Title: Bipartite quantum states admitting a causal explanation

Speakers: Minjeong Song

Collection/Series: Quantum Foundations

Subject: Quantum Foundations

Date: November 10, 2025 - 4:00 PM

URL: https://pirsa.org/25110098

Abstract:

Abstract: In this talk, I will present our recent results in https://arxiv.org/abs/2507.14278: The statistics of local measurements of joint quantum systems can sometimes be used to distinguish the spatiotemporal structure in which they were measured.
We first prove that every bipartite separable density matrix is temporally compatible with direct causal influence for arbitrary finite-dimensional quantum systems and measurements of a tomographically-complete class of observables, which includes all Pauli observables in the case of multi-qubit systems. Equivalently, if a bipartite density matrix is not temporally compatible with direct causal influence, then it must be entangled. We also provide an operational meaning for the temporal evolution consistent with such correlations in terms of a generalized dephasing channel and a pretty good measurement. The two temporal evolutions are Bayesian inverses of each other, which is different from them being Petz recovery maps of each other. Finally, we prove necessary and sufficient conditions for an arbitrary bipartite quantum state to be temporally compatible, thereby providing a temporal analogue of the positive partial transpose criterion valid for quantum systems of any dimension. If time permits, I will also introduce other works that I currently am working on, which is the implementation of Petz recovery maps.

I. Bipartite quantum states admitting a causal explanation

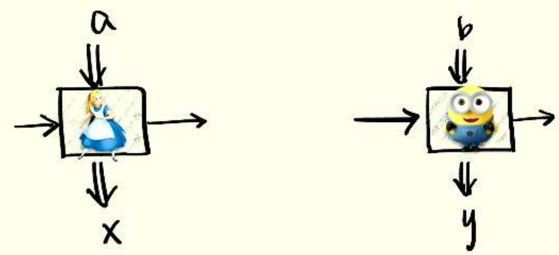
-Minjeong Song-

CQT, Singapore

CENTRE FOR QUANTUM TECHNOLOGIES SINGAPORE

10. Nov. 2025 at Perimeter Institute

Motivation Given the **observed** measurement statistics, can we infer a causal structure?



In pioneer works... some quantum measurement statistics are not compatible with certain causal structures.

ARTICLES
PUBLISHED ONLINE: 23 MARCH 2015 | DOI: 10.1038/NPHYS3266
nature physics

A quantum advantage for inferring causal structure

Katja Ried[1,2,3*†], Megan Agnew[1,2†], Lydia Vermeyden[1,2], Dominik Janzing[4], Robert W. Spekkens[3] and Kevin J. Resch[1,2]

The problem of inferring causal relations from observed correlations is relevant to a wide variety of scientific disciplines. Yet given the correlations between just two classical variables, it is impossible to determine whether they arose from a causal influence of one on the other or a common cause influencing both. Only a randomized trial can settle the issue. Here we consider the problem of causal inference for quantum variables. We show that the analogue of a randomized trial, causal tomography, yields a complete solution. We also show that, in contrast to the classical case, one can sometimes infer the causal structure from observations alone. We implement a quantum-optical experiment wherein we control the causal relation between two optical modes, and two measurement schemes—with and without randomization—that extract this relation from the observed correlations. Our results show that entanglement and quantum coherence provide an advantage for causal inference.

