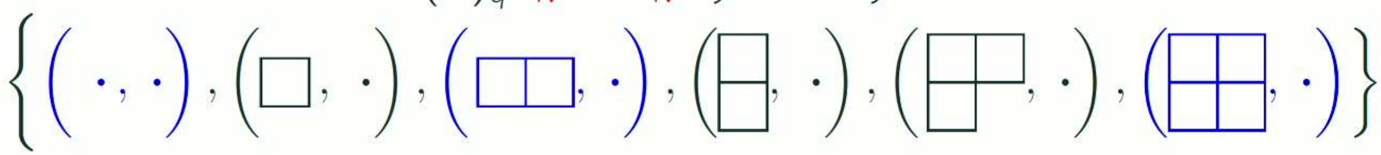
A quick preview: example for $\mathcal{W}_N$

New families: <u>simplest</u> $\mathcal{W}_N(kq+i,q) \xrightarrow{\phi_{(0,k)}} \mathcal{W}_N(kq-i,q)$

$\varphi_{(0,k)} = \Big(\cdot,$ 

$\Big)$, $\qquad k = 1, \cdots, \left\lfloor \dfrac{p-N}{N-1} \right\rfloor$

- $\mathcal{W}_j[\varphi_{(0,k)}] = 0 \Longrightarrow$ Preserved $\mathcal{W}_N$
- Commutes with $SU(N)_{q-N} \supset \mathbb{Z}_N$ symmetry:



Generalizes nicely to $\mathcal{W}_\infty^{(p)}$ (!?)

New questions

We found new flows between Virasoro and W_N models.
Focus on Virasoro

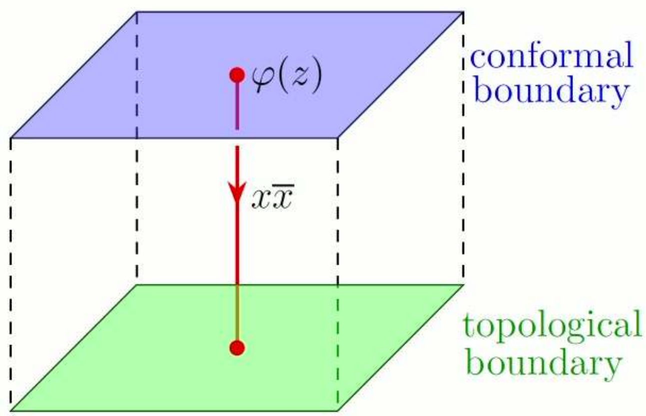
What is special about flows with $\phi_{(1,r)}$?

Remember: $r = 2, 3, 5$ are integrable: red. of BD and SG
We found:[Ambrosino, Runkel, Watts]

