Title: Bounding the asymptotic quantum value of all multipartite compiled non-local games

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Abstract:

Non-local games are a powerful tool to distinguish between correlations possible in classical and quantum worlds. Kalai et al. (STOC'23) proposed a compiler that converts multipartite non-local games into interactive protocols with a single prover, relying on cryptographic tools to remove the assumption of physical separation of the players. While quantum completeness and classical soundness of the construction have been established for all multipartite games, quantum soundness is known only in the special case of bipartite games. In this paper, we prove that the Kalai et al.'s compiler indeed achieves quantum soundness for all multipartite compiled non-local games, by showing that any correlations that can be generated in the asymptotic case correspond to quantum commuting strategies. Our proof uses techniques from the theory of operator algebras, and relies on a characterisation of sequential operationally no-signalling strategies as quantum commuting operator strategies in the multipartite case, thereby generalising several previous results. On the way, we construct universal C*- algebras of sequential PVMs and prove a new chain rule for Radon-Nikodym derivatives of completely positive maps on C*-algebras which may be of independent interest.

Bounding the asymptotic quantum value of all multipartite compiled nonlocal games

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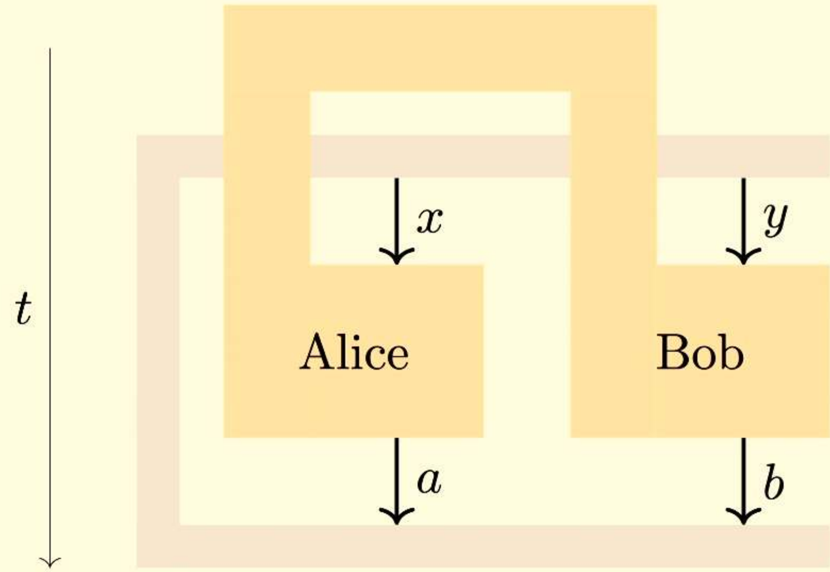
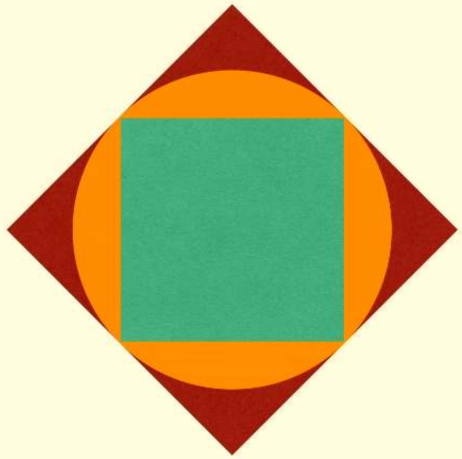
Perimeter Institute

