Title: Lecture - Relativity (Core), PHYS 604

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Collection/Series: Relativity (Core), PHYS 604, November 11 - December, 12 2025

Subject: Cosmology, Strong Gravity

Date: November 11, 2025 - 7:00 PM

URL: https://pirsa.org/25110021

General Relativity:

Energy/Mass $\longleftrightarrow$ Gravity $\longleftrightarrow$ Geometry

Gravity $\longleftrightarrow$ acceleration

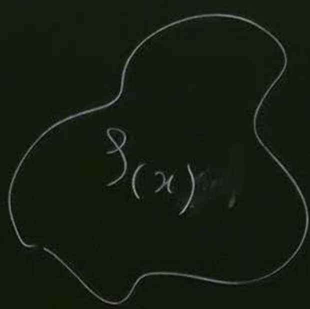
Astrophysics $\longrightarrow$ Geometry $\longrightarrow$ 3+1 Geometry

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

What was wrong with Newton's Gravity?

- @ odds with S.R.

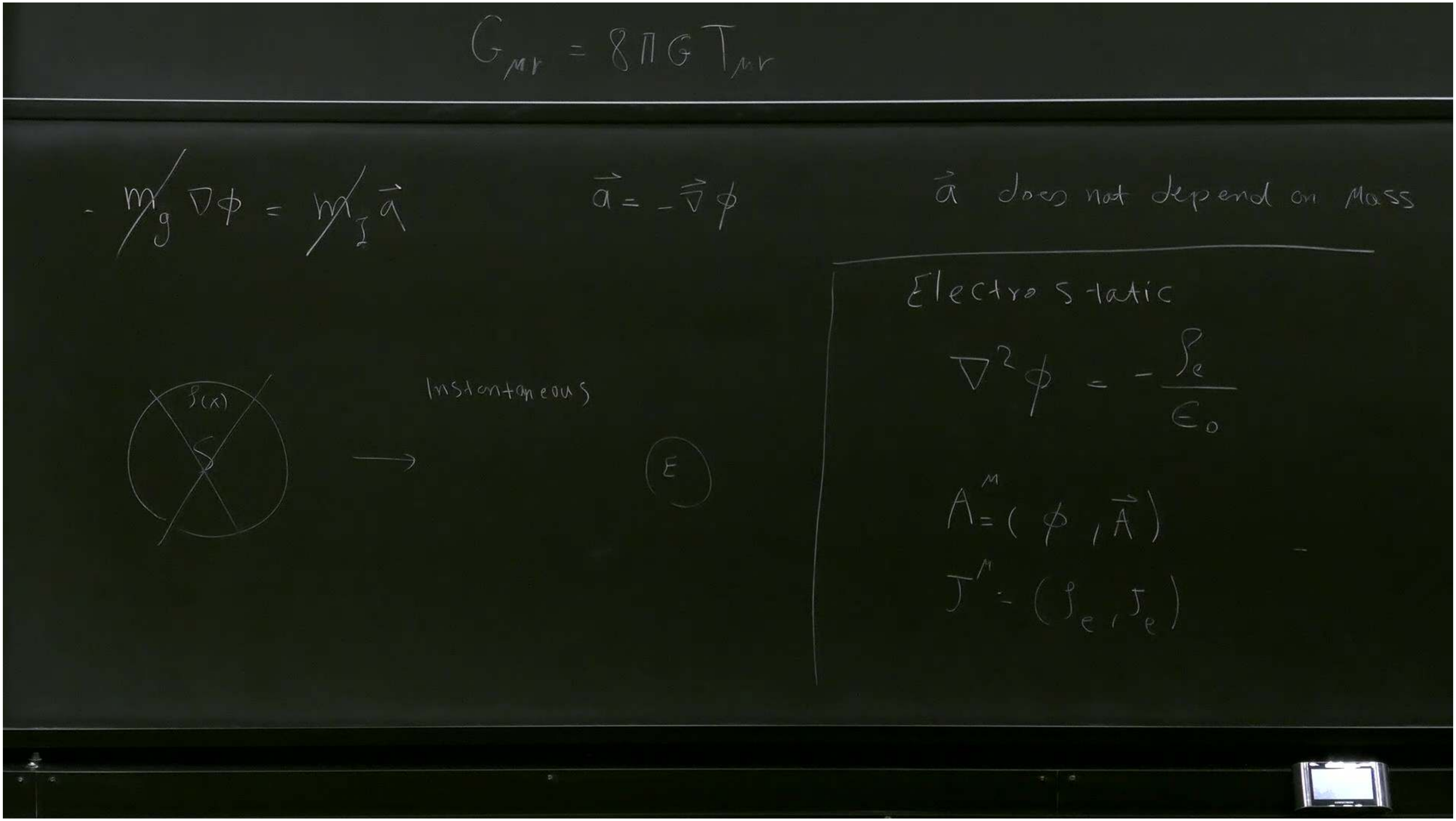
- predicting the orbit of Mercury



$$\nabla^2 \phi = 4 \pi G \rho_{(x)}$$

$$\phi_{(x)} = - \int_V dx^3 \frac{\rho}{|x - x'|}$$

$$F_g = -m_I \nabla \phi \qquad F_g = m a$$


$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
$-\cancel{m}_g \nabla\phi = \cancel{m}_I \vec{a}$
$\vec{a} = -\vec{\nabla}\phi$
$\vec{a}$ does not depend on Mass
Electrostatic
$\nabla^2\phi = -\frac{\rho_e}{\epsilon_0}$
$A^\mu = (\phi, \vec{A})$
$J^\mu = (\rho_e, J_e)$