SCIENTIFIC REPORTS

OPEN

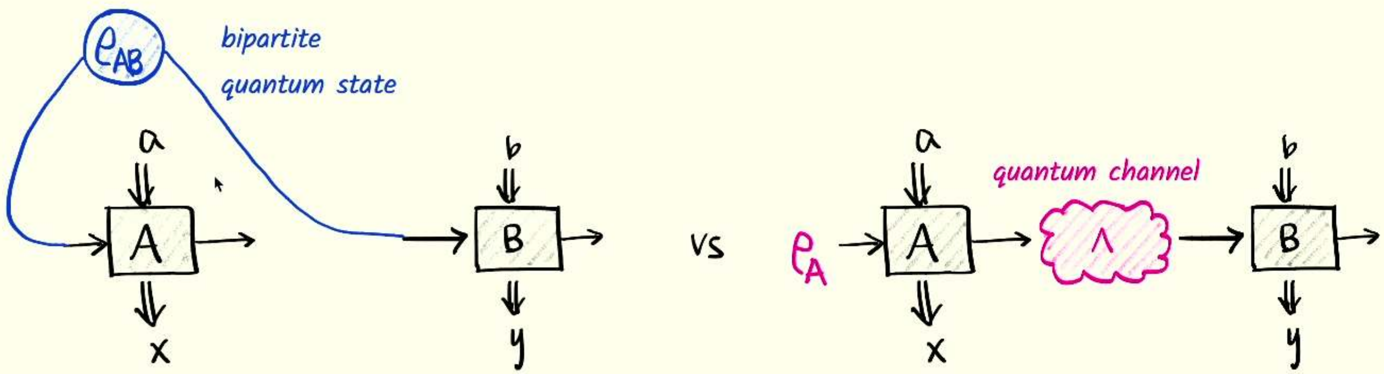
Quantum correlations which imply causation

Joseph F. Fitzsimons[1,2], Jonathan A. Jones[3] & Vlatko Vedral[2,3,4]

Received: 28 September 2015
Accepted: 11 November 2015
Published: 17 December 2015

In ordinary, non-relativistic, quantum physics, time enters only as a parameter and not as an observable: a state of a physical system is specified at a given time and then evolved according to the prescribed dynamics. While the state can, and usually does, extend across all space, it is only defined at one instant of time. Here we ask what would happen if we defined the notion of the quantum density matrix for multiple spatial and temporal measurements. We introduce the concept of a pseudo-density matrix (PDM) which treats space and time indiscriminately. This matrix in general fails to be positive for measurement events which do not occur simultaneously, motivating us to define a measure of causality that discriminates between spatial and temporal correlations. Important properties of this measure, such as monotonicity under local operations, are proved. Two qubit NMR experiments are presented that illustrate how a temporal pseudo-density matrix approaches a genuinely allowed density matrix as the amount of decoherence is increased between two consecutive measurements.

Problem



Common cause vs Direct cause-effect

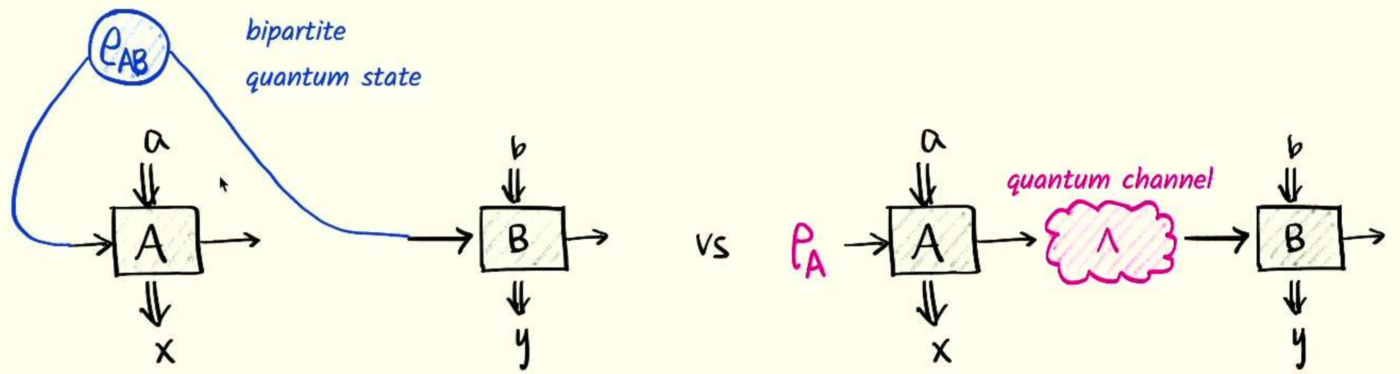
Spatial structure Temporal structure

$A \to B$

- Alice & Bob perform tomographically complete projective measurements independently
- They do not communicate during measurement processes.
- After measurements, the system leaves in the collapsed state to the projector associated with the measurement outcomes.

