Title: Fully Extended 3d Mirror Symmetry

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Abstract:

3d N=4 gauge theories admit topological A- and B-twists; boundary conditions for these twisted theories are expected to form 2-categories. The 3d mirror symmetry duality should identify the A-twist of such a theory with a B-twist of a dual theory. This duality can be used to understand the A-side 2-category in terms of B-side 2-categories which are more well-understood mathematically. We will discuss some progress in describing these 2-categories and formulating 3d mirror symmetry as an equivalence of 2-categories. This is based on ongoing work with Justin Hilburn.

Fully extended 3d mirror symmetry

w/ J. Hilburn

Today. A, B twists of 3d $\mathcal{N}=4$ theories

3d mirror symmetry: A-twist of T = B-twist of $T^\vee$

Goal. Phrase this as an equivalence of 2-cats. of boundary conditions

Q: How to produce dual pairs of 3d $\mathcal{N}=4$ theories?

Idea [Gaiotto, Hilburn-Yoo, Ben-Zvi-Sakellaridis-Venkatesh]:

G reductive grp. $G \overset{\text{Ham}}{\curvearrowright} \mathcal{M}(\overset{?}{=} T^*V) \rightsquigarrow$ b.c. for 4d $\mathcal{N}=4$ SYM$_G$

Interval compactification on 2 of these $\overset{L_1, L_2}{\rightsquigarrow}$ 3d $\mathcal{N}=4$ thy T.

$$L_1 \xrightarrow{4d\ G} L_2 = T$$

$\downarrow$ S-duality

$$L_1^\vee \xrightarrow{4d\ G^\vee} L_2^\vee = T^\vee$$

Ex. $G \curvearrowright T^*G/\!\!/_\chi N$ Nahm pole

$\downarrow$ S-duality

- $G \curvearrowright T^*pt$ Neumann bc
- $G \curvearrowright T^*(G/B) \overset{\text{S-duality}}{\longleftrightarrow} G^\vee \curvearrowright T^*(G^\vee/N)$

$\rightsquigarrow$ Conj. $4d\,A_G(T^*(G/N), T^*(G/N)) \simeq 4d\,B_{G^\vee}(T^*(G^\vee/B), T^*(G^\vee/B))$.

Equality of monoidal 2-cats.

Tamely ramified GLC.

Ex. $G=\mathbb{C}^\times$, $G^\vee=\mathbb{C}^\times$ $\quad G \curvearrowright T^*\mathbb{C} \longleftrightarrow G^\vee \curvearrowright T^*\mathbb{C}$

Conj. $4\mathrm{dA}_{\mathbb{C}^\times}(T^*\mathbb{C}, T^*\mathbb{C}^\times) \simeq 4\mathrm{dB}_{\mathbb{C}^\times}(T^*\mathbb{C}, T^*\mathbb{C}^\times)$