Non-top. defects = Non-local conserved charges

3d TFT for 2d RCFT

$\mathrm{TFT}_{d=3}$ encoding the $\mathrm{QFT}_{d=2}$

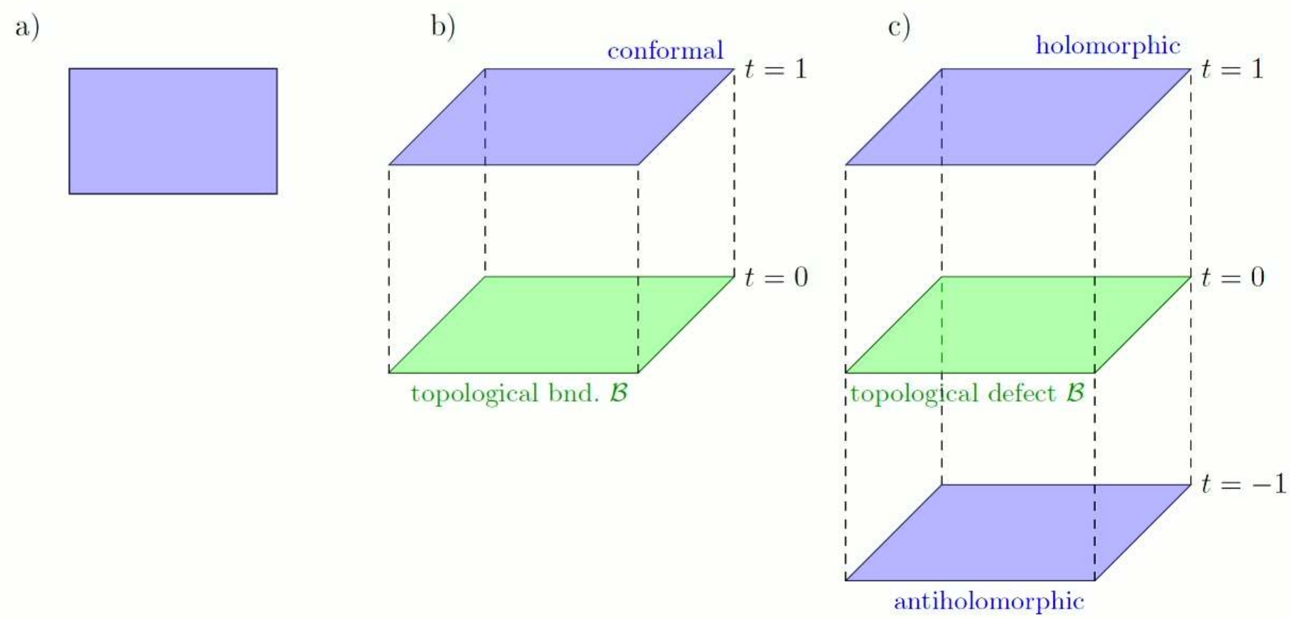


- Topological ops in QFT: Neumann @ top boundary
- Non topological: Dirichlet boundary conditions

All lines = topological anyons of 3D TFT

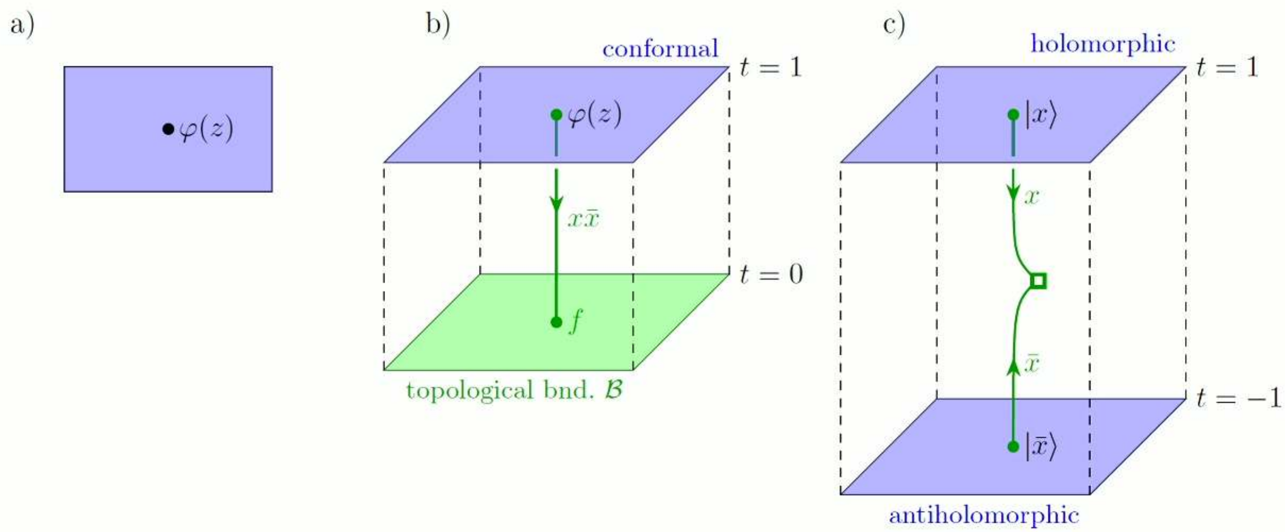
Chiral TFT

In 2d CFT I like chiral TFT:

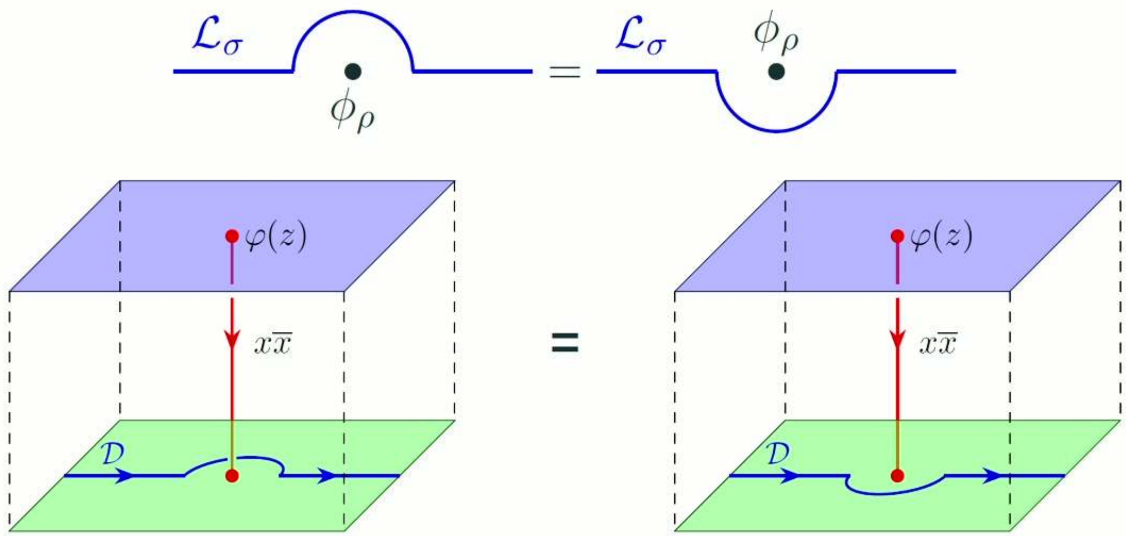


Chiral TFT

In 2d CFT I like chiral TFT:



Example: commutation condition from before



Commuting defects and where to find them

Hamiltonian of perturbed CFT: $H(\mu) = H_0 + H_{\text{pert}}(\mu)$

$$H_0 = \frac{2\pi}{L}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right) \ , \quad H_{\text{pert}}(\mu) = 2i\mu \int_0^L \varphi(s) ds \ .$$

$\mathcal{L}$ topological in H:

$$[H(\mu), \mathcal{D}] = 0$$

$$\underbrace{[H_0, \mathcal{L}] = 0}_{\text{Top. in CFT}}, \qquad \& \qquad \underbrace{[\varphi, \mathcal{L}] = 0}_{\text{Comm. with def.}}$$

Commuting defects and where to find them

Hamiltonian of perturbed CFT: $H(\mu) = H_0 + H_{\text{pert}}(\mu)$

$$H_0 = \frac{2\pi}{L}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right) \ , \quad H_{\text{pert}}(\mu) = 2i\mu \int_0^L \varphi(s) ds \ .$$

$\mathcal{D}$ conserved in H:

$$[H(\mu), \mathcal{D}] = 0 \, ,$$

But: $[H_0, \mathcal{D}] \neq 0, \quad [\varphi, \mathcal{D}] \neq 0$

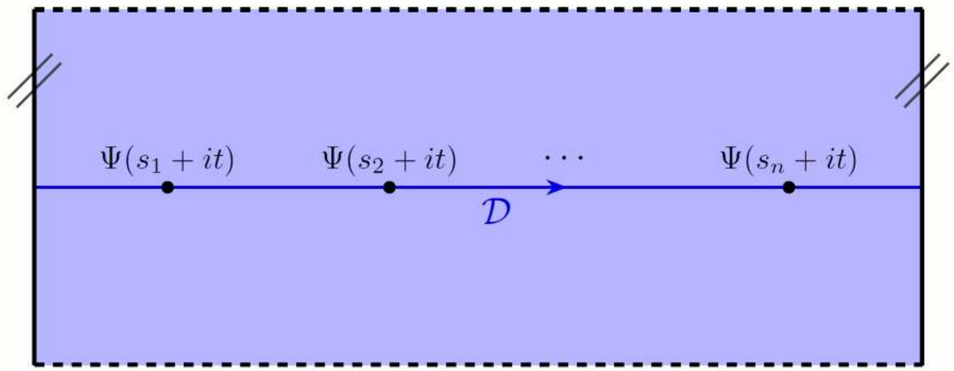
Weaker than topological! $\qquad [T(z), \mathcal{D}] \neq 0$

What kind of animals?

