Title: Lecture - Relativity (Core), PHYS 604

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Subject: Cosmology, Strong Gravity

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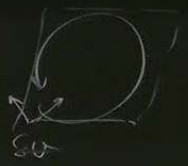
Riemann Tensor 

$$\delta v = -R \ldots ABv$$

Einstein Tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$\nabla_\mu G^{\mu\nu} = 0$$

Rank (2) + $G(g, \partial g, \partial^2 g)$ + Bianch identity

$$G^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$\text{Riemann} = \text{d Ricci } g + \frac{1}{d-2} \text{ Weyl}$$

$$G_{\mu\nu} = 0 \quad \text{in Vacuum} \quad \Rightarrow \quad R_{\mu\nu} = R = 0$$

ity

1) Space-time curv. ⟶ how matter moves

P.P. $u^\mu \nabla_\mu u^\alpha = 0$

EOM $\xrightarrow{\text{Curv.}}$ Matter coupling to gravity

2) Matte ⟶ Space time curvature

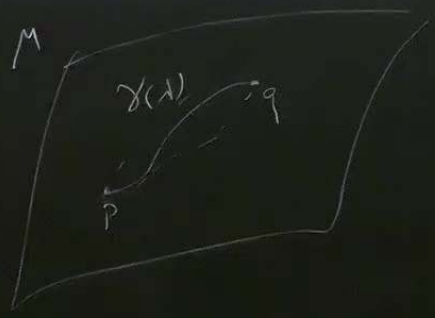
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

3) Complications
- Back-reaction
- Q-Gravity
- Singularities

1) Action for Point-Particle

$$S[x^\mu(\lambda)] = -m\int_P^q d\lambda \sqrt{-g(u,u)} = -m\int_P^q d\lambda \sqrt{-g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}}$$

($-g(u,u) > 0$; $-ds^2$; $\left(\frac{d\tau}{d\lambda}\right)^2$)



$$g(u,u) = g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} \qquad = -m\int_P^q d\tau$$

$$\lambda \to \lambda' = \tau \qquad d\tau^2 := -g(u,u)d\lambda^2 = -g_{\alpha\beta}dx^\alpha dx^\beta$$

$$d\tau = \sqrt{-ds^2} = \sqrt{dt^2 - dx^2} = dt\sqrt{1-v^2}$$

$$\simeq dt\left(1-\tfrac{1}{2}v^2\right)$$

$$S \simeq \int_P^q dt\left(-m + \tfrac{1}{2}mv^2\right)$$

Weak field approx.: $ds^2 = -(1+2\phi)\,dt^2 + (1-2\phi)\,dx^2$

$$S_p = -m\int_p^q d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}} = \int_p^q d\lambda\, L(x^\mu, \dot{x}^\mu) \Longrightarrow$$

$\dot{(\ )} = \frac{d}{d\lambda}$ $\qquad L = -m\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$

$$\simeq dt\left(1-\tfrac{1}{2}v^2\right)$$

$$S = -m\int_P^Q dt\,\left((1+2\phi)-(1-2\phi)v^2\right)^{1/2} \simeq \int_P^Q dt\,\left[-m+\frac{1}{2}mv^2-m\phi+O(\phi v^2)\right]$$

$$\Rightarrow \frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^\alpha}-\frac{\partial L}{\partial x^\alpha}=0 \qquad \frac{d\tau}{d\lambda}=\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta} \qquad f(\lambda)\equiv\frac{d\lambda}{d\tau}=\frac{1}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}$$

$$\frac{\partial L}{\partial \dot{x}^\nu}=+\frac{m\,g_{\alpha\beta}\dot{x}^\beta}{\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}}=f(\lambda)\left(+m g_{\alpha\beta}\dot{x}^\alpha\right)$$

$$\frac{\partial L}{\partial x^\nu}=\frac{m f(\lambda)}{2}\,g_{\mu\nu,\alpha}\,\dot{x}^\mu\dot{x}^\nu$$

$$\Rightarrow f'(\lambda)\,g_{\alpha\beta}\dot{x}^\beta+f(\lambda)\,g_{\alpha\beta,\nu}\dot{x}^\nu\dot{x}^\beta+f\,g_{\alpha\beta}\ddot{x}^\beta-\frac{1}{2}g_{\mu\nu,\alpha}\dot{x}^\mu\dot{x}^\nu f=0$$