20 November 2025

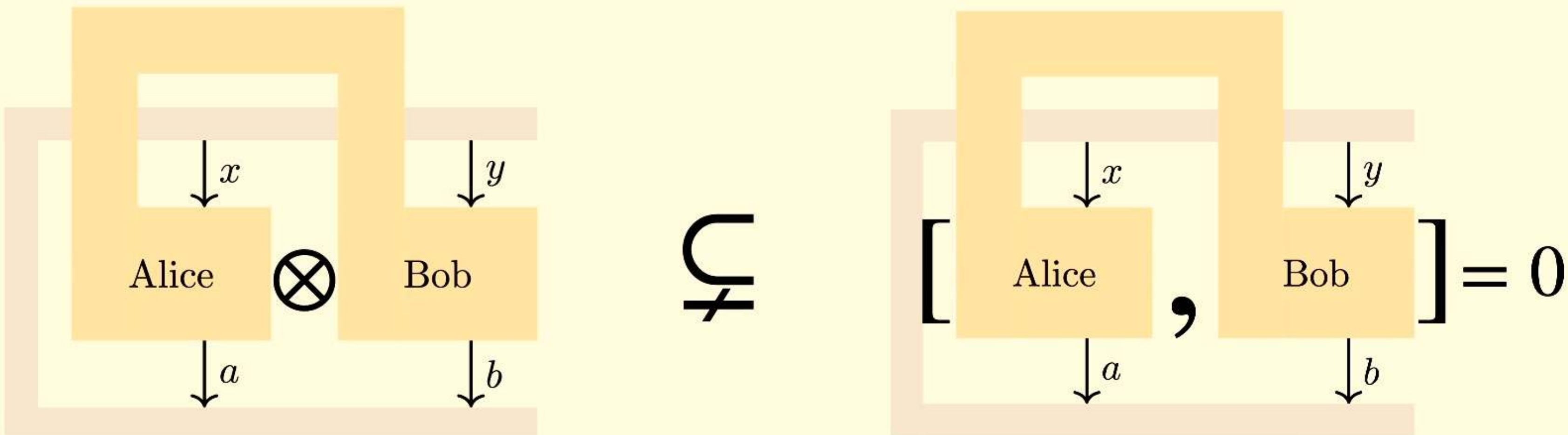
Non-locality 101





Non-locality 102

quantum vs. commuting operator



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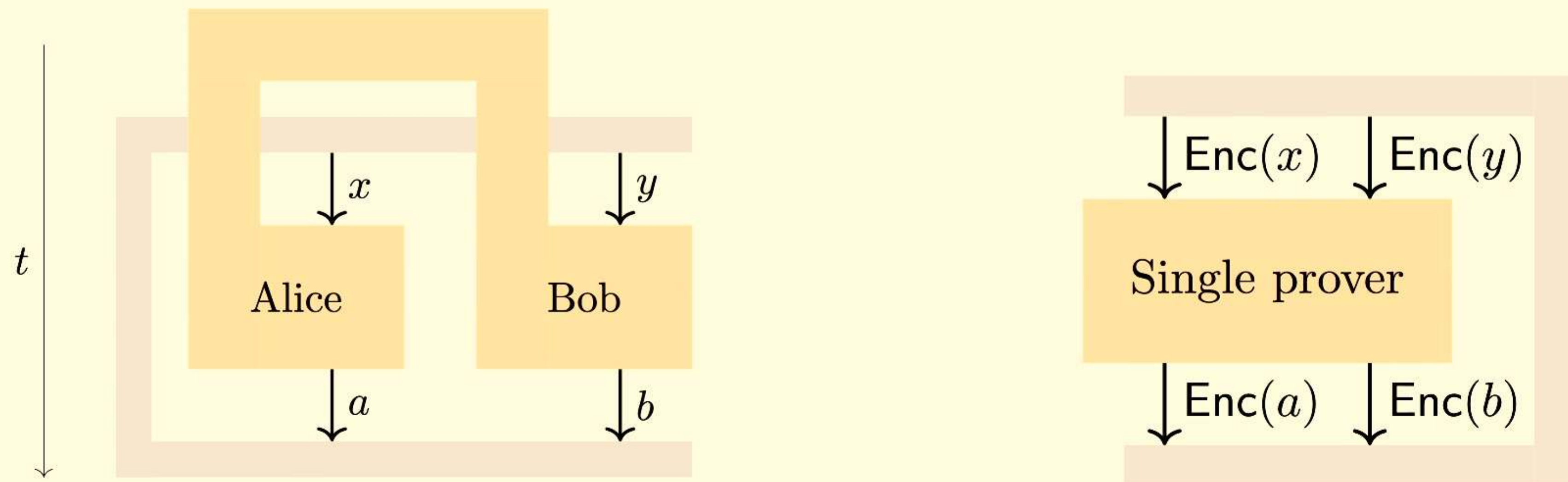
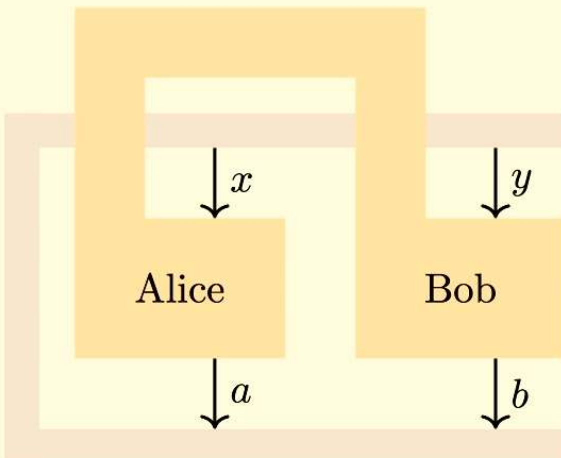
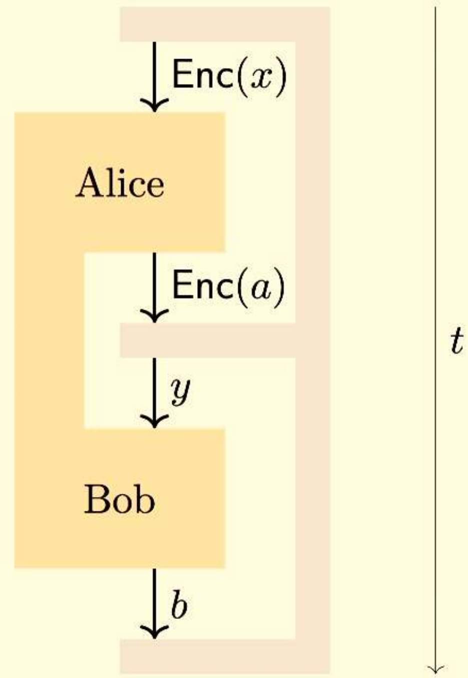
Removing space-like separation using cryptography

