Title: Supplement - Quantum Measurement and Continuous Markov Processes Mini-Course

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Collection/Series: Quantum Measurement and Continuous Markov Processes Mini-Course, Oct 27 - Nov 28, 2025

Subject: Mathematical physics, Other, Quantum Foundations, Quantum Information

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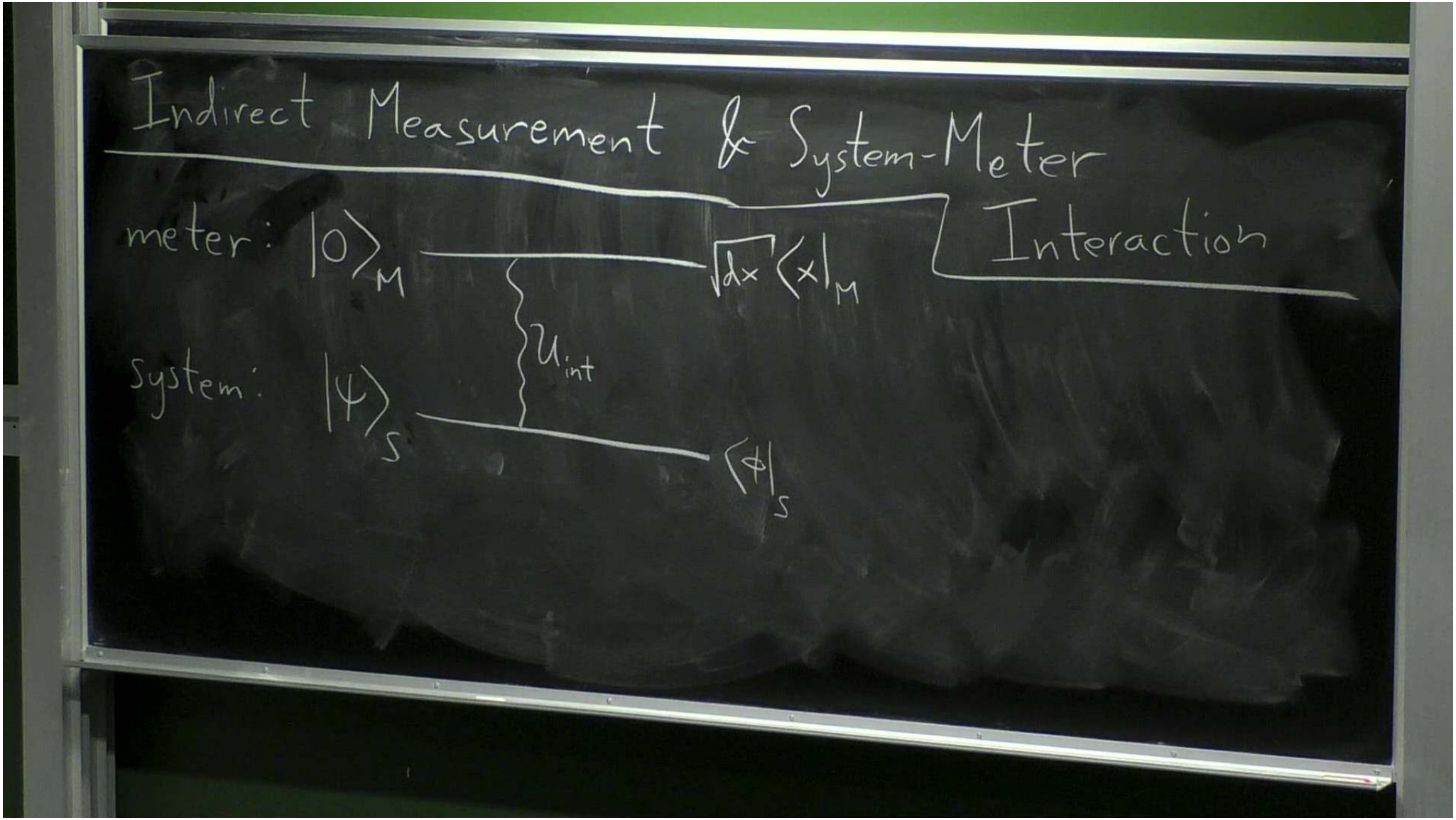
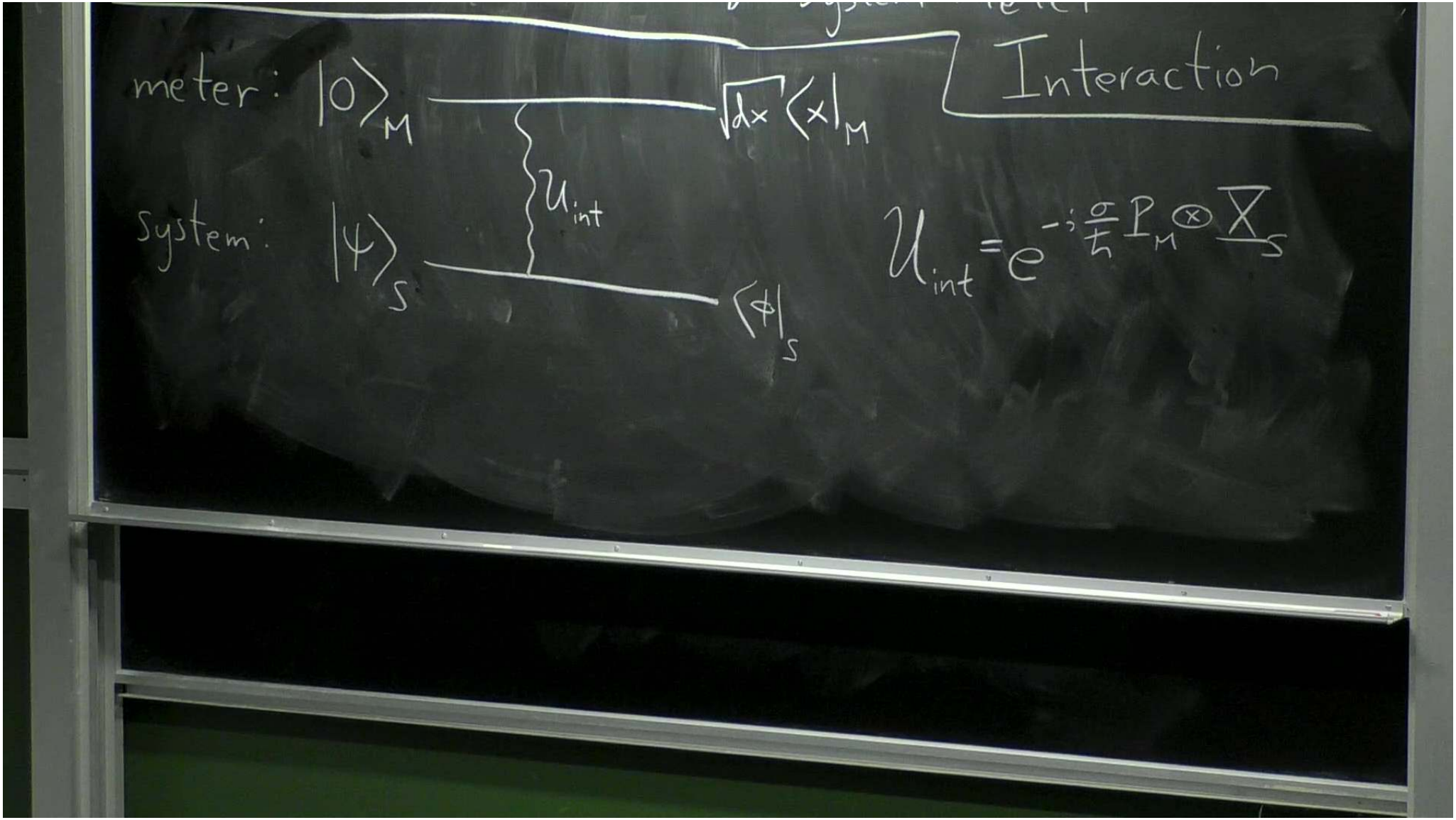
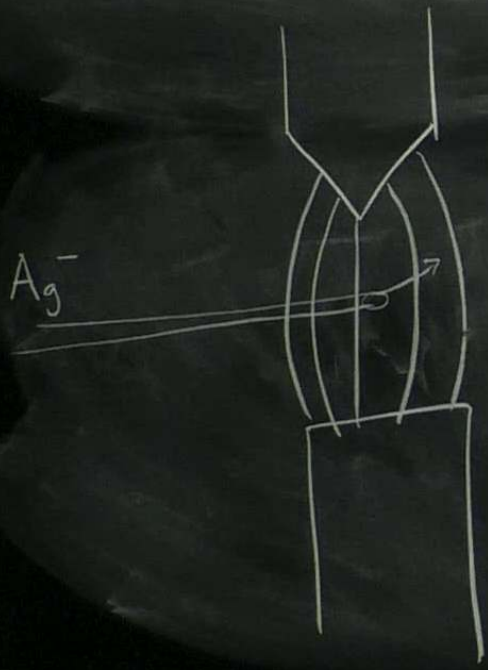
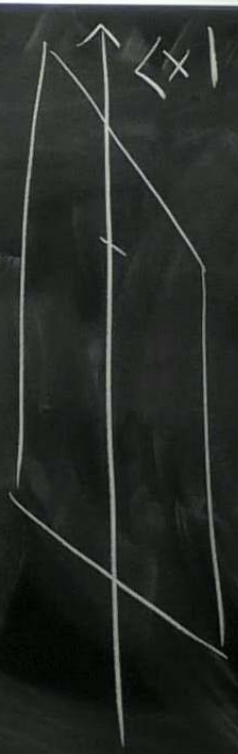