$\rho(x)$
S
Instantaneous
E

$F_g = m_g \nabla\varphi$, $F_g = m_I a$

$\partial_\mu F^{\mu\nu} = J^\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

waves at speed of light!

$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$

$c \to 1$

$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$

$ds^2 = \sum_{\mu,\nu} \eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu$

$\mu, \nu = 0, 1, 2, 3$

t, x, y, z

$x_\mu = \eta_{\mu\nu} x^\nu$ $\quad x^\mu = \eta^{\mu\nu} X_\nu$ $\quad A^\mu \neq A_\mu$ $\quad A^0 = \phi \quad A_0 = -c^2\phi$

$v^i \to 3D$

u^μ

$$v^i = \frac{dx^i}{dt}$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$

$$-\cancel{c^2} d\tau^2 = -c^2 dt^2 + d\vec{x}^2$$

$$= -\cancel{c^2} dt^2 \Big(1 - \frac{1}{c^2}\underbrace{\Big(\frac{d\vec{x}}{dt}\Big)^2}_{\vec{v}^2}\Big)$$

$$d\tau = dt\sqrt{1 - v^2/c^2}$$

$$\frac{dt}{d\tau} = \gamma_v \qquad u^i = \frac{dx^i}{dt}\frac{dt}{d\tau} =$$

$$A^0 = \phi \qquad A_0 = -c^2 \phi$$

$$= -c^2 dt^2 + d\vec{x}^{\,2}$$

$$= -\cancel{c}^2 dt^2 \left(1 - \frac{1}{c^2} \underbrace{\left(\frac{d\vec{x}}{dt}\right)^2}_{\vec{v}^{\,2}}\right)$$