Perturbed defect operators

Perturb a topological defect $\mathcal{D}$ by (chiral) defect operators:

$$\underbrace{\psi(z)}_{\text{hol}}, \qquad \underbrace{\overline{\psi}(\bar{z})}_{\text{antihol.}}, \qquad \psi, \overline{\psi} \in \mathcal{D}, \qquad \varphi \sim \overline{\psi}\psi$$



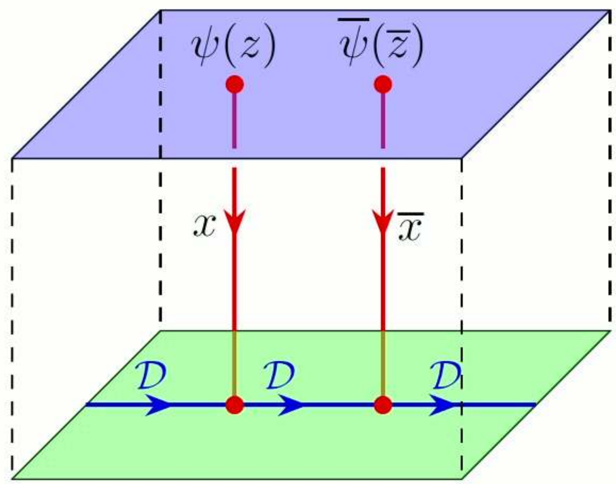
$$\mathcal{D}(\lambda\psi + \widetilde{\lambda}\overline{\psi}) = \exp\left(\int_0^L (\lambda\psi(s+it) + \tilde{\lambda}\bar{\psi}(s+it))\, ds \right),$$

s coordinate along defect, t position of defect

Perturbed defect operators

Perturb a topological defect $\mathcal{D}$ by (chiral) defect operators:

$$\underbrace{\psi(z)}_{\text{hol}}, \qquad \underbrace{\overline{\psi}(\bar{z})}_{\text{antihol.}}, \qquad \psi, \overline{\psi} \in \mathcal{D}, \qquad \varphi \sim \overline{\psi}\psi$$



$$\mathcal{D}(\lambda\psi + \widetilde{\lambda}\overline{\psi}) = \exp\left(\int_0^L (\lambda\psi(s+it) + \tilde{\lambda}\bar{\psi}(s+it))\, ds\right),$$

s coordinate along defect, t position of defect

Bulk commutation condition

Conservation law condition in QFT:

$$\left[\, H(\mu)\,,\ \mathcal{D}(\lambda\psi + \widetilde{\lambda}\overline{\psi})\,\right] \;=\; 0$$

Equivalent to condition in CFT at any order in $\lambda, \widetilde{\lambda}$:

$$\boxed{\mu\,[\mathcal{D},\varphi] \;=\; \lambda\widetilde{\lambda}\,[\psi,\overline{\psi}]}$$

Remember: Topological $\longleftrightarrow [\mathcal{L},\varphi] = 0$

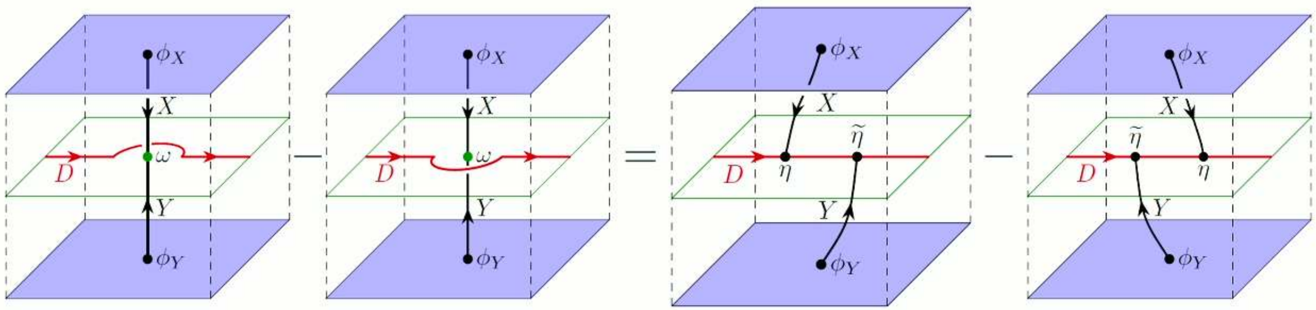
$\rightarrow$ "Adiabatically" deform Line & Bulk simulatenoeusly