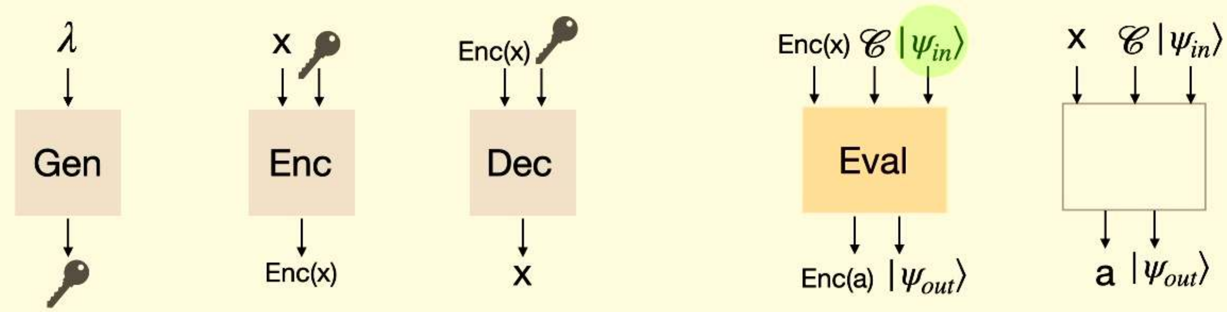


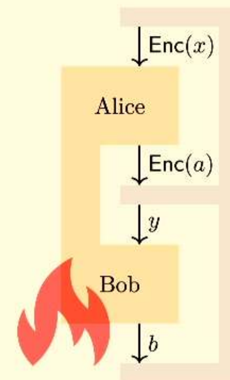
KLVY compiler





KLVY compiler : QFHE



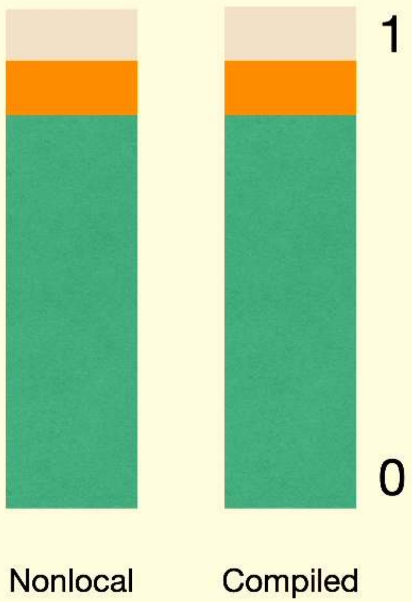


Quantum Fully Homomorphic Encryption scheme (QFHE) with

- correctness with auxiliary input
- IND-CPA security against QPT adversaries $Enc(x) \approx_{\lambda} Enc(x')$

Previous results

1. Classical soundness for all games [KLVY22]
2. Quantum completeness for all games [KLVY22]
3. Quantum soundness for some bipartite games
4. Asymptotic quantum soundness for all bipartite games [KMPSW24]



[KLVY22] arXiv: 2203.15877

[KMPSW24] arXiv: 2408.06711

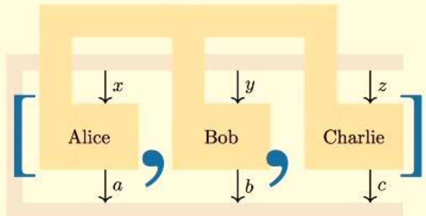
From 2 to 3, to k : why do we care

1. For classical it has already been done, and it was not more complicated

2. Multipartite (>2) quantum correlations are weird (e.g. post-quantum steering)

3. Space-like separation for multiple players is problematic, but multipartite games offer many other advantages

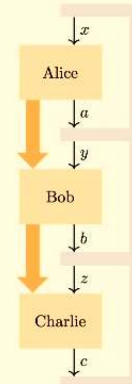
Our results



Non-local game

KLVY compiler

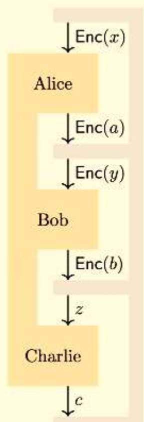
For all k-players games !



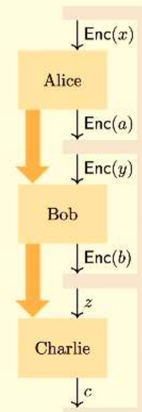
+ Operational-non-signalling states

+ Operational-non-signalling transformations

$\lambda \to \infty$

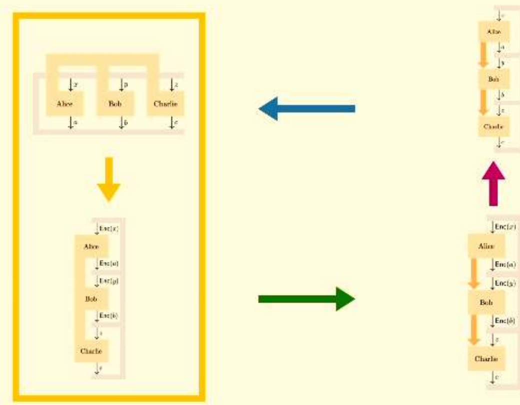


Sequential game



+ Constraints coming from the crypto

1. The compiler

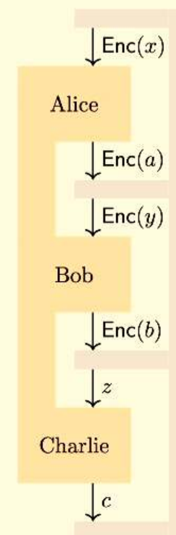


The first two interactions are encrypted, the third is in the clear

✓ Classical soundness

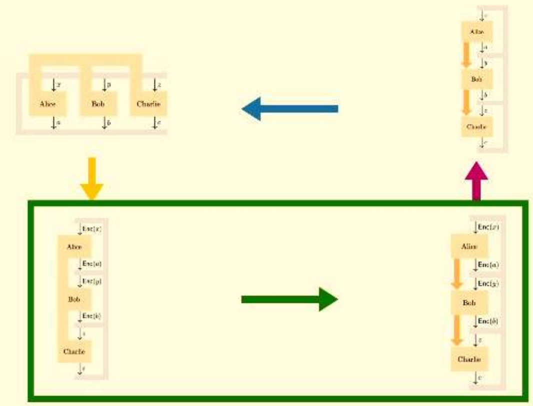
✓ Quantum completeness

✗ Quantum soundness ?