  I.e., the post-measurement state is defined by Luder's rule.

Problem



Common cause

Spatial structure

vs

Direct cause-effect

Temporal structure

$A \to B$

Given the observed statistics $\mathbb{M} = \{ \langle \sigma_a, \sigma_b \rangle = \sum_{x,y} xy \Pr(xy|ab) \}_{a,b}$,

can we definitely infer its causal structure,

or can we rule out certain causal structures?

* σ_a, σ_b : observables

Terminology

- $\mathbb{M} = \{\langle \sigma_a, \sigma_b \rangle\}_{a,b}$ is spatially compatible

  if $\exists$ spatial structure that is compatible with $\mathbb{M}$

  or equivalently if $\exists\ \rho_{AB}$ such that

  $$\langle \sigma_a, \sigma_b \rangle = \mathrm{tr}[(\sigma_a \otimes \sigma_b)\rho_{AB}] \quad \forall a,b$$

- $\mathbb{M} = \{\langle \sigma_a, \sigma_b \rangle\}_{a,b}$ is temporally compatible $A \to B$

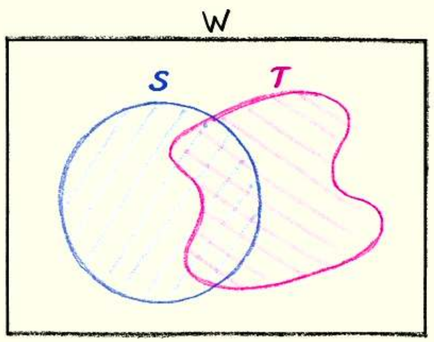
  if $\exists$ temporal structure $A \to B$ that is compatible with $\mathbb{M}$

  or equivalently if $\exists \rho_A,\ \Lambda : A \to B$ such that

  $$\langle \sigma_a, \sigma_b \rangle = \sum_x x\, \mathrm{tr}[\Lambda(P_{x|a}\, \rho_A\, P_{x|a})\, \sigma_b] \quad \forall a,b$$

$* \ \sigma_a \overset{\text{s.d.}}{=} \sum_x x\, P_{x|a}$

Venn Diagram



- S = the set of statistics compatible with a spatial structure
- T = the set of statistics compatible with a temporal structure

Are there statistics that cannot be explained by purely spatial structures? $S^c \neq \phi$?

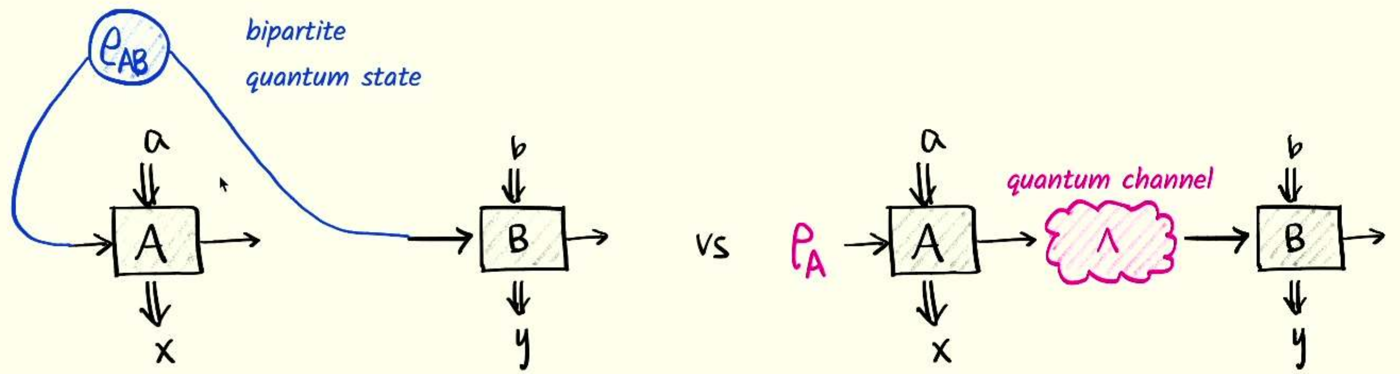
Are there statistics that cannot be explained by purely temporal structures? $T^c \neq \phi$?