$$\frac{1}{2}g_{\alpha\mu,\nu}\dot{x}^\nu\dot{x}^\beta+\frac{1}{2}g_{\alpha\nu,\beta}\dot{x}^\beta\dot{x}^\nu$$

$$g_{\alpha\beta}\ddot{x}^{\beta} + \frac{1}{2}\left(g_{\alpha\beta,\nu} + g_{\alpha\nu,\beta} - g_{\beta\nu,\alpha}\right)\dot{x}^{\beta}\dot{x}^{\nu} = -\frac{f'(\lambda)}{f(\lambda)}\, g_{\alpha\beta}\dot{x}^{\beta} \qquad \times g^{\mu\alpha}$$

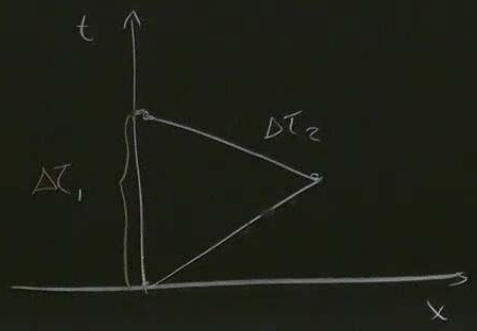
$$\downarrow$$

$$\frac{d^2x^{\mu}}{d\lambda} + \Gamma^{\mu}_{\beta\nu}\dot{x}^{\mu}\dot{x}^{\nu} = \underbrace{\frac{f'}{f}\dot{x}^{\mu}}$$

$$RHS = 0 \quad \Rightarrow \quad f'_{(\lambda)} = 0 \quad \Rightarrow$$

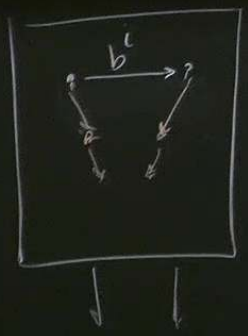
$$\underbrace{u^{\alpha}\nabla_{\alpha}u^{\mu} = 0}$$

$\Rightarrow \lambda = a\tau + b$



$\Delta\tau_2 < \Delta\tau_1$

Geodesic deviation equation:



$$\frac{db^i}{dt^2} = -(\partial_i \partial_j \phi) b^j$$

tidal force $\longleftarrow R^{\alpha}_{\beta\gamma\delta} \neq 0$

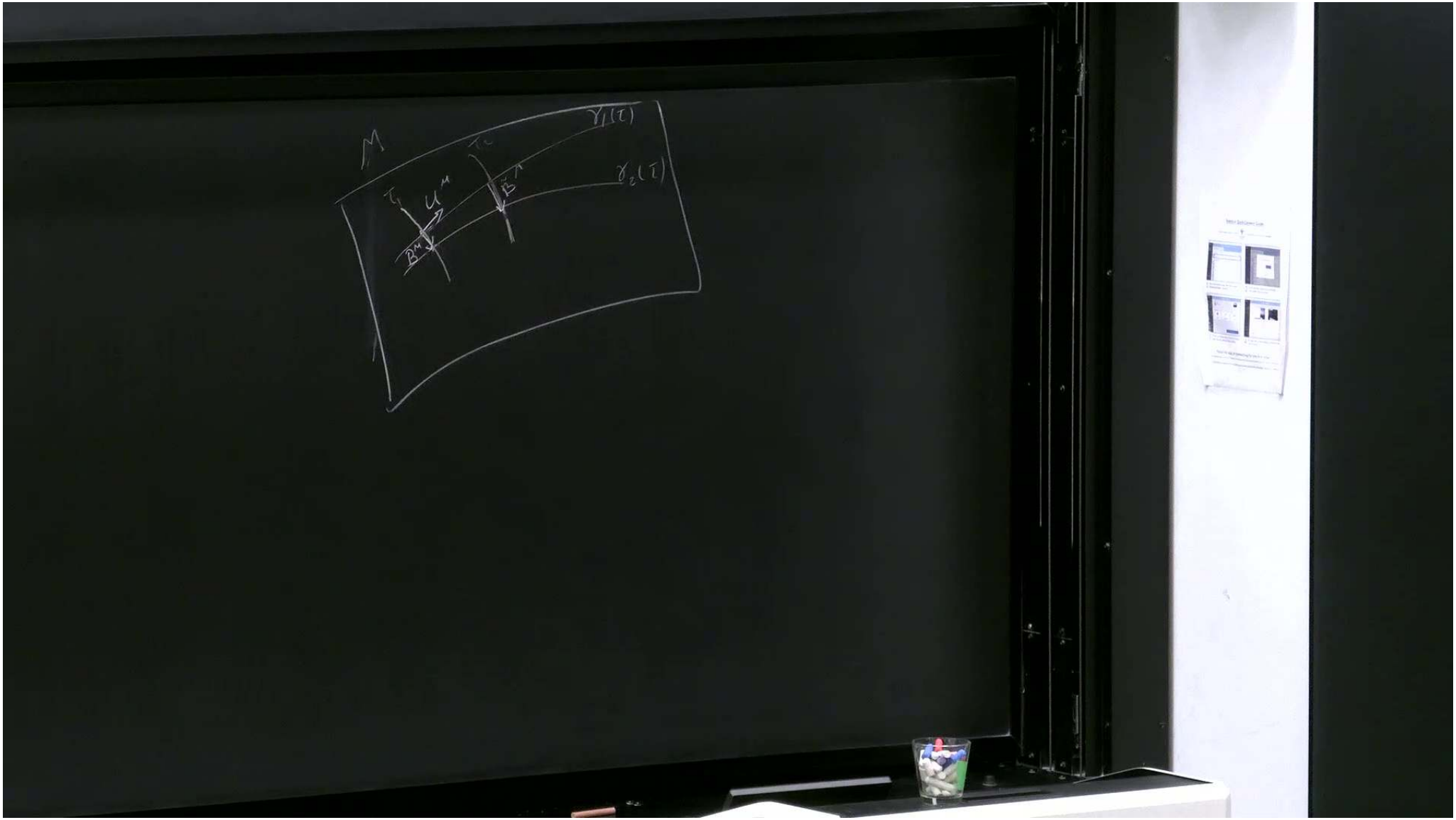
Relative Distance : $B^\mu(\tau)$

$$V^\mu \equiv \frac{DB^\mu}{d\tau} = U^\nu \nabla_\nu B^\mu = \frac{d}{d\tau} B^\mu + \Gamma^\mu_{\sigma\nu} B^\sigma U^\nu$$

$$A^\mu \equiv \frac{D^2 B^\mu}{d\tau} = \frac{DV^\mu}{d\tau} = U^\nu \nabla_\nu V^\mu = \frac{d}{d\tau} V^\mu + \Gamma^\mu_{\sigma\nu} V^\sigma U^\nu$$

$$= \dots = -R^\mu_{\nu\rho\sigma} U^\nu U^\sigma B^\rho$$

$\gamma\delta \neq 0$


M
$\gamma_1(\tau)$
$\gamma_2(\tau)$
τ_1
τ_2
u^μ
E^μ
B^μ

$\eta_{\mu\nu} \to g_{\mu\nu}$, $\partial_\mu \to \nabla_\mu$ $\qquad dv = dx^d \sqrt{-g}$

Example: Scalar field: $\quad S_m = \int dx^d \sqrt{-g}\, \mathcal{L}(\phi, \nabla\phi, g)$

δS for $\delta\phi$ $\qquad \delta S_m = \int_V dx^d \sqrt{-g}\, \frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial \nabla_\mu \phi} \underbrace{\delta \nabla_\mu \phi}_{\nabla_\mu \delta\phi} =$

