Title: Lecture - Quantum Measurement and Continuous Markov Processes Mini-Course

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Collection/Series: Quantum Measurement and Continuous Markov Processes Mini-Course, Oct 27 - Dec 11, 2025

Subject: Quantum Foundations

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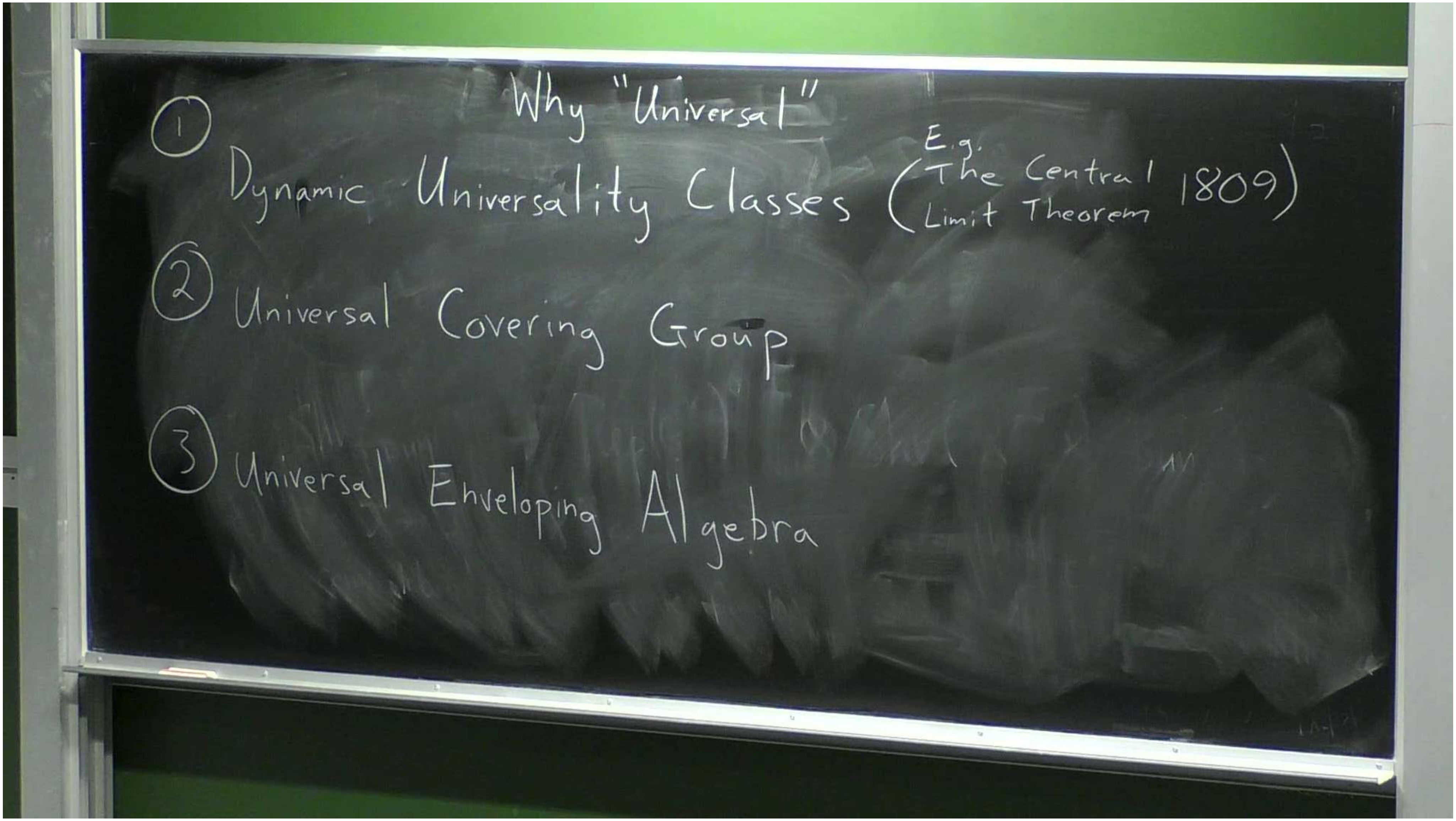
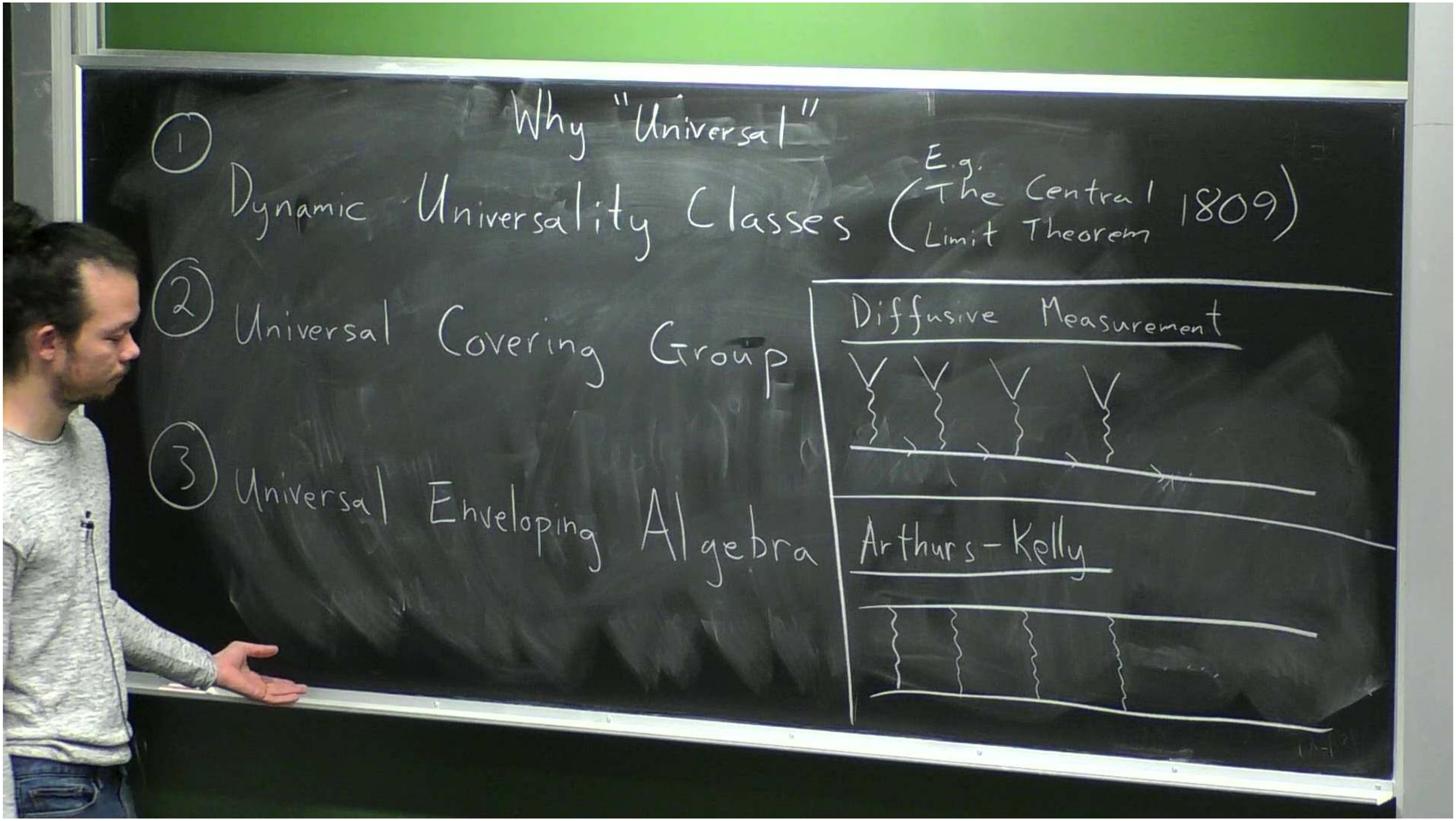