$$= dt \sqrt{1 - v^2/c^2}$$

$$= \gamma_v \qquad u^i = \frac{dx^i}{dt} \frac{dt}{d\tau} = \gamma_v v^i$$

$$-c^2 d\tau^2 = ds^2$$

$$u^\mu = \gamma_v (1, \vec{v})$$

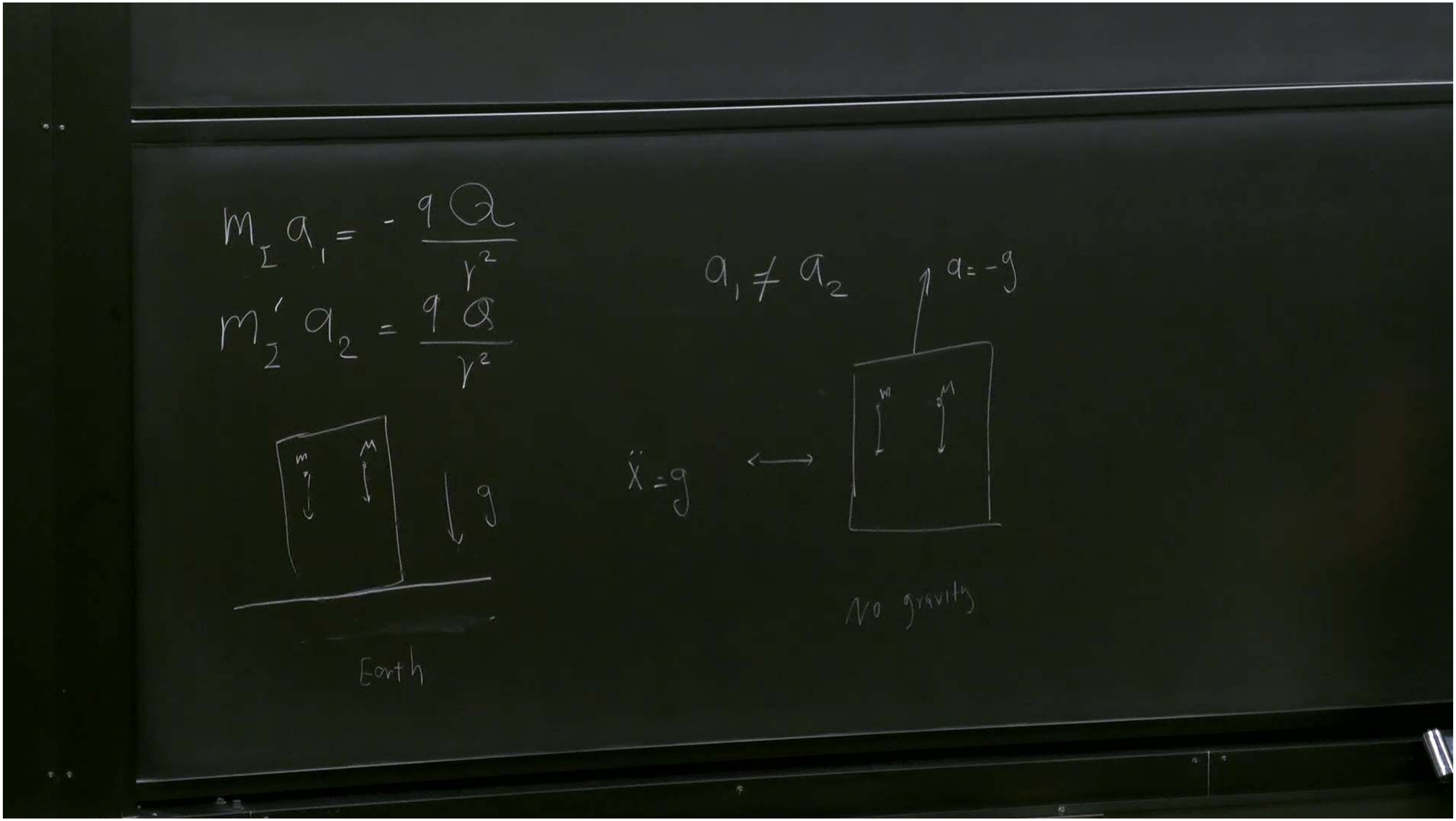
Equivalent Principle

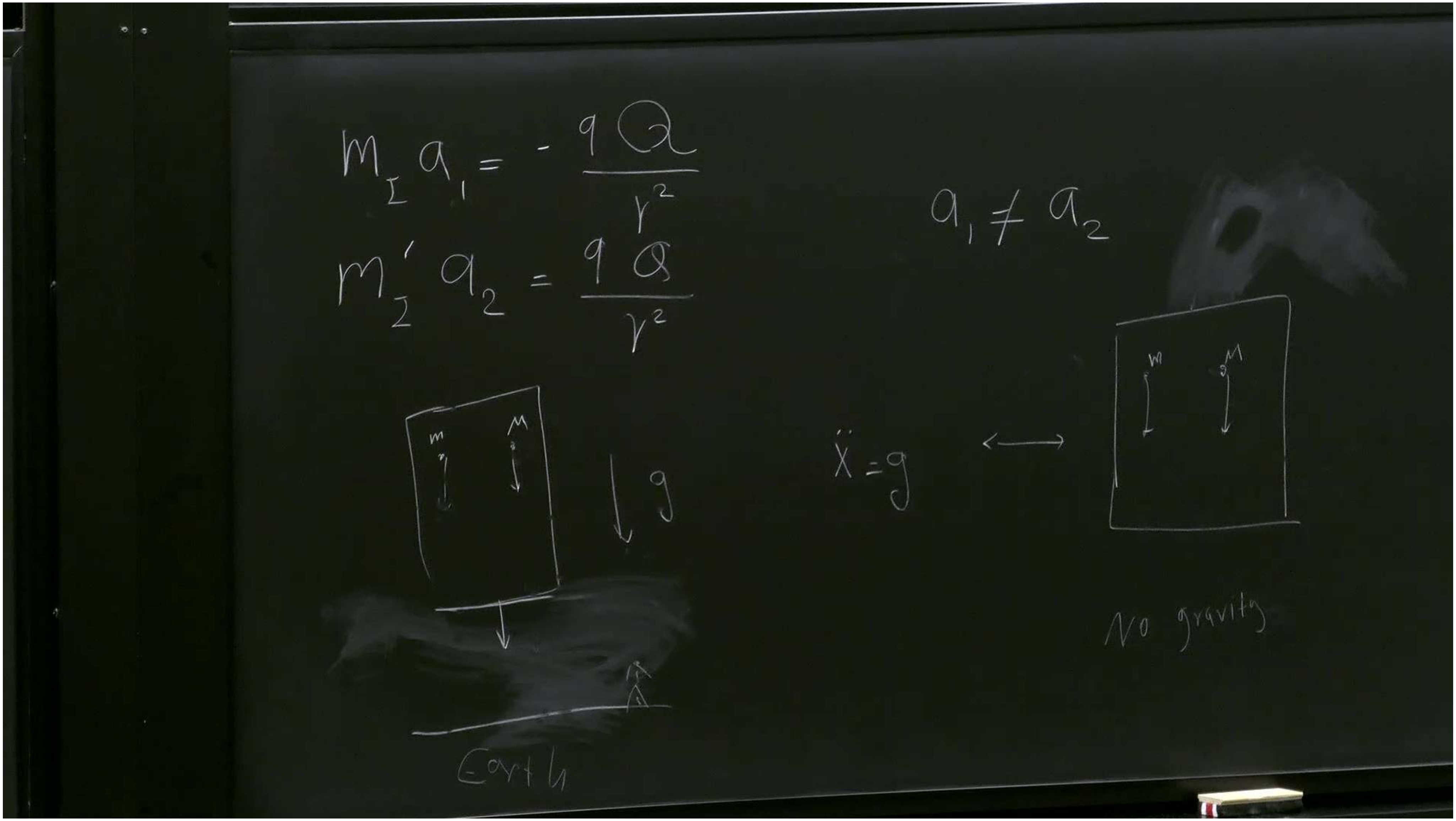
WEP: the trajectory of a free falling test body is independent of its mass & composition