$\rightarrow$ Rigidly translational invariant

$\rightarrow$ Often not renormalized (E.g. $h < \frac{1}{2}$ in $\mathcal{M}(p,q)$)

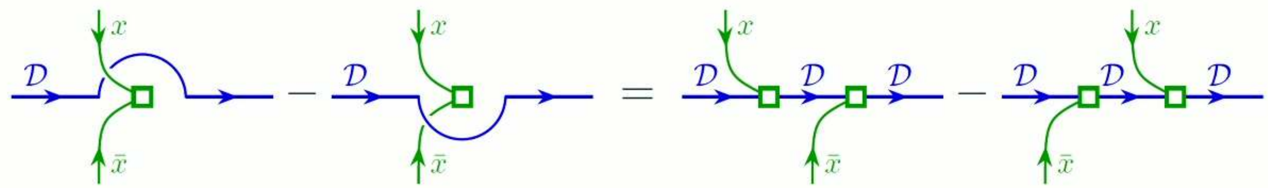
Condition for translation-invariant

$$\boxed{\text{Transl. invariant:} \qquad \mu\,[\mathcal{D}, \varphi] \;=\; \lambda\widetilde{\lambda}\,[\psi, \overline{\psi}]}$$



Conditions

Bulk commutation condition:



Using standard TQFT rules:

$$\boxed{\delta_{ac}\left(\mathrm{R}^{(xa)b}-\frac{1}{\mathrm{R}^{(ax)b}}\right)\mathrm{F}_{1b}^{(ax\bar{x})a} \;=\; \delta_{b\in\mathcal{D}}\,\kappa_{ab}\widetilde{\kappa}_{bc}-\sum_{d\in\mathcal{D}}\mathrm{F}_{db}^{(xa\bar{x})c}\,\widetilde{\kappa}_{ad}\kappa_{dc}}$$

$$\forall a,c\in\mathcal{D},\qquad b\in a\otimes x$$

Solutions in Minimal models

Minimal model $M(p,q)$	perturbing bulk field φ	weights $h=\bar{h}$ of φ	topological defect solving $(\boxed{4})$
$q \geq 3$	$(1,2)$	$h_{1,2}=\frac{3}{4}t-\frac{1}{2}$	$(1,1)\oplus(1,2)$
$q \geq 4$	$(1,3)$	$h_{1,3}=2t-1$	$(1,2)$
$q \geq 6$	$(1,5)$	$h_{1,5}=6t-2$	$(1,3)$
$q=9,10,18$	$(1,7)$	$h_{1,7}=12t-3$	$(1,5)$