In previous work, it has been shown that

- the subset S^c is non-empty
- the subset T^c is non-empty

5

Problem



Common cause vs Direct cause-effect

Spatial structure vs Temporal structure

$A \to B$

Given the observed statistics $\mathbb{M} = \{ \langle \sigma_a, \sigma_b \rangle = \sum_{x,y} xy \Pr(xy|ab) \}_{a,b}$,

can we definitely infer its causal structure,

or can we rule out certain causal structures?

* σ_a, σ_b : observables

3

Terminology

- $\mathbb{M} = \{\langle \sigma_a, \sigma_b \rangle\}_{a,b}$ is spatially compatible

  if $\exists$ spatial structure that is compatible with $\mathbb{M}$

  or equivalently if $\exists$ ρ_{AB} such that

  $$\langle \sigma_a, \sigma_b \rangle = \mathrm{tr}[(\sigma_a \otimes \sigma_b)\rho_{AB}] \quad \forall a,b$$

- $\mathbb{M} = \{\langle \sigma_a, \sigma_b \rangle\}_{a,b}$ is temporally compatible $A \to B$

  if $\exists$ temporal structure $A \to B$ that is compatible with $\mathbb{M}$

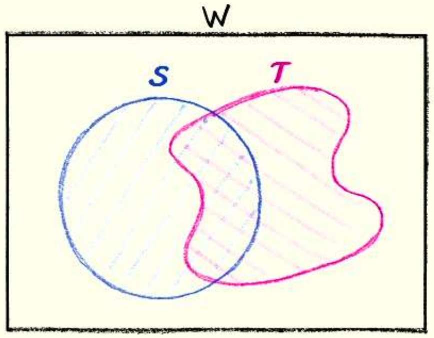
  or equivalently if $\exists \rho_A$, $\Lambda : A \to B$ such that

  $$\langle \sigma_a, \sigma_b \rangle = \sum_x x \, \mathrm{tr}[\Lambda(P_{x|a} \rho_A P_{x|a}) \sigma_b] \quad \forall a,b$$

4

$* \; \sigma_a \overset{\text{s.d.}}{=} \sum_x x \, P_{x|a}$

Venn Diagram



- S = the set of statistics compatible with a spatial structure
- T = the set of statistics compatible with a temporal structure

Are there statistics that cannot be explained by purely spatial structures? $S^c \neq \phi$?

Are there statistics that cannot be explained by purely temporal structures? $T^c \neq \phi$?

In previous work, it has been shown that

- the subset S^c is non-empty
- the subset T^c is non-empty

5

Research questions In this work, we were interested in characterizing the set T in two-qubit cases with Pauli measurements

Causal Classification of Spatiotemporal Quantum Correlations

Minjeong Song,[1,*] Varun Narasimhachar,[1,2,†] Bartosz Regula,[3] Thomas J. Elliott,[4,5] and Mile Gu[1,6,7,‡]

[1] *Nanyang Quantum Hub, School of Physical and Mathematical Sciences, Nanyang Technological University, 637371, Singapore*
[2] *Institute of High Performance Computing, Agency for Science, Technology and Research (A*STAR), 1 Fusionopolis Way, Singapore 138632*
[3] *Mathematical Quantum Information RIKEN Hakubi Research Team, RIKEN Cluster for Pioneering Research (CPR) and RIKEN Center for Quantum Computing (RQC), Wako, Saitama 351-0198, Japan*
[4] *Department of Physics & Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[5] *Department of Mathematics, University of Manchester, Manchester M13 9PL, United Kingdom*
[6] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543, Singapore*
[7] *MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654, Singapore 117543, Singapore*

From correlations in measurement outcomes alone, can two otherwise isolated parties establish whether such correlations are atemporal? That is, can they rule out that they have been given the same system at two different times? Classical statistics says no, yet quantum theory disagrees. Here, we introduce the necessary and sufficient conditions by which such quantum correlations can be identified as atemporal. We demonstrate the asymmetry of atemporality under time reversal, and reveal it to be a measure of spatial quantum correlation distinct from entanglement. Our results indicate that certain quantum correlations possess an intrinsic arrow of time, and enable classification of general quantum correlations across space-time based on their (in)compatibility with various underlying causal structures.