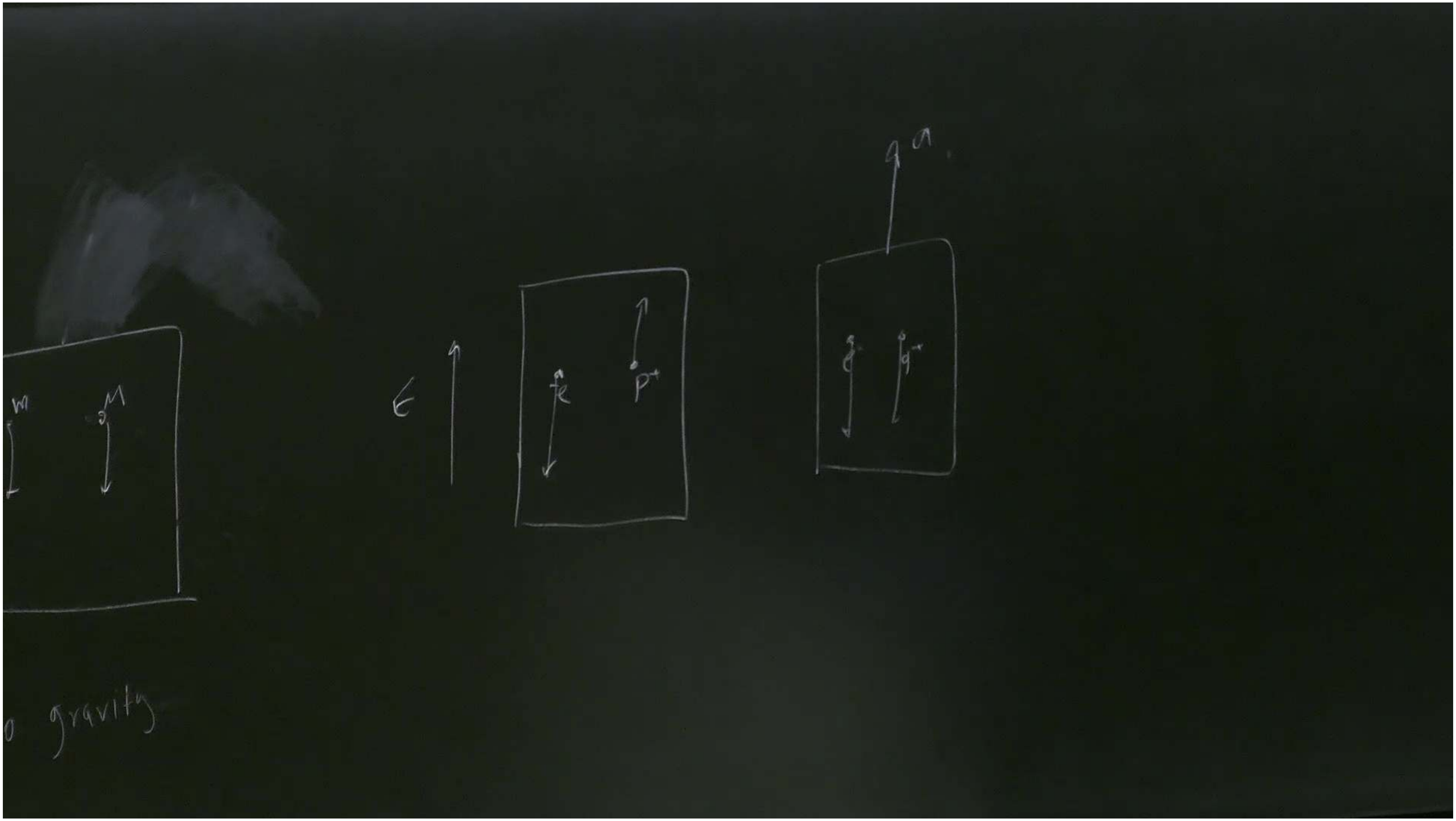
M & m $\longrightarrow$ Same acceleration

$$\boxed{m_I = m_g}$$

$$\frac{m_I}{m_g} = 1 \pm 10^{-13}$$


$m_I a_1 = -\frac{qQ}{r^2}$
$m_I' a_2 = \frac{qQ}{r^2}$
$a_1 \neq a_2$
$a = -g$
m
M
g
$\ddot{x} = g \longleftrightarrow$
Earth
No gravity


$m_I a_1 = -\frac{qQ}{r^2}$
$m_I' a_2 = \frac{qQ}{r^2}$
$a_1 \neq a_2$
m
M
g
Earth
$\ddot{x} = g$
m
M
No gravity



m
M
E
e^-
p^+
a
q^-
q^+
gravity

$\boxed{\text{local}}$ inertial frame (L.I.)