Indirect Measurement & System-Meter Interaction
meter: $|0\rangle_M$
$\int dx \langle x|_M$
U_{int}
system: $|\psi\rangle_S$
$\langle \phi|_S$

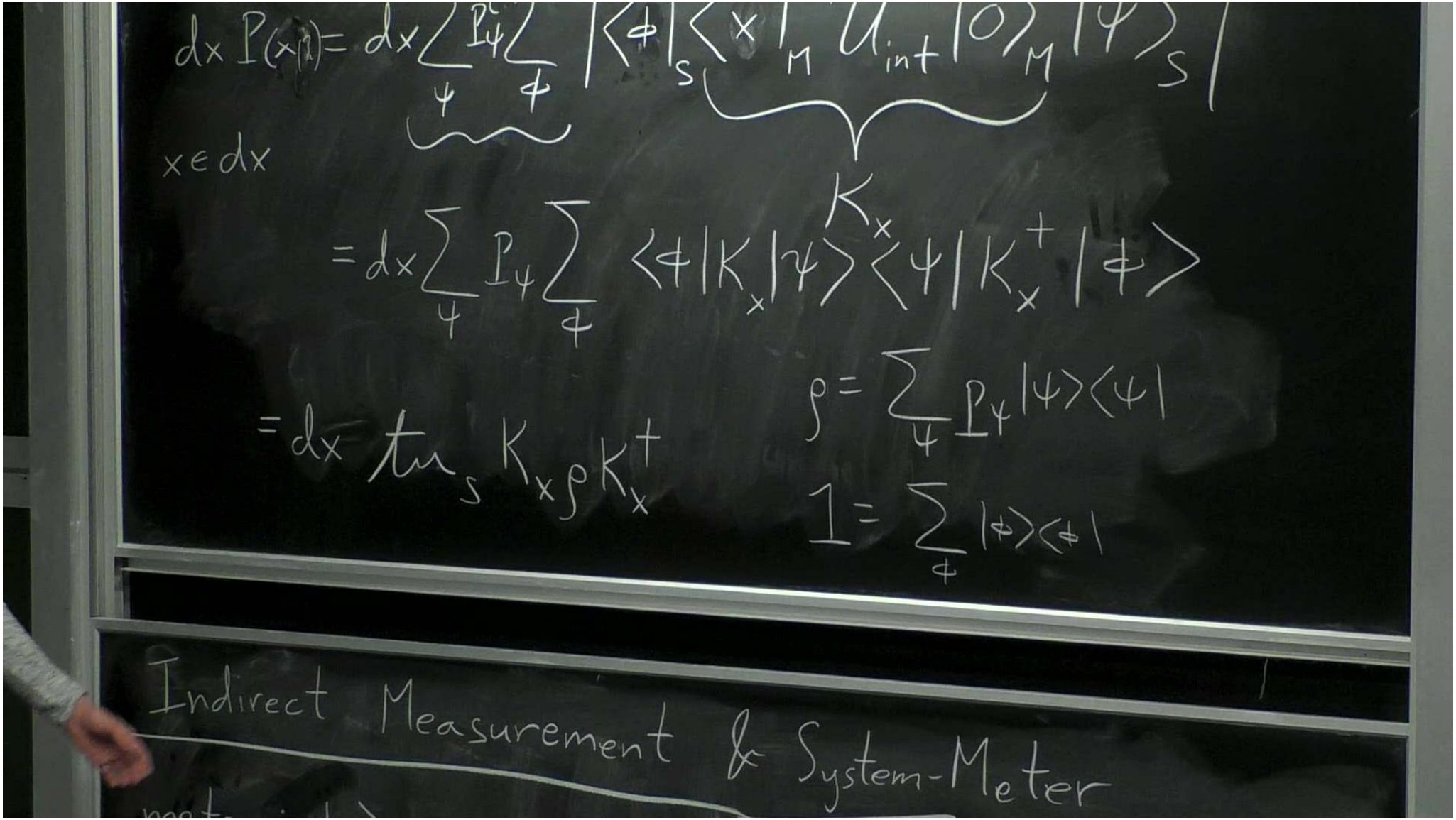

meter: $|0\rangle_M$
$\int dx \langle x|_M$
Interaction
System: $|\psi\rangle_S$
U_{int}
$\langle \phi|_S$
$U_{int} = e^{-i\frac{\sigma}{\hbar} P_M \otimes X_S}$