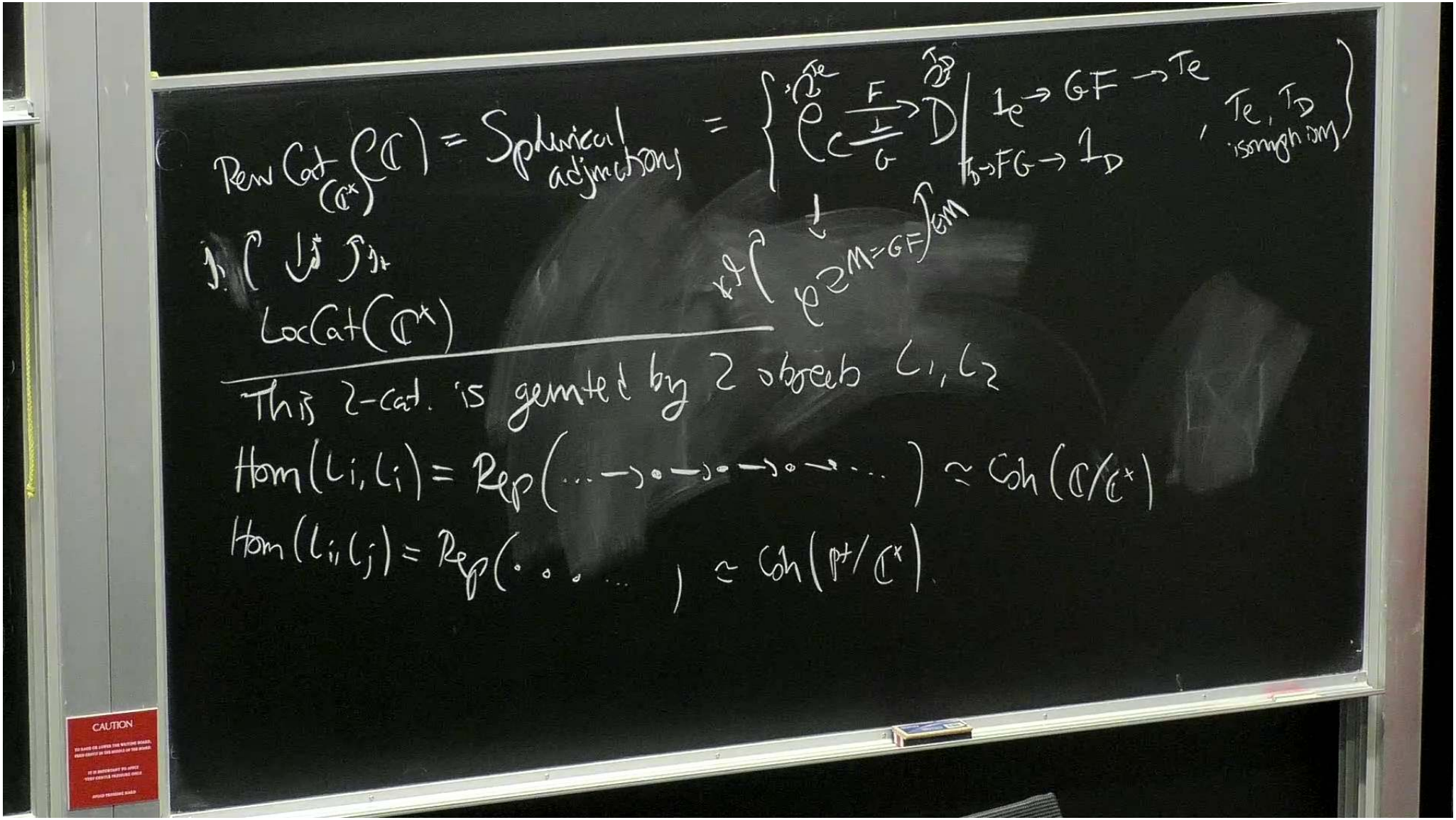
$\| \qquad\qquad \|$

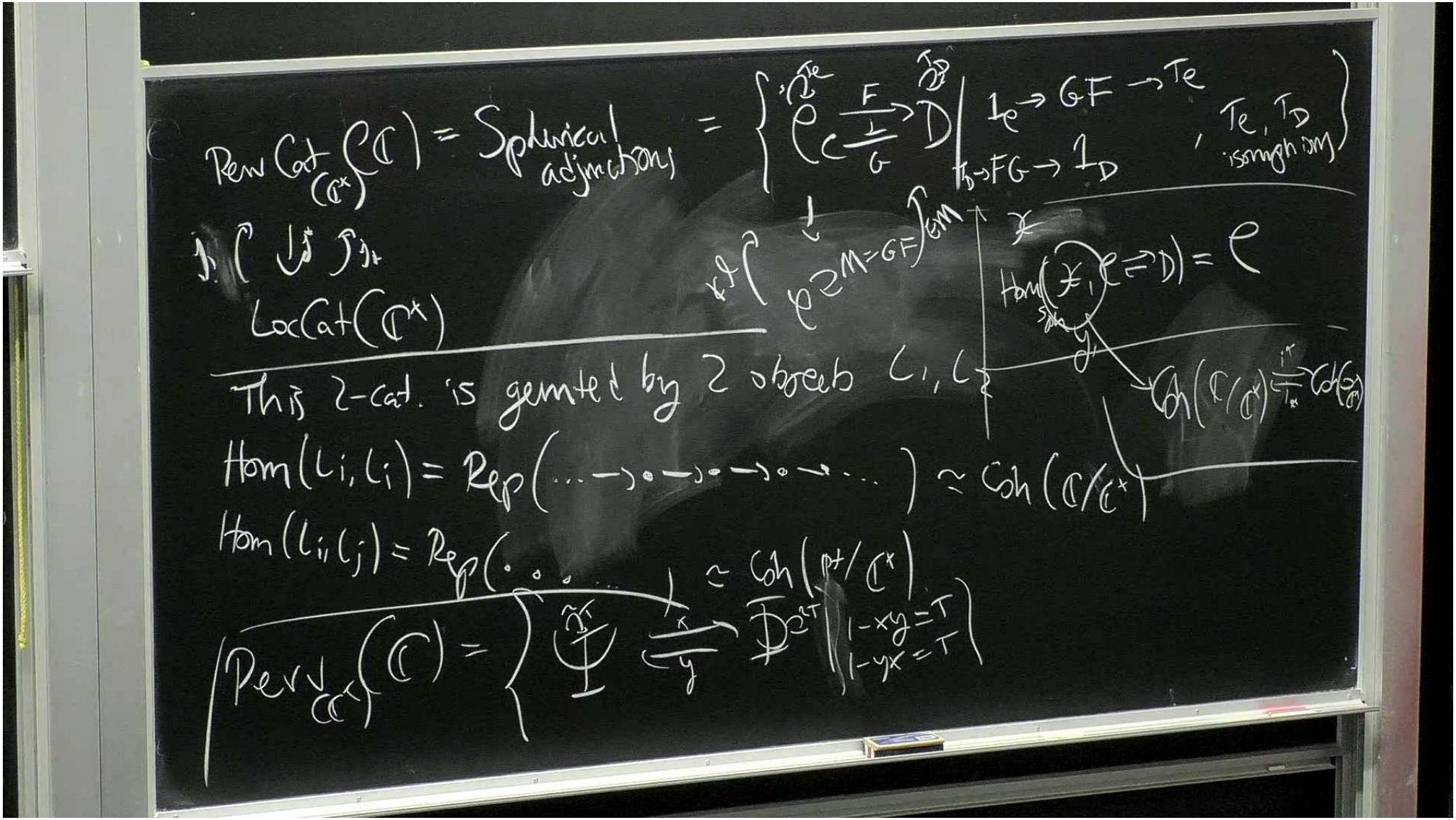
$3\mathrm{dA}(T^*\mathbb{C}) \simeq 3\mathrm{dB}(T^*(\mathbb{C}/\mathbb{C}^\times))$

Thm [G.-Hilburn-Mazel-Gee]: $\mathrm{PervCat}_{(\mathbb{C}^\times)^2}(\mathbb{C}) \simeq \mathrm{IndCoh}(\mathrm{at}^{\mathbb{C}^\times}(\mathbb{C}))$

Thm [G.-Hilburn]: $\mathrm{Sh}_{(\mathbb{R}^\times\mathbb{C}^\times)}(\mathbb{C}) \simeq \mathrm{IndCoh}(\mathrm{Maps}(S^1, \mathbb{C}/\mathbb{C}^\times))$

$t \int S^1$ [BZNP]


Rem Cat$_{(\mathbb{C}^*)}$(C) = Spherical adjunctions = { $\mathcal{C} \rightleftarrows \mathcal{D}$ | $1_{\mathcal{C}} \to GF \to T_{\mathcal{C}}$, $1_{\mathcal{D}} \to FG \to T_{\mathcal{D}}$ | $T_{\mathcal{C}}, T_{\mathcal{D}}$ isomorphisms }
LocCat($\mathbb{P}^1$)
This 2-cat. is generated by 2 objects L_1, L_2
Hom(L_i, L_i) = Rep(... → • → • → • → ...) ≃ Coh($\mathbb{P}^1/\mathbb{C}^*$)
Hom(L_i, L_j) = Rep(• • • ...) ≃ Coh($\mathbb{P}^1/\mathbb{C}^*$)
CAUTION


Rem Cat_{C*}(C) = Spherical adjunctions = { C ⇄ D | 1_C → GF → T_C , T_D → FG → 1_D , T_C, T_D isomorphisms }
Loc Cat(C*)
This 2-cat. is gen'ted by 2 objects C_1, C_2
Hom(C_i, C_i) = Rep(... → • → • → • → ...) ≃ Coh(C/C*)
Hom(C_i, C_j) = Rep(• • ... •) ≃ Coh(pt/C*)
Perv_{C*}(C) = { X ⇄ Y | -xy = T, -yx = T }