Why "Universal"
(1) Dynamic Universality Classes (E.g. The Central Limit Theorem 1809)
(2) Universal Covering Group
(3) Universal Enveloping Algebra


Why "Universal"
1 Dynamic Universality Classes (E.g. The Central Limit Theorem 1809)
2 Universal Covering Group
3 Universal Enveloping Algebra
Diffusive Measurement
Arthurs-Kelly

Weak Two-Outcome POVM

$$E_b = \frac{1}{2}\left(1 + 2b\epsilon X\right) \qquad \epsilon \ll 1,\ b = \pm 1,\ X \text{ is Hermitian}$$

$$\Omega_b = \sqrt{\frac{1}{2}}\left(1 + b\epsilon X\right)$$

$$\xi = \sum_b \Omega_b \odot \Omega_b^\dagger = \sum_b \frac{1}{2}\left(1 \odot 1 + b\epsilon\left(X \odot 1 + 1 \odot X\right) + b^2 \epsilon^2 X^2\right)$$

Weak Two-Outcome POVM

$$E_b = \frac{1}{2}\left(\mathbb{1} + 2b\epsilon X\right) \qquad \epsilon \ll 1,\quad b = \pm 1,\quad X \text{ is Hermitian}$$

$$\Omega_b \stackrel{?}{=} \sqrt{\frac{1}{2}}\left(\mathbb{1} + b\epsilon X\right) \qquad (1+\epsilon)^2 = (1+2\epsilon)$$

$$\zeta = \sum_b \Omega_b \otimes \Omega_b^\dagger = \sum_b \frac{1}{2}\left(\mathbb{1}\otimes\mathbb{1} + b\epsilon\left(X\otimes\mathbb{1} + \mathbb{1}\otimes X\right) + b^2\epsilon^2 X^2\right)$$

$(1+\epsilon) = (1+2\epsilon)$

$$\Omega_b \overset{?}{=} \sqrt{\frac{1}{2}}\left(1 + b\epsilon X\right)$$

$$\mathcal{J} = \sum_b \Omega_b \odot \Omega_b^\dagger = \sum_b \frac{1}{2}\left(1 \odot 1 + b\epsilon\left(X \odot 1 + 1 \odot X\right) + b^2 \epsilon^2 X \odot X\right)$$

$$= 1 \odot 1 + 0 + \epsilon^2 X \odot X$$

So, actually

$$\Omega_b = \sqrt{\frac{1}{2}}\left(1 + b\epsilon X - \frac{1}{2}\epsilon^2 X^2\right)$$