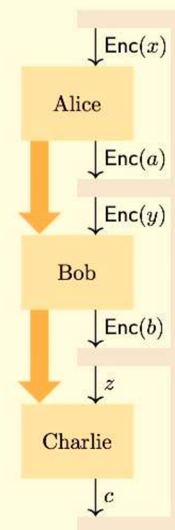


2. Constraints on the correlations

Quantum strategies



$$p_\lambda(a,b,c|x,y,z) = \mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{b|y}(\rho^\lambda_{a|x})\right]$$

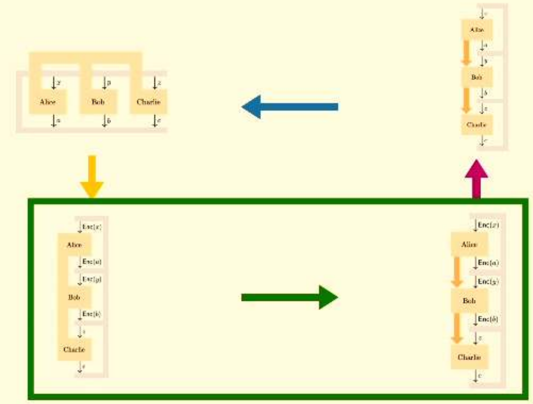


$\sum_c$: $\left|\mathrm{Tr}\left[\mathbb{1}\tilde{B}^\lambda_{b|y}(\rho^\lambda_{a|x})\right] - \mathrm{Tr}\left[\mathbb{1}\tilde{B}^\lambda_{b|y}(\rho^\lambda_{a|x})\right]\right| = 0$

Enc(y) $\approx_\lambda$ *Enc*(y') $\sum_b$: $\left|\mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{y}(\rho^\lambda_{a|x})\right] - \mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{y'}(\rho^\lambda_{a|x})\right]\right| \leq \mathsf{negl}(\lambda)$

Enc(x) $\approx_\lambda$ *Enc*(x') $\sum_a$: $\left|\mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{b|y}(\rho^\lambda_{x})\right] - \mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{b|y}(\rho^\lambda_{x'})\right]\right| \leq \mathsf{negl}(\lambda)$

2. Constraints on the correlations



Block Encodings and efficient estimations

Stronger constraints from IND-CPA

$Enc(x) \approx_\lambda Enc(x')$

$$\left|\mathrm{Tr}\left[B^\lambda_{b|y}\rho^\lambda_x\right] - \mathrm{Tr}\left[B^\lambda_{b|y}\rho^\lambda_{x'}\right]\right| \leqslant \mathsf{negl}(\lambda)$$

Th. Efficient estimation of QPT block-encodable operators

$$\rho^\lambda_x \approx_\lambda \rho^\lambda_{x'}$$

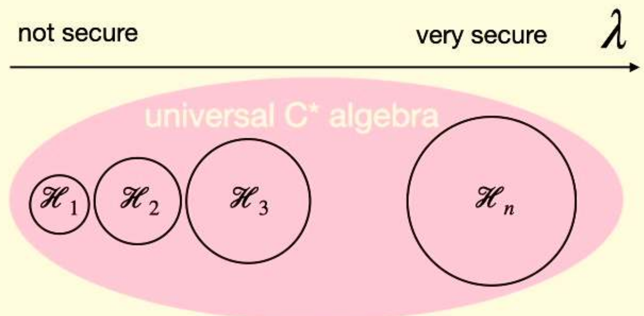
$Enc(y) \approx_\lambda Enc(y')$

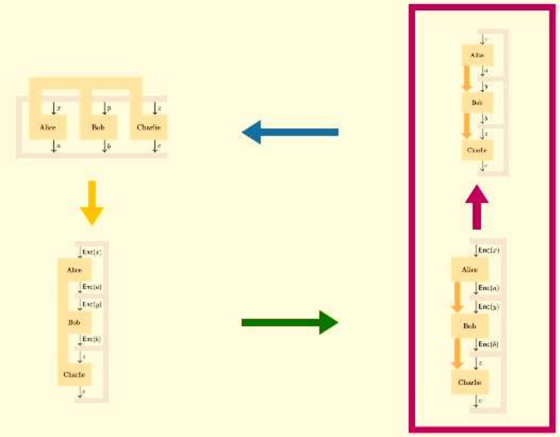
$$\left|\mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_y(\rho^\lambda_{a|x})\right] - \mathrm{Tr}\left[C^\lambda_{c|z}\tilde{B}^\lambda_{y'}(\rho^\lambda_{a|x})\right]\right| \leq \mathsf{negl}(\lambda)$$

$$\left|\mathrm{Tr}\left[P(\{C^\lambda_{c|z}\})\tilde{B}^\lambda_y\left(\mathcal{R}^\lambda\rho^\lambda_{a|x}\mathcal{L}^{\lambda,*}\right)\right] - \mathrm{Tr}\left[P(\{C^\lambda_{c|z}\})\tilde{B}^\lambda_{y'}\left(\mathcal{R}^\lambda\rho^\lambda_{a|x}\mathcal{L}^{\lambda,*}\right)\right]\right| \leq \mathsf{negl}(\lambda)$$