NATURE PHYSICS DOI: 10.1038/NPHYS3266

a non-Markovian evolution, the state of the system at some time t_2 is influenced not only directly by the state at time t_1, but also by some environmental degrees of freedom, which act as a reservoir of information about the state of the system at some earlier time t_0—the common cause. Our inference schemes may also help to detect initial correlations between system and environment, which, if unaccounted for, can lead to errors in the characterization of processes[25–30].

Our results suggest several interesting avenues for future research. What can be inferred from observational data when the state is not pure and the channel is not unitary? What is the most general sort of quantum measurement scheme that falls into the observational class and what can be inferred from it? How does one generalize such schemes from pairs of systems to arbitrary numbers of systems?

6

Temporal Incompatibility

We introduced a witness f of temporal incompatibility such that

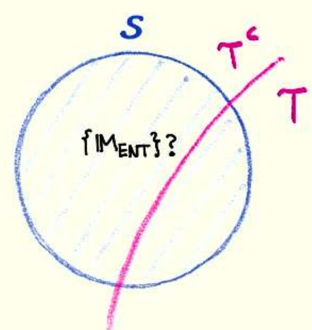
$$f(\mathbb{M}) \begin{cases} = 0 & \text{iff } \mathbb{M} \in T \\ > 0 & \text{iff } \mathbb{M} \in T^c \end{cases}$$

Any relation between temporal incompatibility and entanglement?

Does temporal incompatibility implies entanglement, and vice versa?

$\{\mathbb{M}_{ENT}\}, \quad T^c \quad ???$

$\{\mathbb{M}_{SEP}\}, \quad T \quad ???$



7

Characterization of T

In this work, we proved this conjecture to be true in arbitrary dimensions

Bipartite quantum states admitting a causal explanation

Minjeong Song*
Centre for Quantum Technologies, National University of Singapore

Arthur J. Parzygnat†
Experimental Study Group, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

The statistics of local measurements of joint quantum systems can sometimes be used to distinguish the spatiotemporal structure in which they were measured. We first prove that every bipartite separable density matrix is temporally compatible with direct causal influence for arbitrary finite-dimensional quantum systems and measurements of a tomographically-complete class of observables, which includes all Pauli observables in the case of multi-qubit systems. Equivalently, if a bipartite density matrix is not temporally compatible with direct causal influence, then it must be entangled. We also provide an operational meaning for the temporal evolution consistent with such correlations in terms of a generalized dephasing channel and a pretty good measurement. The two temporal evolutions are Bayesian inverses of each other, which is different from them being Petz recovery maps of each other. Finally, we prove necessary and sufficient conditions for an arbitrary bipartite quantum state to be temporally compatible, thereby providing a temporal analogue of the positive partial transpose criterion valid for quantum systems of any dimension.

Keywords: Temporal compatibility, causal compatibility, quantum state over time, quantum conditioning, pseudo-density matrix, Schur channel, Hadamard channel, generalized dephasing channel, pretty good measurement, Petz recovery map, quantum Bayes' rule, light-touch observable, Jordan product

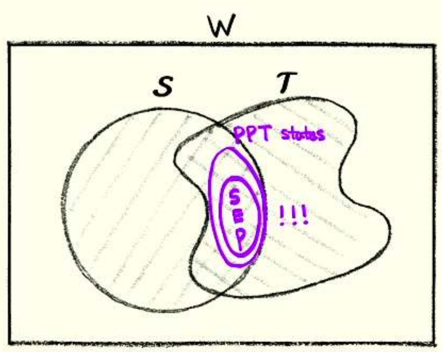
Under the measurement scheme such that

- *tomographically complete binary projective measurements, e.g., Pauli measurements*
- *post-measurement states are defined by Luder's rule*