Weak Two-Outcome POVM

$$E_b = \frac{1}{2}(1 + 2b\epsilon X)$$

So, actually

$$\Omega_b = \sqrt{\frac{1}{2}}\left(1 + b\epsilon X - \frac{1}{2}\epsilon^2 X^2\right).$$

Ironically

$$\Omega^\dagger{}_b \Omega_b = \frac{1}{2}\left(1 + 2\epsilon X + \mathcal{O}(\epsilon^3)\right) \quad \text{still}.$$

Weak Two-Outcome POVM

$$E_b = \frac{1}{2}(1 + 2b\epsilon X)$$

$$\Omega_{b_n} \cdots \Omega_{b_1} = \sqrt{\frac{1}{2^n}}\left(1 + \epsilon(b_1 + \dots + b_n)X + \underbrace{\epsilon^2 \sum_{i \neq j} b_i b_j X^2}_{\text{Ugh.}} - \frac{1}{2} n \epsilon^2 X^2\right)$$

$$\Omega_{b_n}\cdots\Omega_{b_1} = \sqrt{\frac{1}{2^n}}\left(1+\epsilon(b_1+\dots+b_n)X + \epsilon^2 \underbrace{\sum_{i\neq j} b_i b_j X^2}_{\text{Ugh.}} - \frac{1}{2} n\epsilon^2 X^2\right)$$

"Call on Euler", i.e. consider

$$\Omega_b = \sqrt{\frac{1}{2}}\, e^{1+b\epsilon X - \epsilon^2 X^2}$$

$$\Omega_{b_n}\cdots\Omega_{b_1} = \sqrt{\frac{1}{2^n}}\Big(1+\epsilon(b_1+\dots+b_n)X+\epsilon^2\underbrace{\sum_{i\neq j} b_i b_j X^2}_{\text{Ugh.}}-\frac{1}{2}n\epsilon^2X^2\Big)$$

"Call on Euler", i.e. consider

$$\Omega_b = \sqrt{\frac{1}{2}}\, e^{1+b\epsilon X-\epsilon^2X^2}$$

Then $\Omega_{b_n}\cdots\Omega_{b_1} = \sqrt{\frac{1}{2^n}}\, e^{1+(b_1+\dots+b_n)\epsilon X-\epsilon^2X^2}$

So, actually

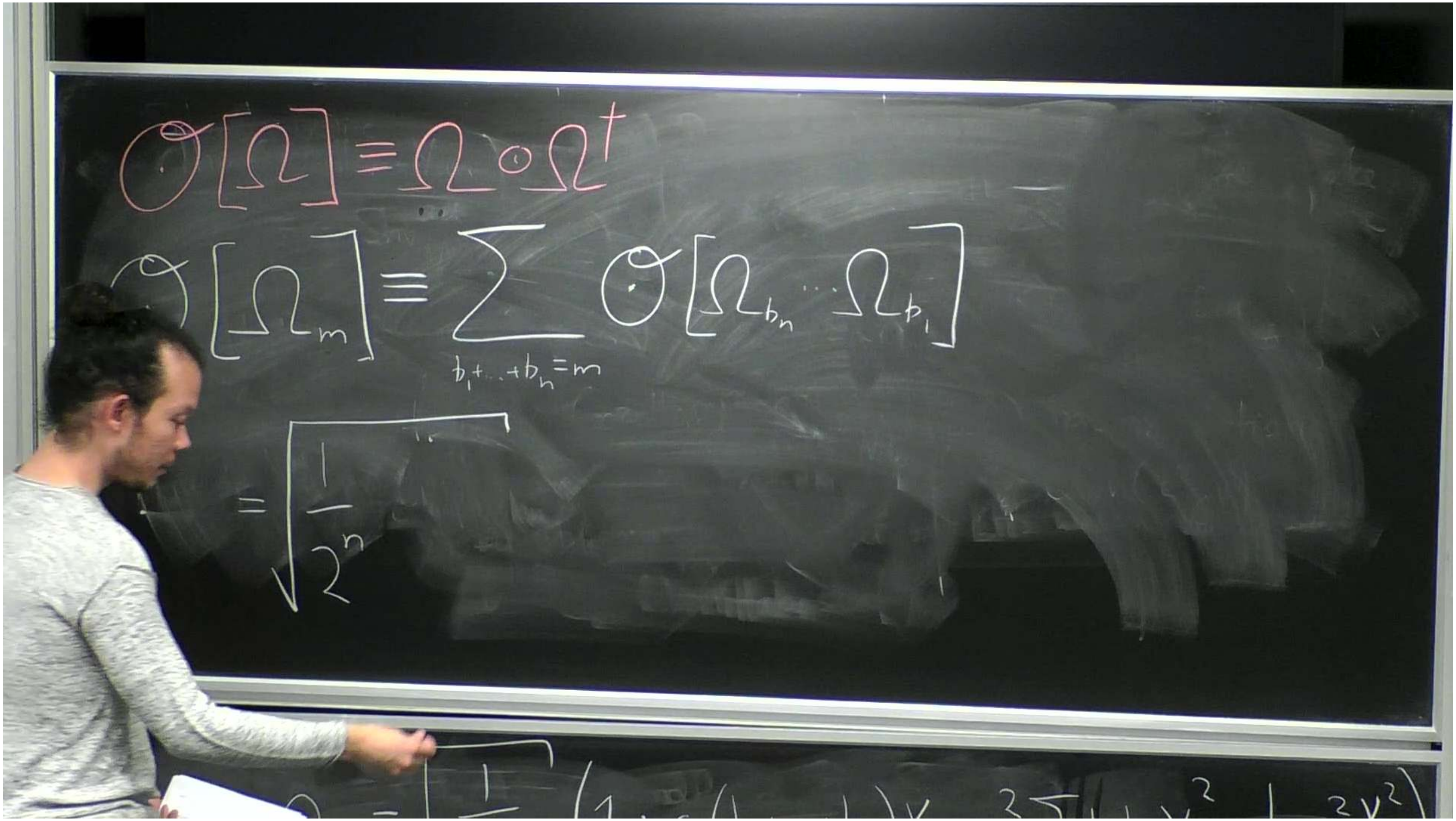
$$\Omega_b = \sqrt{\frac{1}{2}}\left(1 + b\epsilon X - \frac{1}{2}\epsilon^2 X^2\right).$$