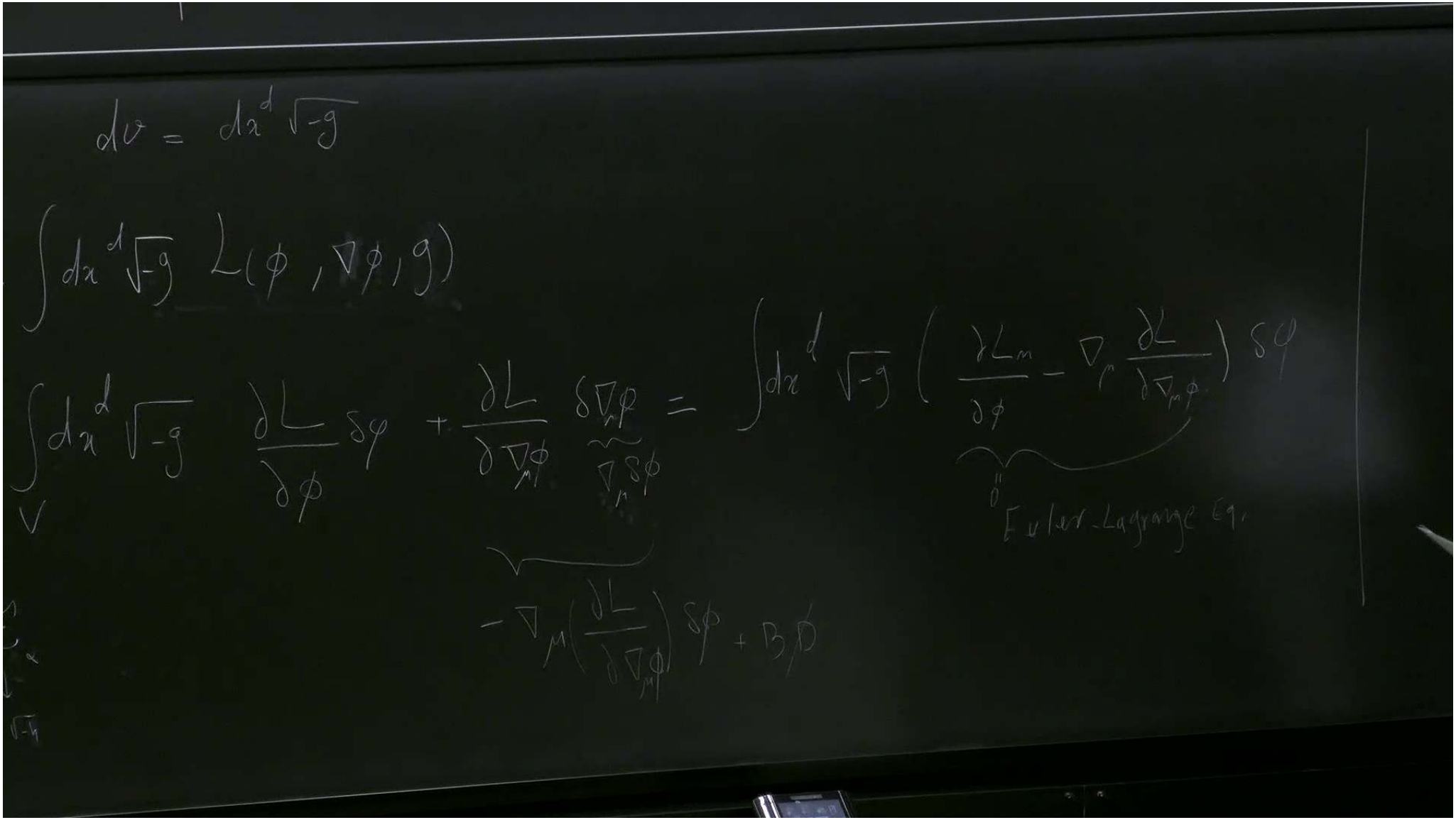
Gauss-stokes theorem

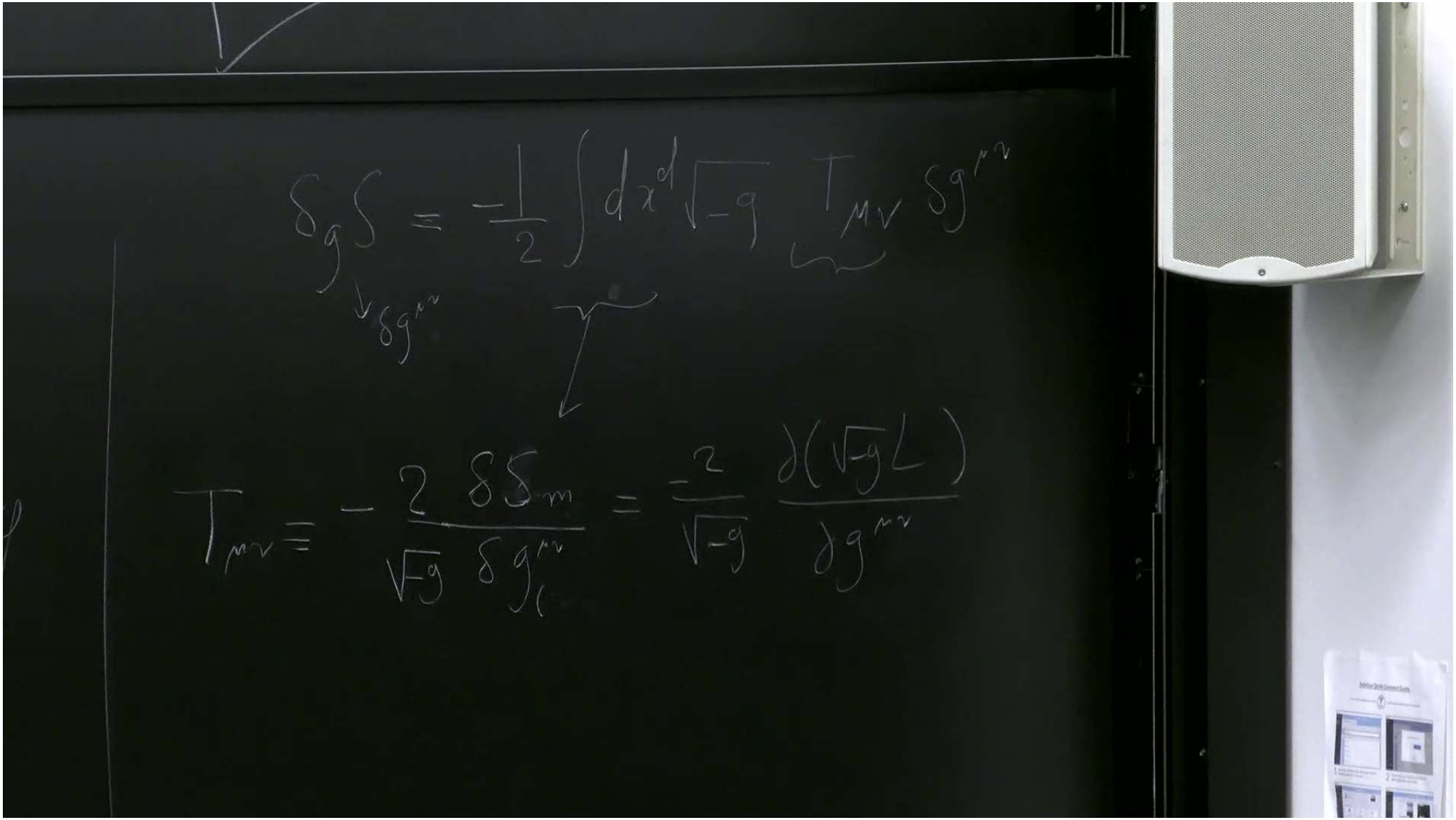
$$\int_V dx^d \sqrt{-g}\, \nabla_\alpha v^\alpha = \oint_{\partial V} v^\alpha d\Sigma_\alpha$$

$$\left(\frac{\partial \mathcal{L}}{\partial \nabla_\mu \phi} \delta\phi\right) \qquad \sqrt{-g}$$

$$- \nabla_\mu \left(\frac{\partial \mathcal{L}}{\partial \nabla_\mu \phi}\right) \delta\phi + BD$$

V


$dv = dx^d \sqrt{-g}$
$\int dx^d \sqrt{-g}\, L(\phi, \nabla\phi, g)$
$\int dx^d \sqrt{-g}\, \frac{\partial L}{\partial \phi} \delta\phi + \frac{\partial L}{\partial \nabla_\mu \phi} \delta \nabla_\mu \phi = \int dx^d \sqrt{-g} \left(\frac{\partial L_m}{\partial \phi} - \nabla_\mu \frac{\partial L}{\partial \nabla_\mu \phi} \right) \delta\phi$
$\nabla_\mu \delta\phi$
$= 0$ Euler-Lagrange Eq.
$- \nabla_\mu \left(\frac{\partial L}{\partial \nabla_\mu \phi} \right) \delta\phi + B.D$


$\delta_g S = -\frac{1}{2}\int d^4x \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$
$\delta g^{\mu\nu}$
$T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S_m}{\delta g^{\mu\nu}} = \frac{-2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\mu\nu}}$