Space-time coordinate frame in which one is weightless

A. free particle moves on a straight line with constant velocity

$+ \left\{ \begin{array}{l} \text{geometry of space-time is locally Minkowski} \Rightarrow \text{locally Lorentz frame} \\ \text{laws of special relativity apply} \end{array} \right.$

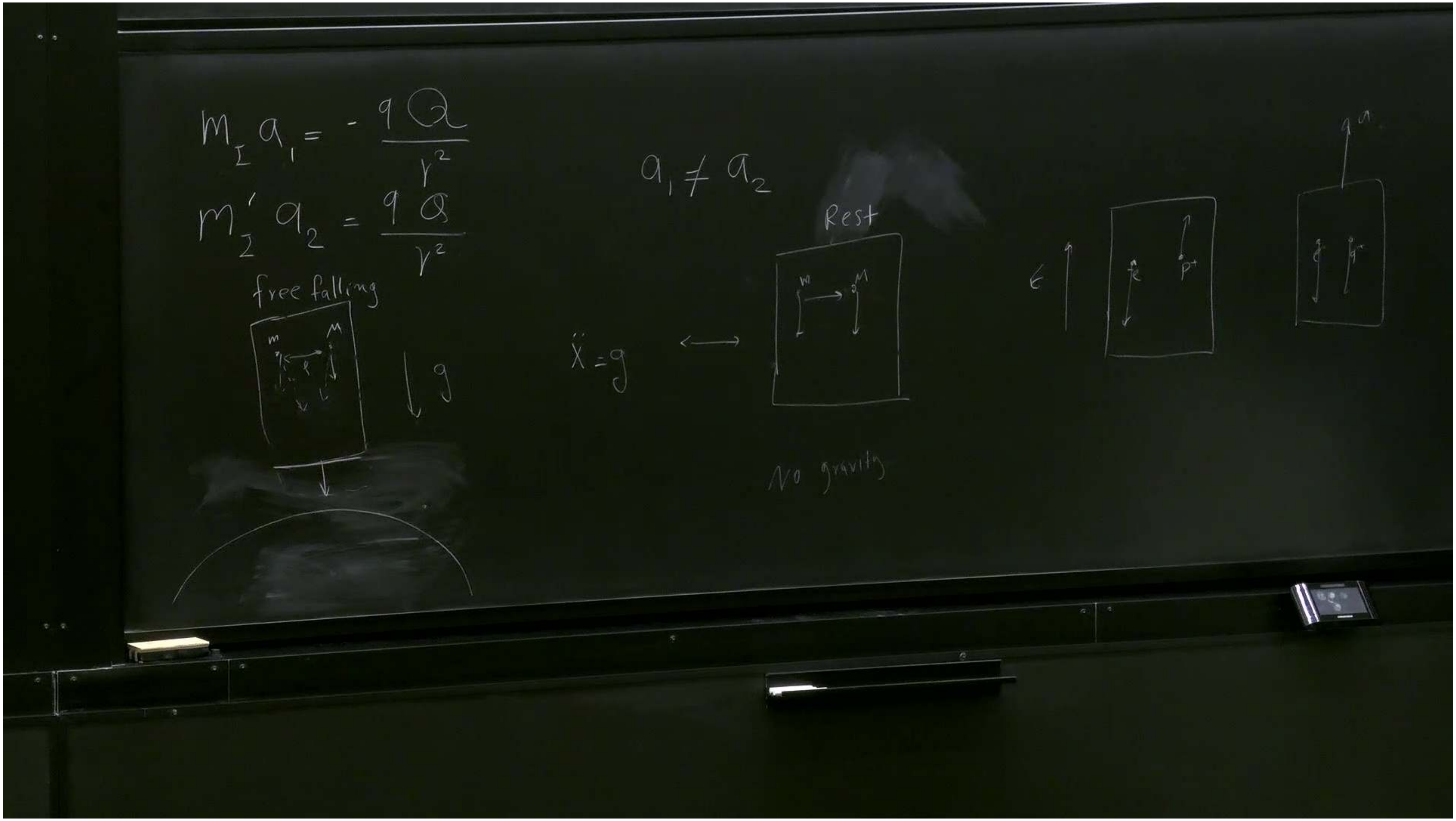
\+ anywhere & anytime in Universe

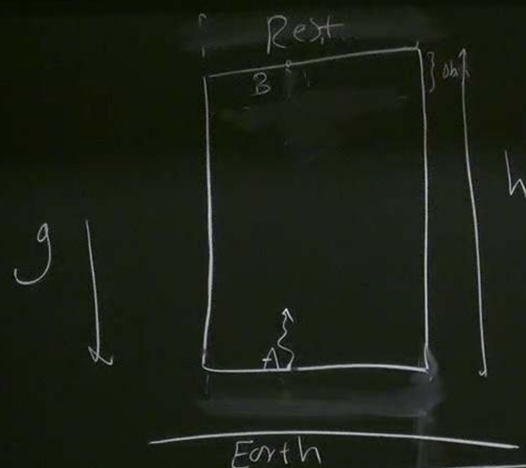
t, x, y, z

...tz frame

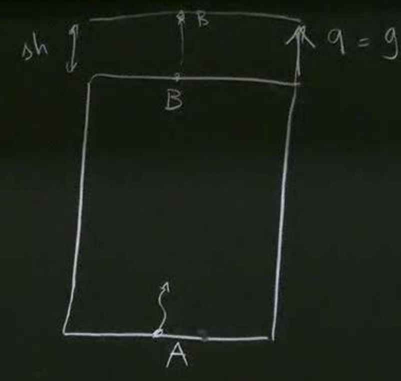
Einstein Equivalence principle

Anywhere & anytime in the universe there exists a local inertial frame where locally all laws of physic reduce to their form in S.R.


m_I a_1 = - qQ / r^2
m_I' a_2 = qQ / r^2
a_1 ≠ a_2
free falling
Rest
No gravity



EP



No gravity

$g \to \text{const}$

ν_{source}

$\Delta\nu = \nu_{obs} - \nu_s = g\,\Delta t \quad > 0$

$h + \Delta h = c\,\Delta t \qquad \Delta h \ll h$

$\Rightarrow \Delta\nu = \frac{g}{c}(h + \Delta h)$

$\frac{\lambda_{obs} - \lambda_{source}}{\lambda_s} = \frac{\Delta\nu}{c} = \frac{gh}{c^2}$ Redshift