Ironically

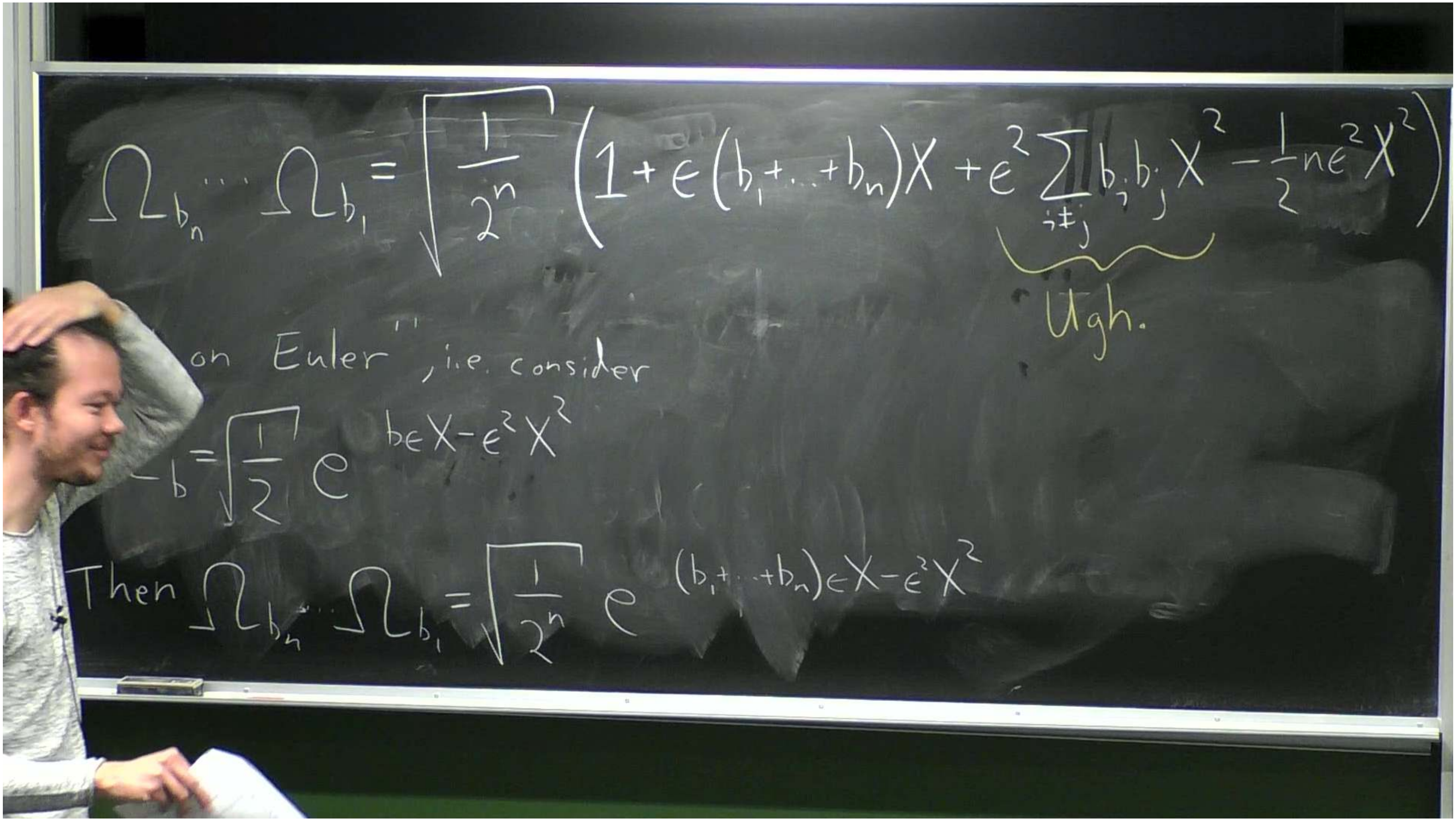
$$\Omega^\dagger_b \Omega_b = \frac{1}{2}\left(1 + 2\epsilon X + \mathcal{O}(\epsilon^3)\right) \quad \text{still}.$$