3. The asymptotic limit

Algebraic strategies

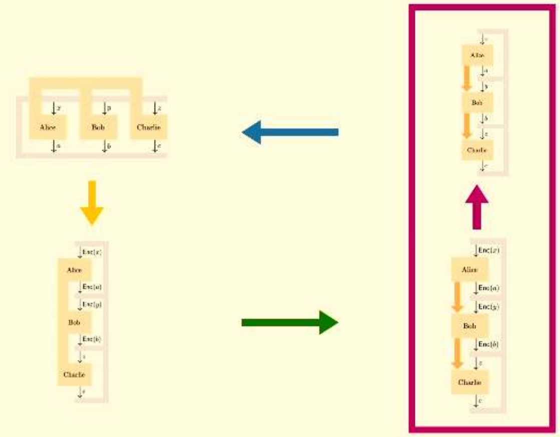


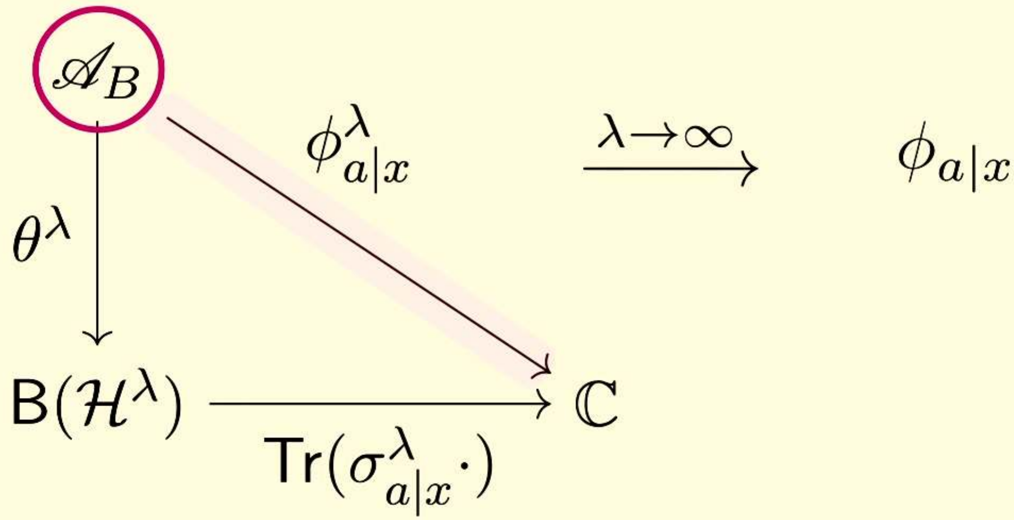


Space	Hilbert space $\mathcal{H}^{\lambda}_{\mathsf{A,B}}$	C*-algebra $\mathscr{A}, \mathscr{B}$	
Measurements	$C^{\lambda}_{c\|z} \in \mathsf{B}(\mathcal{H}^{\lambda}_{\mathsf{B}})$	$\mathfrak{m}_{c\|z} \in \mathscr{A}$	$\mathfrak{m}_{c\|z} \sim C_{c\|z}$
States	$\rho^{\lambda}_{a\|x} \in \mathsf{B}(\mathcal{H}^{\lambda}_{\mathsf{A}})$	$\phi^{\lambda}_{a\|x} : \mathscr{B} \to \mathbb{C}$	$\phi_{a\|x}(\cdot) \sim \mathrm{Tr}(\cdot \rho_{a\|x})$
Transformations	$\tilde{B}^{\lambda}_{b\|y} \in \mathsf{CP}(\mathcal{H}^{\lambda}_{\mathsf{A}}, \mathcal{H}^{\lambda}_{\mathsf{B}})$	$T_{b\|y} : \mathscr{A} \to \mathscr{B}$	$T_{b\|y} \sim \tilde{B}^{*}_{b\|y}$
Correlations	$\mathrm{Tr}(C^{\lambda}_{c\|z} \tilde{B}^{\lambda}_{b\|y}(\rho^{\lambda}_{a\|x}))$	$\phi^{\lambda}_{a\|x}(T_{b\|y}(\mathfrak{m}_{c\|z}))$	

3. The asymptotic limit

Universal C* algebras of PVMs [KMPSW]





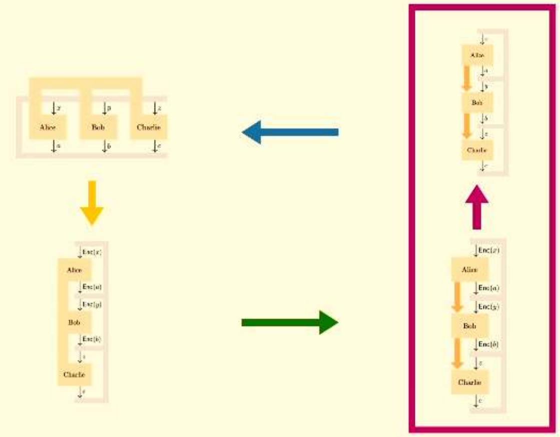
Universal C* algebras of PVMs

Generated by $e_{c|z} \in \mathscr{A}_C$ s.t.

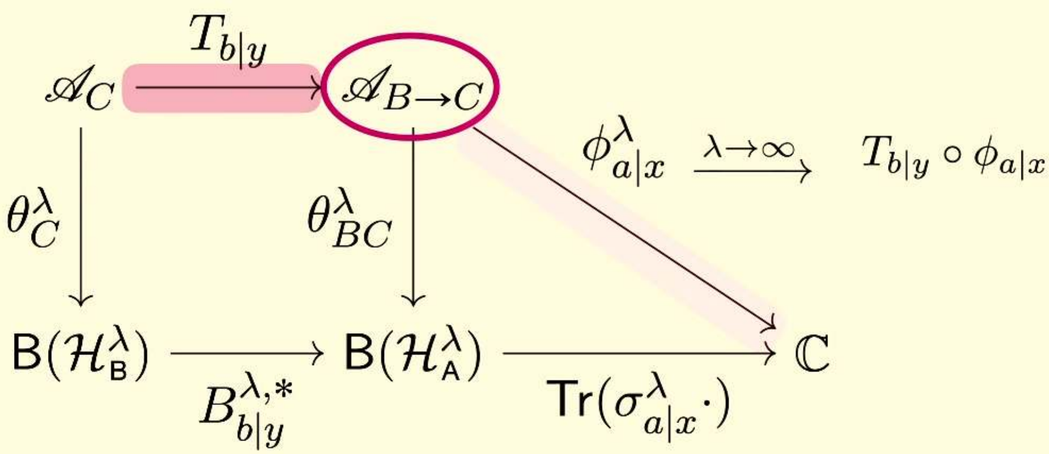
1. Self-adjoint
2. Positive
3. $\sum_c$ sums to identity
4. (Projections)

3. The asymptotic limit

Universal C* algebras of sequential PVMs



$$T_y \circ \phi^\lambda_{a|x} \approx_\lambda T_{y'} \circ \phi^\lambda_{a|x}$$



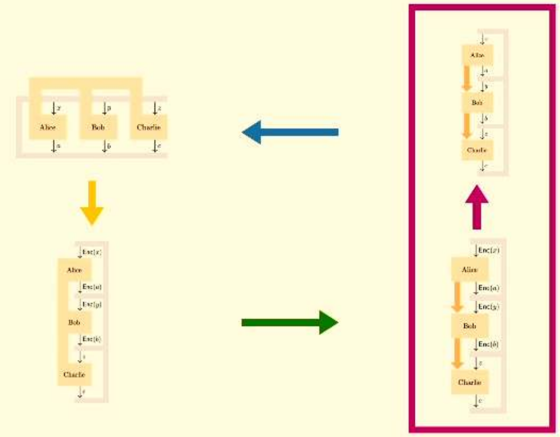
Universal C* algebras of sequential PVMs

Generated by $f_{bc|yz} \in \mathscr{A}_{B\to C}$ s.t.