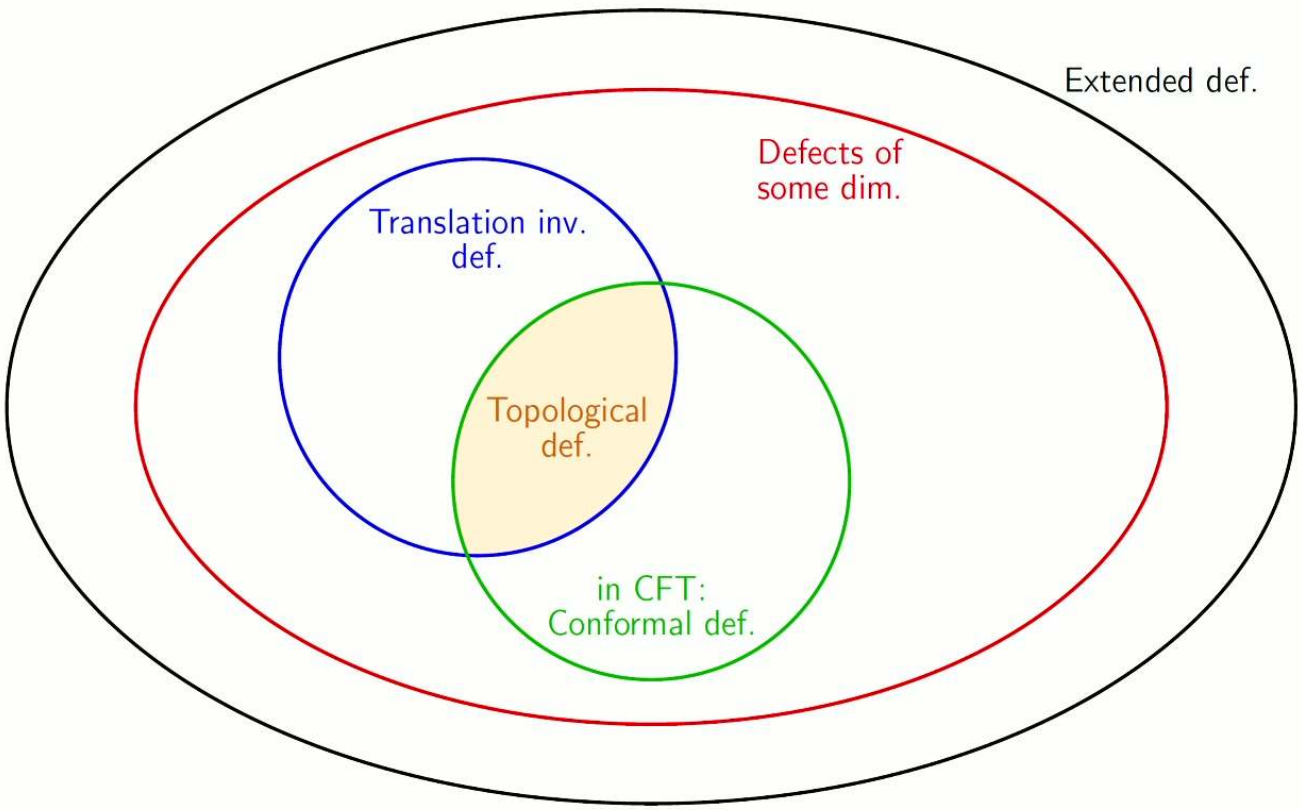
And many others. . . E.g. (1,9) on (1,5), (1,6) (1,7), etc...

$$\left[\mathcal{D}(\lambda,\mu/\lambda),\mathcal{D}(\lambda',\mu/\lambda')\right]=0,\qquad \lambda\neq\lambda'$$

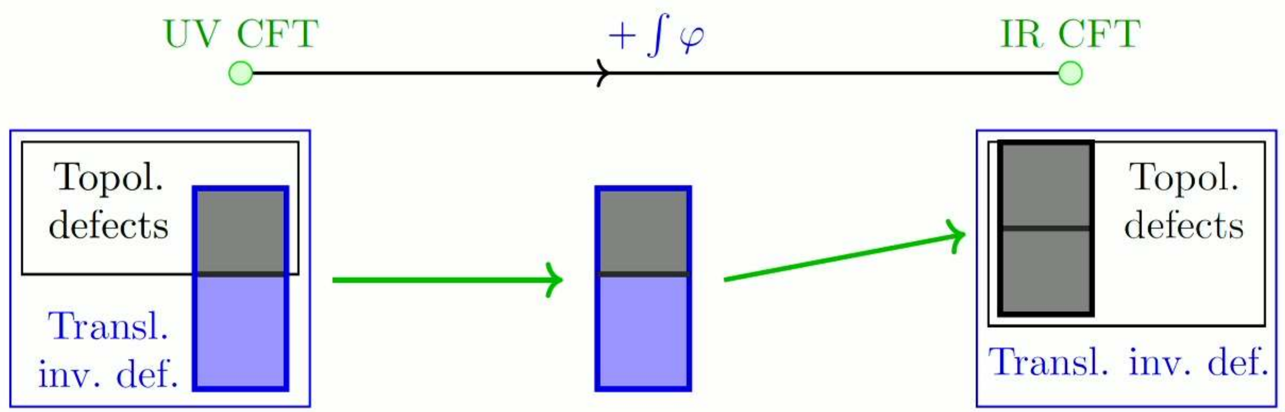
Infinitely many non-local conserved charges!
(Complicated condition)

$(1,7)$ deformation has no local conserved charges [BLZ]
But NEW non-local conserved charges!!