Weak Two-Outcome POVM

$$E_b = \frac{1}{2}(1 + 2b\epsilon X)$$



$\mathcal{O}[\Omega] \equiv \Omega \circ \Omega^{\dagger}$
$\mathcal{O}[\Omega_m] \equiv \sum_{p_1+\dots+p_n=m} \mathcal{O}[\Omega_{p_n}\cdots\Omega_{p_1}]$
$= \sqrt{\frac{1}{2^n}}$



$\Omega_{b_n} \cdots \Omega_{b_1} = \sqrt{\frac{1}{2^n}} \left(1 + \epsilon (b_1 + \ldots + b_n) X + \epsilon^2 \underbrace{\sum_{i \neq j} b_i b_j X^2}_{\text{Ugh.}} - \frac{1}{2} n \epsilon^2 X^2 \right)$
on Euler", i.e. consider
$\Omega_b = \sqrt{\frac{1}{2}}\, e^{b\epsilon X - \epsilon^2 X^2}$
Then $\Omega_{b_n} \cdots \Omega_{b_1} = \sqrt{\frac{1}{2^n}}\, e^{(b_1 + \ldots + b_n)\epsilon X - \epsilon^2 X^2}$

$$\mathcal{O}[\Omega] \equiv \Omega \circ \Omega^{\dagger}$$

$$\mathcal{O}[\Omega_m] \equiv \sum_{p_1+\dots+p_n=m} \mathcal{O}[\Omega_{p_n} \cdots \Omega_{p_1}]$$

$$= \frac{1}{2^n}\binom{n}{k} e^{m\epsilon X - \epsilon^2 X^2}$$

$$\boxed{\text{where } m = 2k-n}$$

$$\mathcal{O}[\Omega] \equiv \Omega \circ \Omega^{\dagger}$$

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$k=\#$

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$$\boxed{\text{where } m = 2k-n}$$

$$\sum_{b_1+\dots+b_n=m} 1 = \binom{n}{k} \qquad k = \text{"}\# b_i = +1\text{"}$$

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$$= \frac{1}{2^n}\binom{n}{k}\mathcal{O}\left[m\epsilon X - \epsilon^2 X^2\right]$$

where

$$\boxed{m = 2k - n}$$

$m: -n, -n+2, \dots, n$

$k: 0, 1, \dots, n$

$$\sum_{b_1+\dots+b_n=m} 1 = \binom{n}{k} \qquad k = \text{``}\# b_i = +1\text{''}$$

$$\Omega_m = \sqrt{\frac{1}{2^n}\binom{n}{k}}\, e^{m\epsilon X - n\epsilon^2 X^2} \quad \text{where } m = 2k - n$$

$$\mathcal{O}[\Omega] \equiv \Omega \circ \Omega^{\dagger}$$

$$\mathcal{O}[\Omega_m] \equiv \sum_{b_1+\dots+b_n=m} \mathcal{O}[\Omega_{b_n} \cdots \Omega_{b_1}]$$