S-G $\hat{X}_S = \hat{J}_z$





$Q_M = T_{flight} P_{OM}$


$dx\, P(x) = dx \sum_\psi P_\psi \sum_\phi |\langle\phi|_S \langle x|_M U_{int} |0\rangle_M |\psi\rangle_S|$
$x \in dx$
K_x
$= dx \sum_\psi P_\psi \sum_\phi \langle\phi|K_x|\psi\rangle\langle\psi|K_x^\dagger|\phi\rangle$
$\rho = \sum_\psi P_\psi |\psi\rangle\langle\psi|$
$= dx\, tr_S K_x \rho K_x^\dagger$
$1 = \sum_\phi |\phi\rangle\langle\phi|$
Indirect Measurement & System-Meter

$\sum_x \int K_x$ $\quad 1 = \sum_\phi |\phi\rangle\langle\phi|$

$\Omega_x = \sqrt{dx}\, K_x$ "Kraus Operator" or "Measurement Operator"

$\{\Omega_x\}$ "Generalized Measurement"

$\{\Omega_x^\dagger \Omega_x\}$ "Generalized Observable" "positive-operator-valued-measure"

POVM

A G.M. is Luders–von Neumann if Ω_x are positive. $\quad \Omega_x = \sqrt{\Omega_x^\dagger \Omega_x}$

Indirect Measurement & System-Meter

$A \circ B^\dagger(X) = A X B^\dagger$

$= \Omega_x \circ \Omega_x^\dagger(\rho)$

$\{\Omega_x \circ \Omega_x^\dagger\}$ "Instrument" (Kraus-rank one) "QOVM or quantum-operation-valued measure"

elements are called "Instrument e'ts", "selective operations", or just "operations"

S-G $\overline{X}_S = \overline{J}_z$

$\langle x |$

POVM elements $(\Omega_x^\dagger \Omega_x)$ a.k.a. "effects"

$$\mathcal{E} = \sum_x \Omega_x \circ \Omega_x^\dagger$$

"decoherence channel", "channel", "CPTP map", "nonselective operation", "the quantum operation".

where $\mathcal{E}$ is the primary object

In communication and state evolution meter = "environment"

"operator-sum decomposition" "unraveling" "simulating quantum trajectories" "ostensible distribution"

What is the meter?

In measurement meter = "observer"

Gaussian Instruments

$$\sqrt{dx}\langle x|0\rangle_M = \sqrt{\frac{dx}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{x^2}{2\sigma^2}}}$$

POVM elements $(\Omega_x^\dagger \Omega_x)$ a.k.a. "effects"

"channel", CPTP map",

$\sqrt{dx}\langle x|0\rangle_M = \sqrt{\frac{dx}{\sqrt{2\pi\sigma^2}}} e^{-\frac{x^2}{2\sigma^2}}$

$\langle x|Q_M = x\langle x|$

$\langle x|\frac{1}{\hbar}P_M = -i\frac{\partial}{\partial x}\langle x|$

$K_x = \langle x|e^{-i\frac{\sigma}{\hbar}P_M \otimes 2\overline{X}_S\theta}|0\rangle_M$

$= e^{-\sigma\frac{\partial}{\partial x}2\overline{X}_S\theta}\langle x|0\rangle_M$

POVM elements $(\Omega_x^\dagger\Omega_x)$ a.k.a. "effects"

"coherence channel", "channel", CPTP map",

$$= \sum_{n=0} \frac{1}{n!} a^n f^{(n)}(x)$$

$$= \sqrt{\frac{1}{\sqrt{2\pi\sigma^2}}} e^{-\frac{1}{2\sigma^2}(x - 2\sigma\theta \mathbb{X}_s)^2}$$

$$K_x = \frac{1}{(2\pi\sigma^2)^{1/4}} e^{-\theta^2\left(\frac{x}{2\sigma\theta} - \mathbb{X}_s\right)^2} \to 0 \text{ unless } x = 2\sigma\theta\lambda \text{ for some } \lambda$$

$$\mathbb{X}_s = \sum_\lambda \lambda \Pi_\lambda$$

$$\int dx\, K_x^\dagger K_x = 1_S$$

$$\Sigma = \int dx\, K_x \circ K_x^\dagger$$

POVM elements $(\Omega_x^\dagger \Omega_x)$ a.k.a. "effects"

"coherence channel", "channel", CPTP map",

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$$\xi = \int dx\, K_x \circ K_x^\dagger$$

POVM elements $(\Omega_x^\dagger \Omega_x)$ a.k.a. "effects"

"channel", CPTP map",