1. Self-adjoint
2. Positive
3. $\sum_{bc}$ sums to identity
4. (Projections)
5. $\sum_c f_{bc|yz} = \sum_c f_{bc|yz'}$ ⟵

3. The asymptotic limit

Asymptotic constraints



Approximate constraints from IND-CPA

$\lambda \to \infty$

Exact constraints

$$\phi_x^\lambda \approx_\lambda \phi_{x'}^\lambda$$

$$\phi_x = \phi_{x'}$$

Operational-non-signalling states

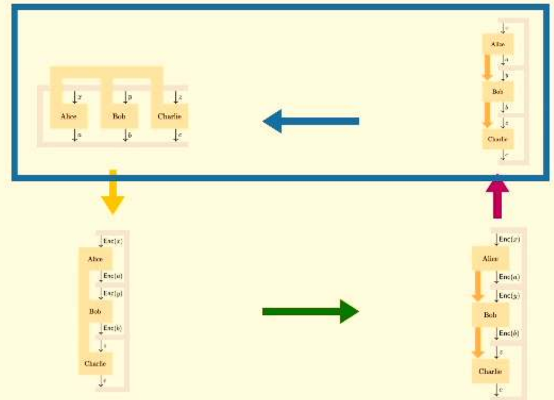
$$T_y \circ \phi_{a|x}^\lambda \approx_\lambda T_{y'} \circ \phi_{a|x}^\lambda$$

$$T_y = T_{y'}$$

Operational-non-signalling transformations

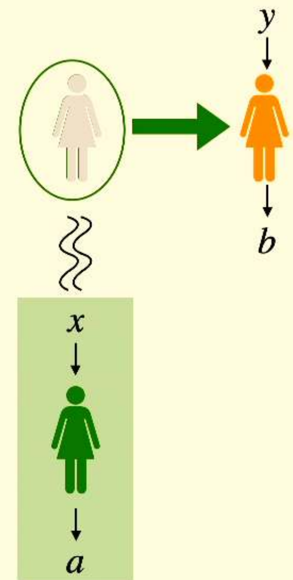
4. From k-sequential to k-non-local

2 players : prepare and measure



S-G-HJW theorem

$$\sum_a \rho_{a|x} = \rho \qquad \forall x \qquad \Leftrightarrow \qquad \rho_{a|x} = \mathrm{Tr}_k\left((A_{a|x} \otimes \mathbb{1})\sigma\right)$$

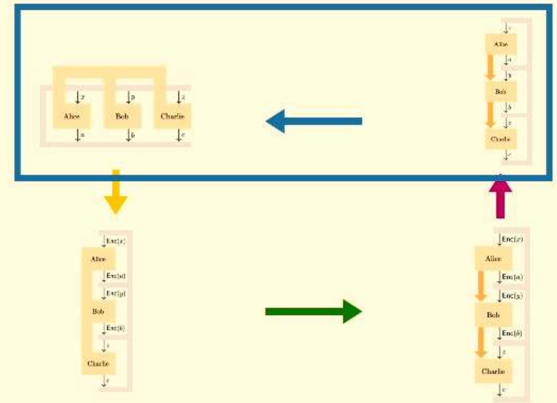


Radon-Nikodym (RN) theorem for states

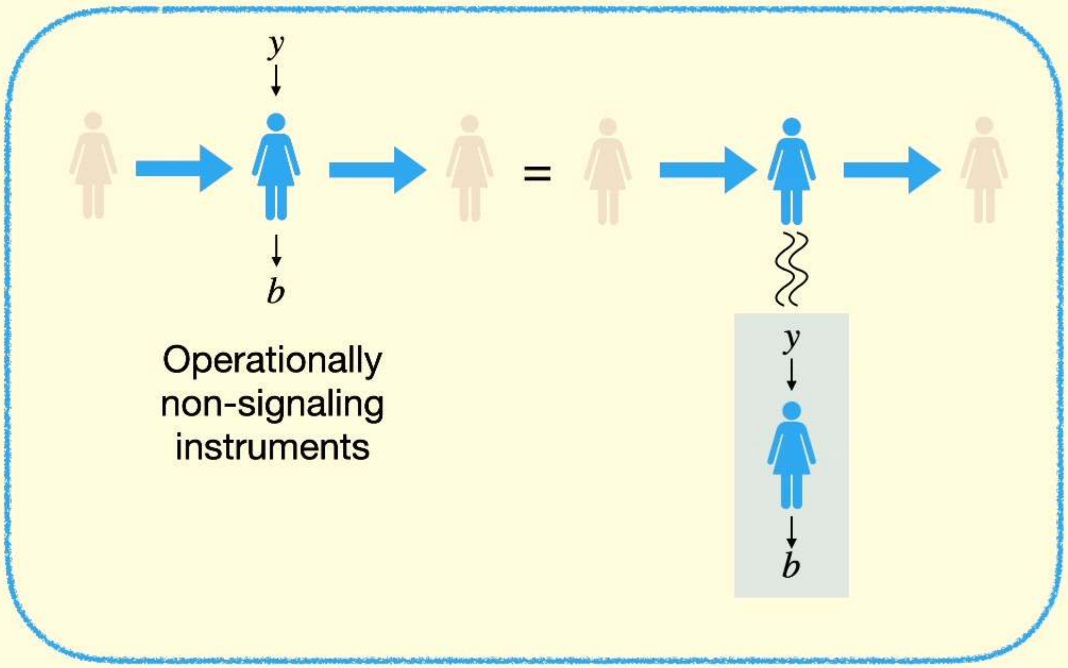
$$\sum_a \phi_{a|x}(\mathfrak{a}) = \phi(\mathfrak{a}) \qquad \forall x \qquad \Leftrightarrow \qquad \phi_{a|x}(\mathfrak{a}) = \langle \Omega_\phi | \, D_{a|x} \pi_\phi(\mathfrak{a}) \, | \Omega_\phi \rangle$$