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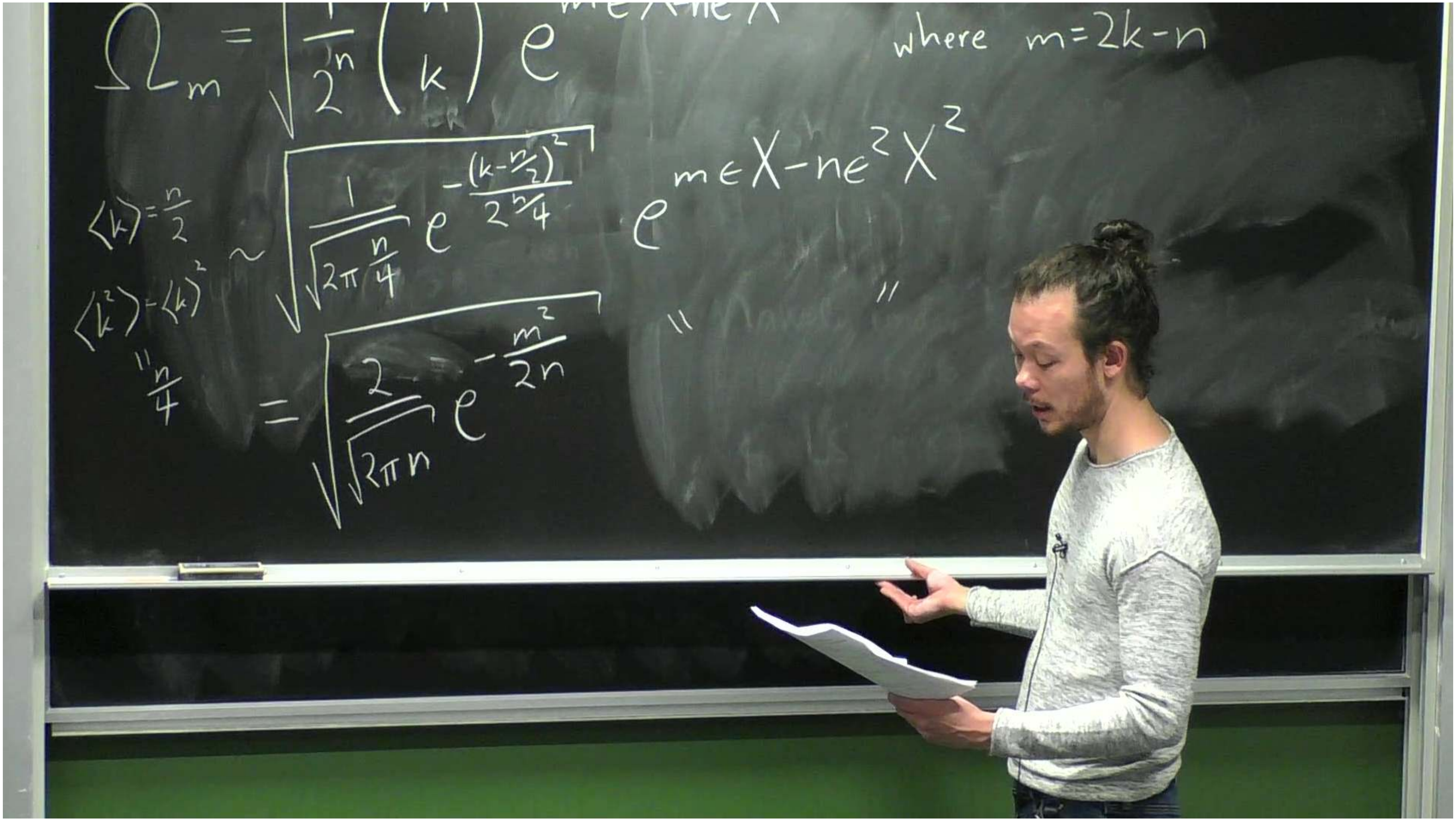
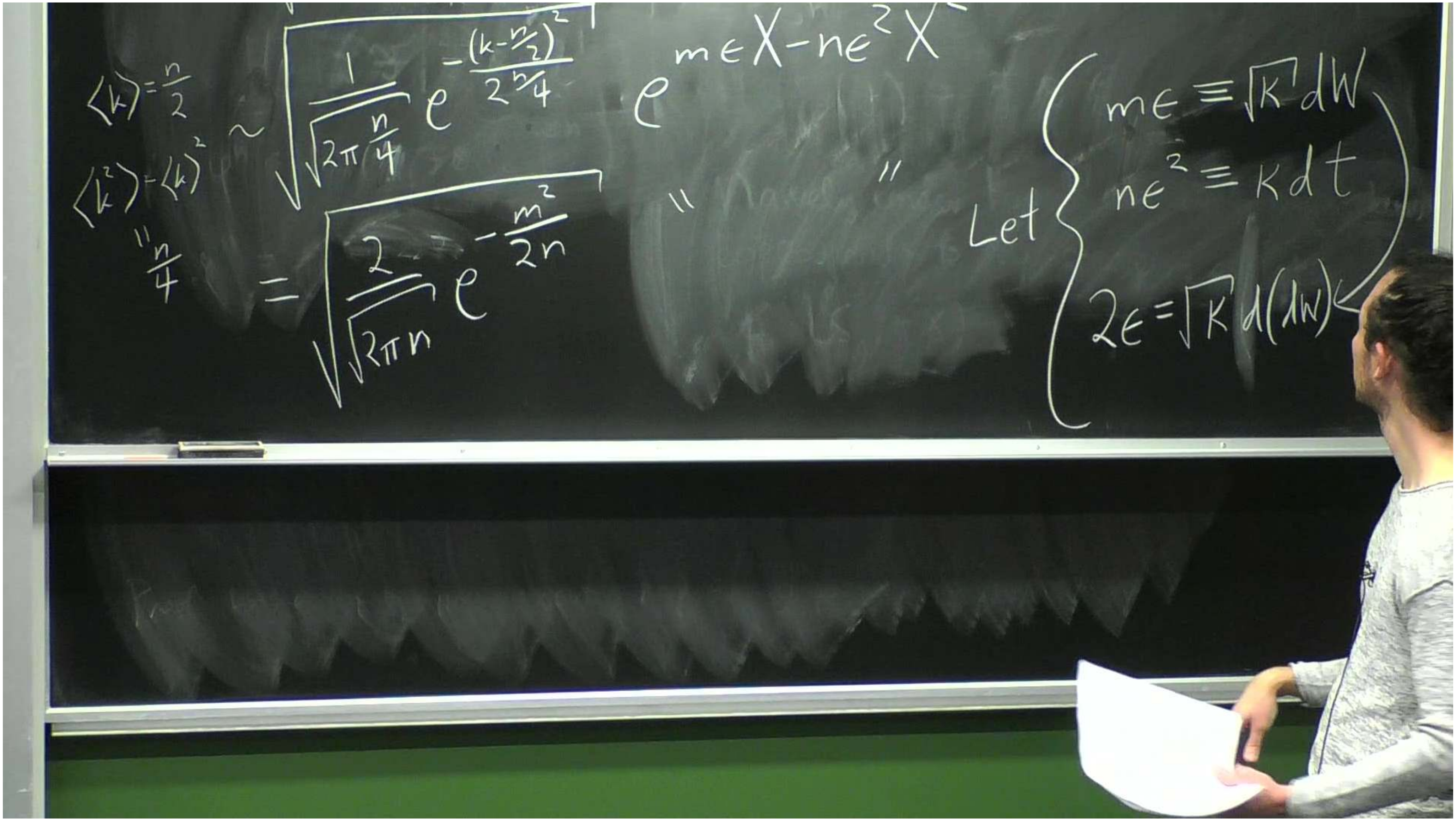