Characterization of T

Results

1. All separable states are temporally compatible.
2. Interpretation of a temporal map associated with a separable state
3. Relation between such temporal map and Petz recovery map in the context of Bayesian retrodiction.
4. Characterization of the statistics in $S \cap T$



11

Characterization of T

Results

1. All separable states are temporally compatible.
2. Interpretation of a temporal map associated with a separable state
3. Relation between such temporal map and Petz recovery map in the context of Bayesian retrodiction.
4. Characterization of the statistics in $S \cap T$

Let τ_{AB} be a separable state s.t. $\tau_{AB} = \sum_{\theta} \lambda_\theta \rho_{A,\theta} \otimes \rho_{B,\theta}$

τ_{AB} admits a temporal explanation $A \to B$

with initial state being ρ_A and temporal evolution being $\vec{\Lambda} : A \to B$ given by

$$\vec{\Lambda} = \vec{G} \circ \vec{D}_{\rho_A}$$

where $\vec{G}$ is a pretty good measure-prepare channel that discriminates the ensemble $\{\lambda_\theta, \rho_{A,\theta}\}$

$\vec{D}_{\rho_A}$ is a (generalized) dephasing channel in the eigenbasis of ρ_A

*reduced states $\rho_A = \mathrm{tr}_B\, \tau_{AB}$

$\rho_B = \mathrm{tr}_A\, \tau_{AB}$

12

Characterization of T

Results

1. All separable states are temporally compatible.
2. Interpretation of a temporal map associated with a separable state
3. Relation between such temporal map and Petz recovery map in the context of Bayesian retrodiction.
4. Characterization of the statistics in $S \cap T$

matrix representation

$$\rho_A \rightarrow \boxed{\vec{\mathcal{E}}} \rightarrow \rho_B = \vec{\mathcal{E}}(\rho_A) \rightsquigarrow \mathbb{M} = \{\langle \sigma_a, \sigma_b \rangle\}_{a,b} \Leftrightarrow \vec{\mathcal{E}} \star \rho_A = \sum_{a,b} \frac{\langle \sigma_a, \sigma_b \rangle}{d_A d_B} \sigma_a \otimes \sigma_b$$

$$\rho_A \leftarrow \boxed{\overleftarrow{\mathcal{E}}} \leftarrow \rho_B \qquad \overleftarrow{\mathcal{E}} : \text{reverse temporal map} \qquad \mathrm{SWAP}(\vec{\mathcal{E}} \star \rho_A) = \overleftarrow{\mathcal{E}} \star \rho_B$$

Meanwhile, in Ref [1],

$$\mathrm{SWAP}(\vec{\mathcal{E}} \star_{LS} \rho_A) = \overleftarrow{\mathcal{E}} \star_{LS} \rho_B \rightarrow \overleftarrow{\mathcal{E}}_{LS} = \text{Petz recovery map}$$

Relation between two reverse maps $\quad \vec{D}_{\rho_A} \circ \overleftarrow{\mathcal{E}} = \overleftarrow{\mathcal{E}}_{LS} \cdot \vec{D}_{\rho_B}$

Ref [1]. Leifer, M. S., & Spekkens, R. W. (2013). Towards a formulation of quantum theory as a causally neutral theory of Bayesian inference. Physical Review A, 88(5), 052130.

Characterization of T

Results

1. All separable states are temporally compatible.
2. Interpretation of a temporal map associated with a separable state
3. Relation between such temporal map and Petz recovery map in the context of Bayesian retrodiction.
4. Characterization of the statistics in $S \cap T$

Let τ_{AB} be a bipartite quantum state. Then,

$$(\vec{D}_{\rho_A} \otimes \mathrm{id})(\tau_{AB}^{T_A}) \geq 0 \quad \text{if and only if } \tau_{AB} \text{ is temporally compatible } A \to B$$

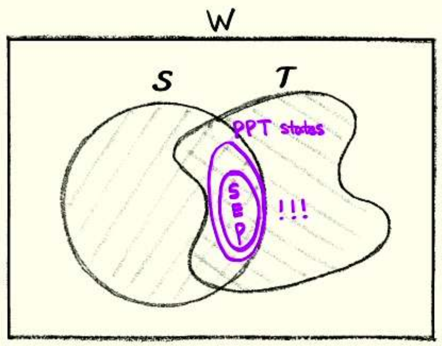
Remark. If $\rho_A = \frac{\mathbb{1}}{d_A}$, $\vec{D}_{\rho_A}$ reduces to the identity channel.