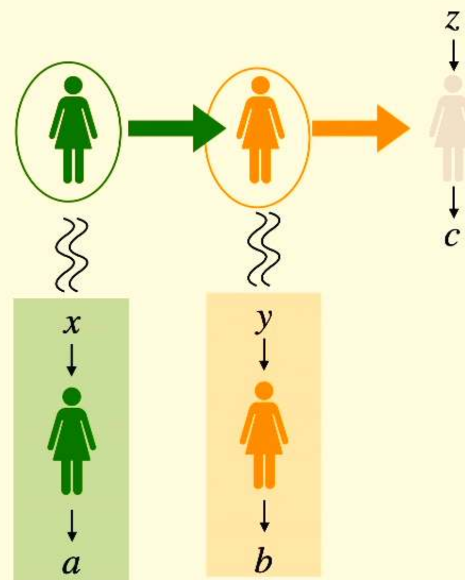
$$[D_{a|x}, \pi_\phi(\mathfrak{a})] = 0 \qquad \forall \mathfrak{a} \in \mathscr{A}$$

4. From k-sequential to k-non-local



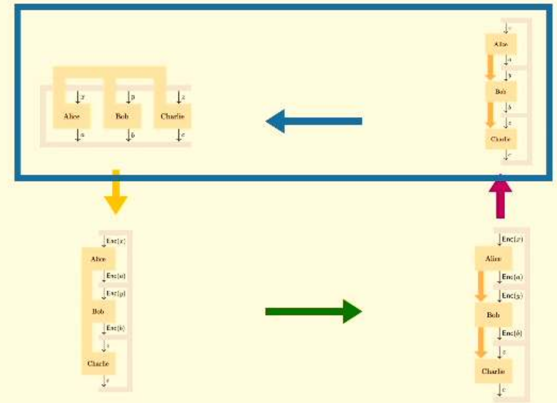
3 players : prepare, transform, and measure

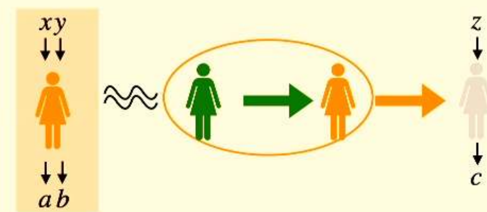




4. From k-sequential to k-non-local

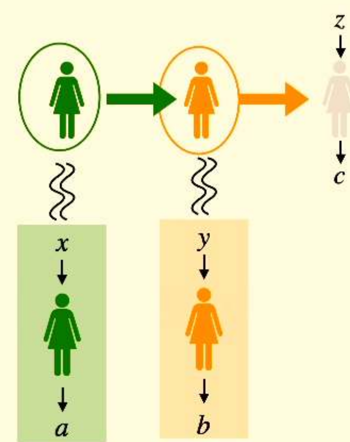
3 players : the need for the chain rule





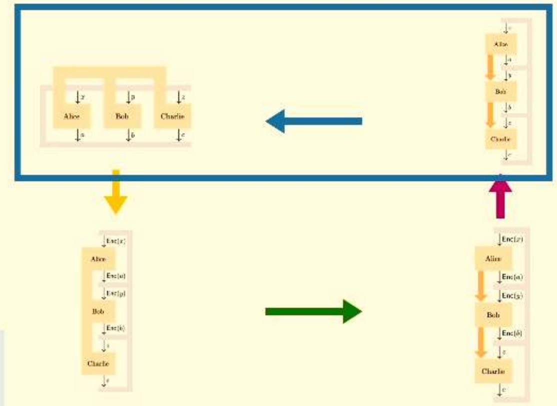
$$\phi_{a|x}(T_{b|y}(\mathfrak{m}_{c|z})) = \langle\Omega_{\phi\circ T}|\, D_{ab|xy}\pi_{\phi\circ T}(\mathfrak{m}_{c|z})\,|\Omega_{\phi\circ T}\rangle$$

$$= \langle\Omega_{\phi}|\, D_{a|x} V_T^* D_{b|y}\pi_T(\mathfrak{m}_{c|z}) V_T\,|\Omega_{\phi}\rangle$$



$$= \langle\Omega_{\phi}|\, V_T^* \bar{D}_{a|x} D_{b|y}\pi_T(\mathfrak{m}_{c|z}) V_T\,|\Omega_{\phi}\rangle$$