$\Omega_m = \sqrt{\frac{1}{2^n}\binom{n}{k}}\, e$
where $m=2k-n$
$\langle k \rangle = \frac{n}{2}$
$\langle k^2 \rangle - \langle k \rangle^2 = \frac{n}{4}$
$\sim \sqrt{\frac{1}{\sqrt{2\pi \frac{n}{4}}} e^{-\frac{(k-\frac{n}{2})^2}{2\,n/4}}}\; e^{m\epsilon X - n\epsilon^2 X^2}$
$= \sqrt{\frac{2}{\sqrt{2\pi n}} e^{-\frac{m^2}{2n}}}$



$\langle k \rangle = \frac{n}{2}$
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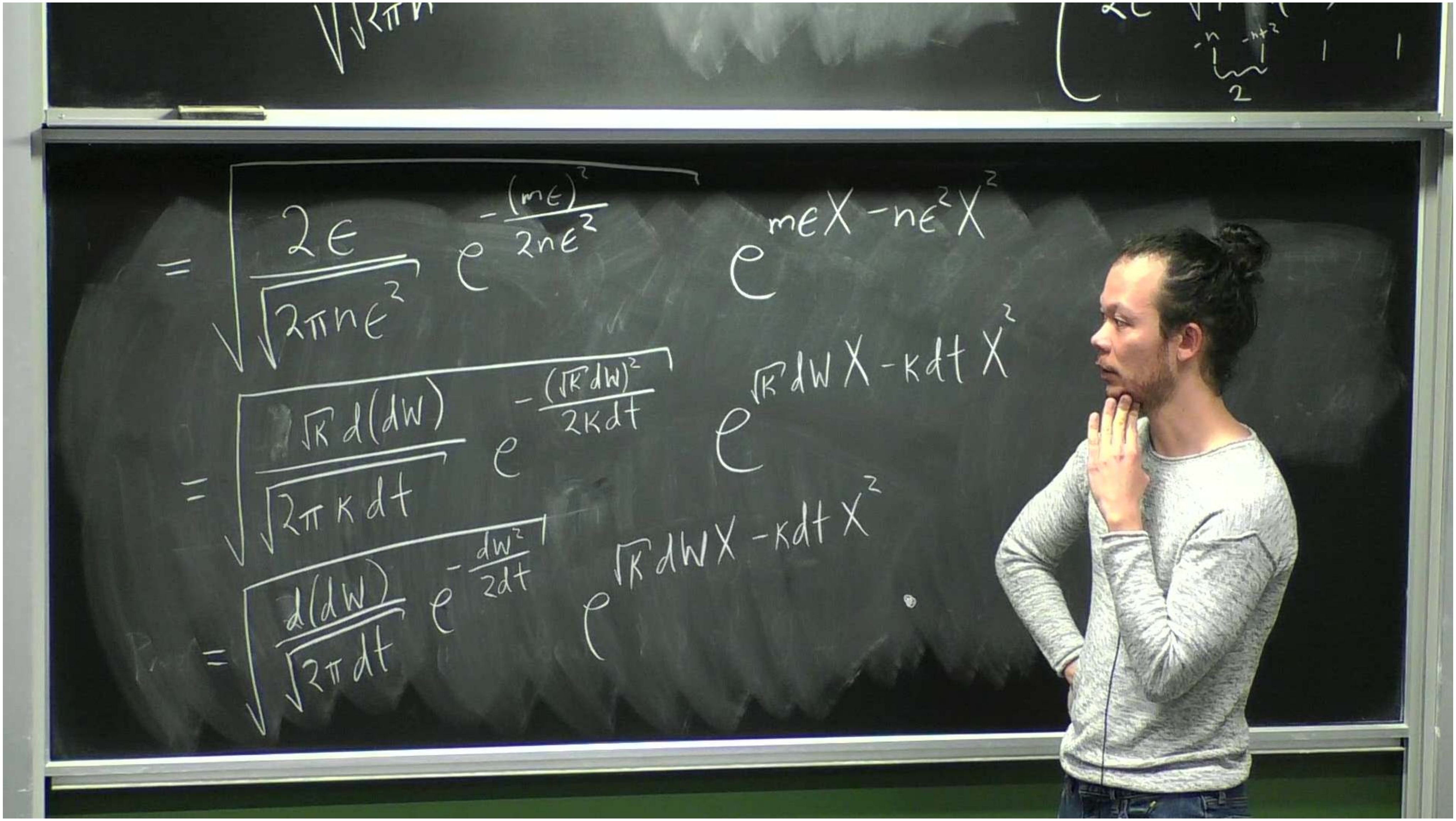
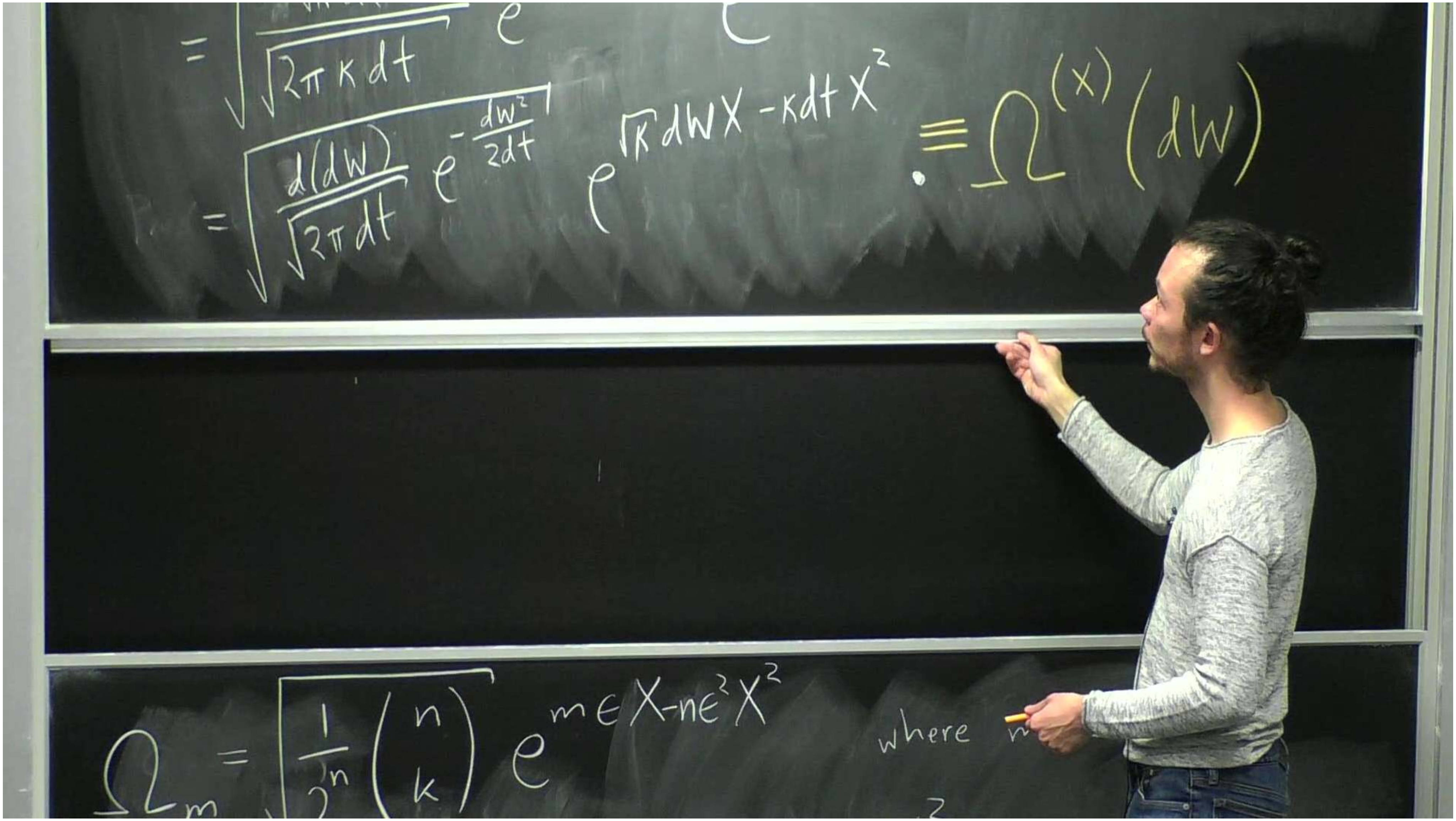
$\sim \sqrt{\frac{1}{\sqrt{2\pi \frac{n}{4}}} e^{-\frac{(k-\frac{n}{2})^2}{2\frac{n}{4}}}} \; e^{m\epsilon X - n\epsilon^2 X^2}$
$= \sqrt{\frac{2}{\sqrt{2\pi n}} e^{-\frac{m^2}{2n}}}$
Let
$m\epsilon \equiv \sqrt{\kappa}\, dW$
$n\epsilon^2 \equiv \kappa\, dt$
$2\epsilon = \sqrt{\kappa}\, d(\lambda W)$