General picture



General picture



Where do those new defect flow?

New defects = New constraints

[FA, Runkel, Watts, Konenchy, Runkel '07]

Warm up Chiral deformation of $\mathcal{M}(p,q)$:

E.g. chiral $\psi = \psi_{(1,3)}(z)$, $\mathcal{D}_s(\lambda) := \mathcal{L}_{(1,s)}(\lambda\psi)$

$\mathcal{D}(\lambda\psi)$ satisfies Y-systems!

$$\boxed{\mathcal{D}_2(\lambda)\mathcal{D}_s(\zeta^{\pm s}\lambda) = \mathcal{D}_{s-1}(\zeta^{\pm(s+1)}\lambda) + \mathcal{D}_{s+1}(\zeta^{\pm(s-1)}\lambda)\,, \qquad \zeta = e^{i\pi p/q}}$$

Implies Hirota: $\mathcal{D}_s(\zeta\lambda)\mathcal{D}_s(\zeta^{-1}\lambda) = 1 + \mathcal{D}_{s-1}(\lambda)\mathcal{D}_{s+1}(\lambda)$

- ⋆ $p-1$ Stokes lines in $\mathcal{M}(p,-)$
- ⋆ In the IR: $\mathcal{D}_s(\lambda) = \mathcal{D}_s^\infty \exp(a_s\lambda^\alpha)$ $\qquad \alpha = \frac{1}{2(1-p/q)}$
- ⋆ We can solve asymptotically!

Solutions of Hirota in $\mathcal{M}(p,q)$

$$\boxed{\mathcal{D}_s(\zeta\lambda)\mathcal{D}_s(\zeta^{-1}\lambda) = 1 + \mathcal{D}_{s-1}(\lambda)\mathcal{D}_{s+1}(\lambda)}$$

Form asymptotic semi-ring: $\mathcal{D}_s(\lambda) = \mathcal{D}_s^{\infty} \exp(a_s\lambda^{\alpha})$

- $\mathcal{M}(2,-)$ all flow to identity: $(\theta = \arg(\lambda))$

  $$\mathcal{D}_s(\lambda) \to \mathrm{Id}\exp\big(a_s\cos(\overline{\alpha\theta})\big), \qquad a_s{=}a_s\frac{\sin((s-1)\pi p/q)}{\sin(\pi p/q)}$$

- $\mathcal{M}(3,-)$:
  $\mathrm{Id}\exp\big(a_s\cos(\overline{\alpha\theta})\big) + \mathcal{D}_{q-1}\exp\Big(a_{q-s}\cos\Big(\overline{\alpha(\theta-\pi)}\Big)\Big)$
- etc... We can do any chiral on any defects ...
- We (**Gerard**) check with TCSA!

Concluding remarks

News from Minimal Models!! [Belavin, Polyakov Zamolodchikov, 1984]

- New flows predicted by non-invertible symmetries
  - → From Virasoro to $\mathcal{W}_N$ and beyond! 
  - → Truncations $Y_{N,M,L} \cap Y_{0,P,Q}$, Grassmanian VOA [Eberhardt, Prochazka], compact irrational CFT [Antunes, Behan, Rong], WZW? [Levine. . .] etc...
- Putative new NLIE [Ambrosino, Negro] (Backup if interested!!)
- Virasoro ADE classification [Cappelli,Itzykson,Zuber][Nakayama,Tanaka]
  - → Modular invariant classification for $\mathcal{W}_N$?
- Beyond the topological framework → Transl. invariant
- IR defects from solving Y-systems

Thanks for your attention!