4. From k-sequential to k-non-local

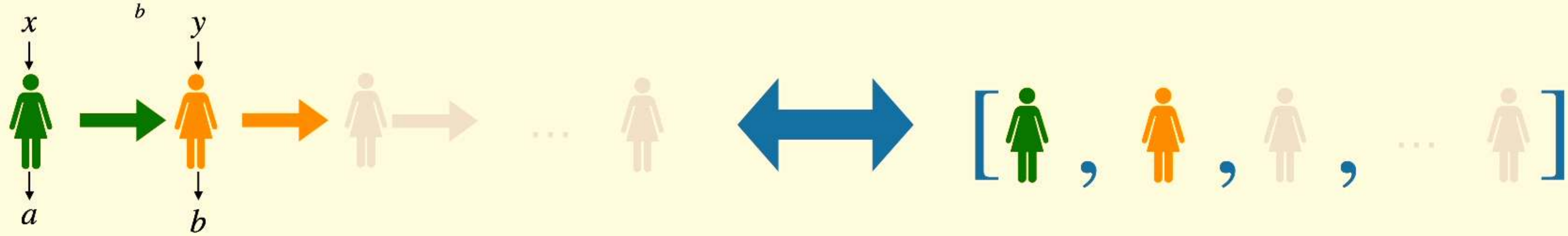


k players

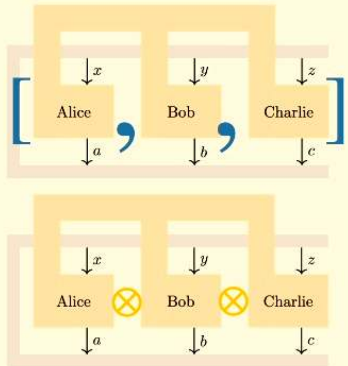
Theorem

Sequential k-player correlations, where every players implements operationallyNS operations, always admit a k-partite quantum model.

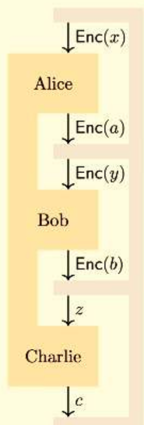
$$\sum_a \rho_{a|x} = \rho \quad \sum_b B_{b|y}(\cdot) = B(\cdot)$$



To sum up



KLVY compiler

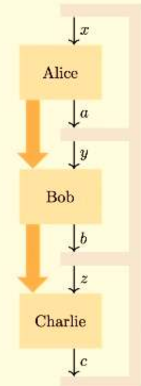


Non-local game

New chain rule for RN

Sequential game

New block-encoding theorem

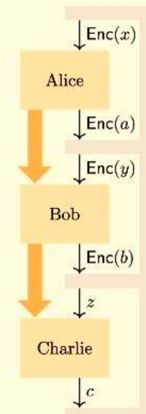


\+ ONS states

\+ ONS transformations

$\lambda \to \infty$

Universal C*algebra of sequential PVMs



\+ λ-approximate operational-non-signalling

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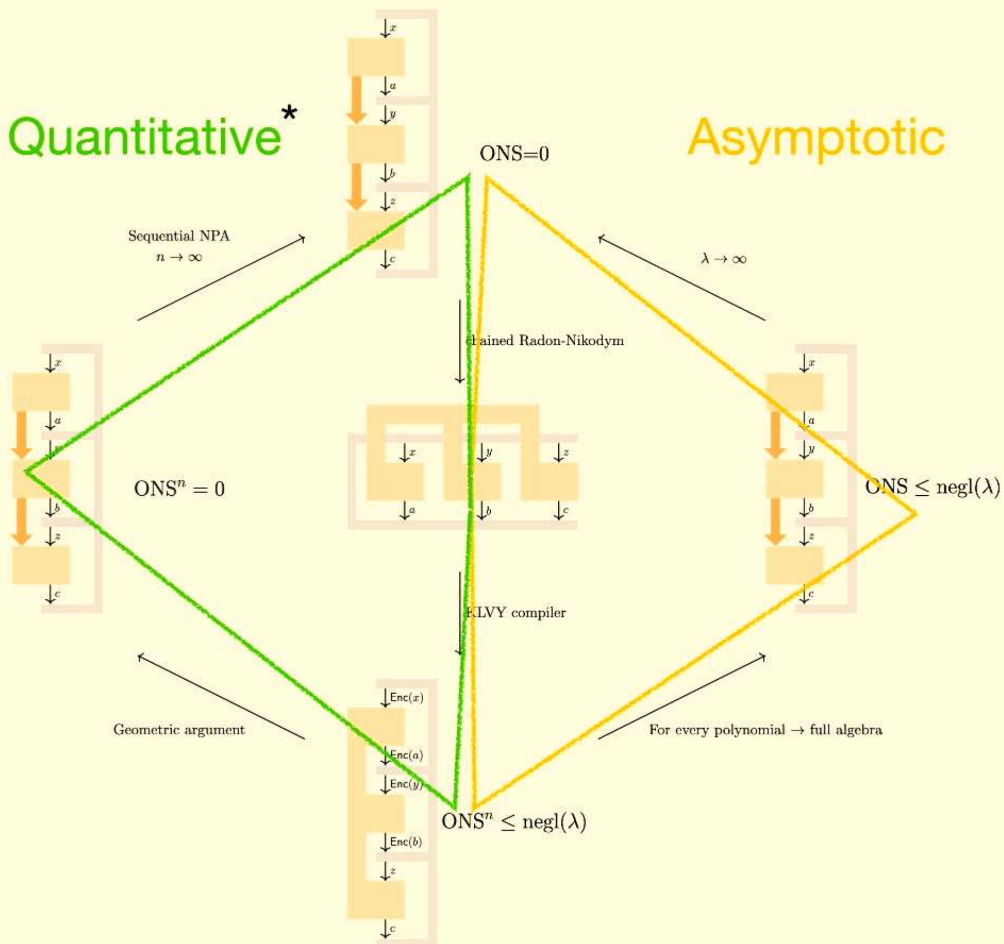
Conclusions

- Asymptotic quantum soundness of the KLVY compiler for all multipartite games
- Many new techniques to characterize q-instruments
- Convergence speed for finite levels of security?
- Is this the tightest bound we can get?



* arXiv:2509.25145,
Quantitative quantum soundness for all multipartite compiled nonlocal games,
M. Baroni, I. Klep, D. Leichtle, M.O. Renou, I. Šupić, L. Tendick, X. Xu

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Thanks !

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[KMPSW24] *A bound on the quantum value of all compiled nonlocal games*.
A. Kulpe, G. Malavolta, C. Paddock, S. Schmidt, M. Walter

[BLJŠ25] *Bounding the asymptotic quantum value of all multipartite compiled nonlocal games*.
M. Baroni, D. Leichtle, S. Janković, I. Šupić