$$\boxed{\frac{\Delta\lambda}{\lambda} \simeq \frac{gh}{c^2}}$$

Gravitational Redshif

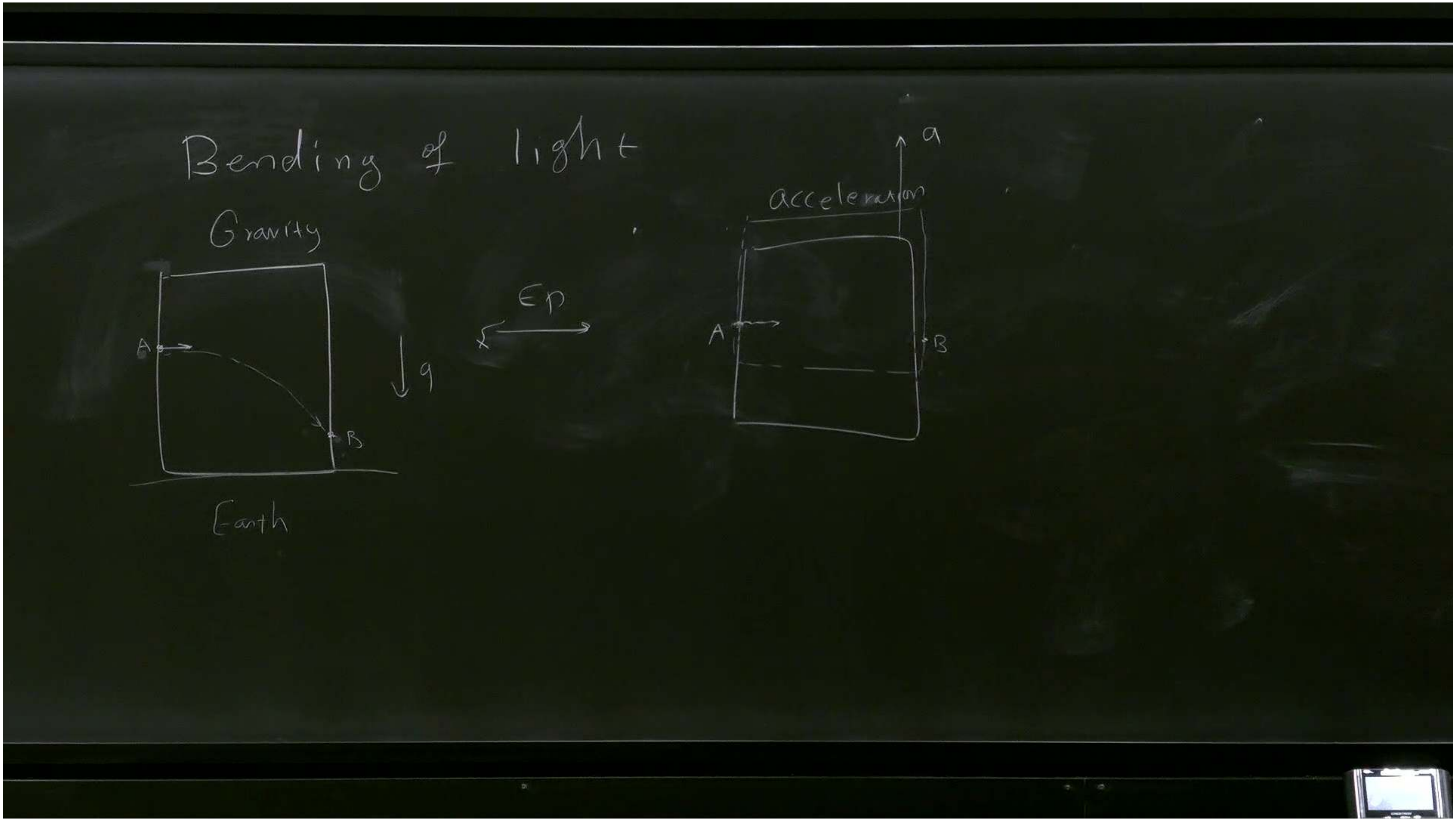
$$\nabla \phi_g = g\hat{z} \simeq \frac{\phi_B - \phi_A}{h} = g \Rightarrow \frac{\Delta\lambda}{\lambda} = \Delta\phi/c^2 > 0$$

$$\frac{T_A}{T_B} = \frac{\lambda_A}{\lambda_B} = \frac{\lambda_A}{\lambda_A + \Delta\lambda} = \frac{1}{1 + \Delta\phi/c^2} \approx 1 - \Delta\phi/c^2 \Rightarrow T_A < T_B$$

> 0

$\Delta h \ll h$

$= \frac{gh}{c^2}$ Redshift


Bending of light
Gravity
A
B
g
Earth
Ep
x
acceleration
a
A
B