Consequently, the condition becomes the positive partial transpose (PPT) criterion, i.e., a PPT state τ_{AB} such that $(\tau_{AB}^{T_A}) \geq 0$ is temporally compatible $A \to B$

14

In summary

1. All separable states are temporally compatible.
2. Interpretation of a temporal map associated with a separable state
3. Relation between such temporal map and Petz recovery map in the context of Bayesian retrodiction.
4. Complete characterization of the statistics in $S \cap T$



Under the following assumptions;

A1) Device-dependent

A2) Tomographically complete binary projective measurements, e.g., Pauli measurements

A3) Post-measurement states are defined by Luder's rule

Open questions More to be answered. Specifically, what measurements and update rule (or instruments) are sufficient to infer causal structures?

NATURE PHYSICS DOI: 10.1038/NPHYS3266

a non-Markovian evolution, the state of the system at some time t_2 is influenced not only directly by the state at time t_1, but also by some environmental degrees of freedom, which act as a reservoir of information about the state of the system at some earlier time t_0—the common cause. Our inference schemes may also help to detect initial correlations between system and environment, which, if unaccounted for, can lead to errors in the characterization of processes[25–30].

Our results suggest several interesting avenues for future research. What can be inferred from observational data when the state is not pure and the channel is not unitary? What is the most general sort of quantum measurement scheme that falls into the observational class and what can be inferred from it? How does one generalize such schemes from pairs of systems to arbitrary numbers of systems?

1. Causal Classification

2. Temporally compatible quantum states

3. Gaussian extension

2. Petz recovery map and its implementation

Appearance

- Quantum Information Theory (cf. D. Petz in 1986, 2004)
  ~data processing inequality, conditional mutual information...
- Quantum Error Correction (cf. Barnumm-Knill in 2002)
  ~near-optimal recovery
- Quantum Fluctuation Theorem (cf. J. Aberg in 2018)
- Quantum Bayesian Retrodiction and Quantum State Over Time (cf. Liefer-Spekkens in 2013)

Def. Let $\mathcal{N}$ be a quantum channel. Its **Petz recovery map** $\hat{\mathcal{N}}_\gamma$ is defined as

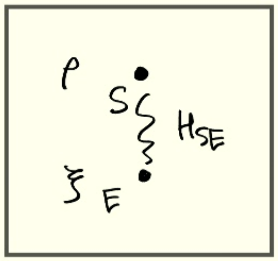
$$\hat{\mathcal{N}}_\gamma(\cdot) = \sqrt{\gamma}\, \mathcal{N}^\dagger\left(\frac{1}{\sqrt{\mathcal{N}(\gamma)}} \cdot \frac{1}{\sqrt{\mathcal{N}(\gamma)}}\right)\sqrt{\gamma}$$

for a reference state γ

18

Tabletop Reversibility

In lab,



Noise = Interaction with uncontrolled environment

$$\mathcal{N}: \rho \mapsto \mathcal{N}(\rho) = \mathrm{tr}_E\left[\, U\, (\rho_S \otimes \xi_E)\, U^\dagger \right], \quad U = e^{-i\Delta t H_{SE}}$$

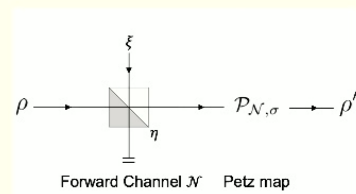
Approximate recovery with Petz maps by using the same resources (e.g. H_{SE})

Mathematically, $\hat{\mathcal{N}}_\gamma(\rho) \overset{?}{=} \mathrm{tr}_E\left[U^\dagger (\rho_S \otimes \xi'_E)\, U \right]$

Example. Thermal operations

Tabletop Reversibility

Recovery of optical losses with Petz recovery map



19

Thank you for your attention!