$= \sqrt{\frac{2\epsilon}{\sqrt{2\pi n\epsilon^2}} e^{-\frac{(m\epsilon)^2}{2n\epsilon^2}}} \quad e^{m\epsilon X - n\epsilon^2 X^2}$
$= \sqrt{\frac{\sqrt{\kappa}\, d(dW)}{\sqrt{2\pi \kappa\, dt}} e^{-\frac{(\sqrt{\kappa}\, dW)^2}{2\kappa\, dt}}} \quad e^{\sqrt{\kappa}\, dW X - \kappa\, dt X^2}$
$= \sqrt{\frac{d(dW)}{\sqrt{2\pi\, dt}} e^{-\frac{dW^2}{2dt}}} \quad e^{\sqrt{\kappa}\, dW X - \kappa\, dt X^2}$


$= \sqrt{\frac{d(dW)}{\sqrt{2\pi\,dt}}\, e^{-\frac{dW^2}{2dt}}}\; e^{\sqrt{\kappa}\,dW X - \kappa\,dt X^2} \equiv \Omega^{(x)}(dW)$
$\Omega_m = \sqrt{\frac{1}{2^n}\binom{n}{k}}\, e^{m\epsilon X - n\epsilon^2 X^2}$
where

$\sqrt{2\pi dt}$

$$\Omega^{(X)}(dW)\,\Omega^{(Y)}(dV) =$$

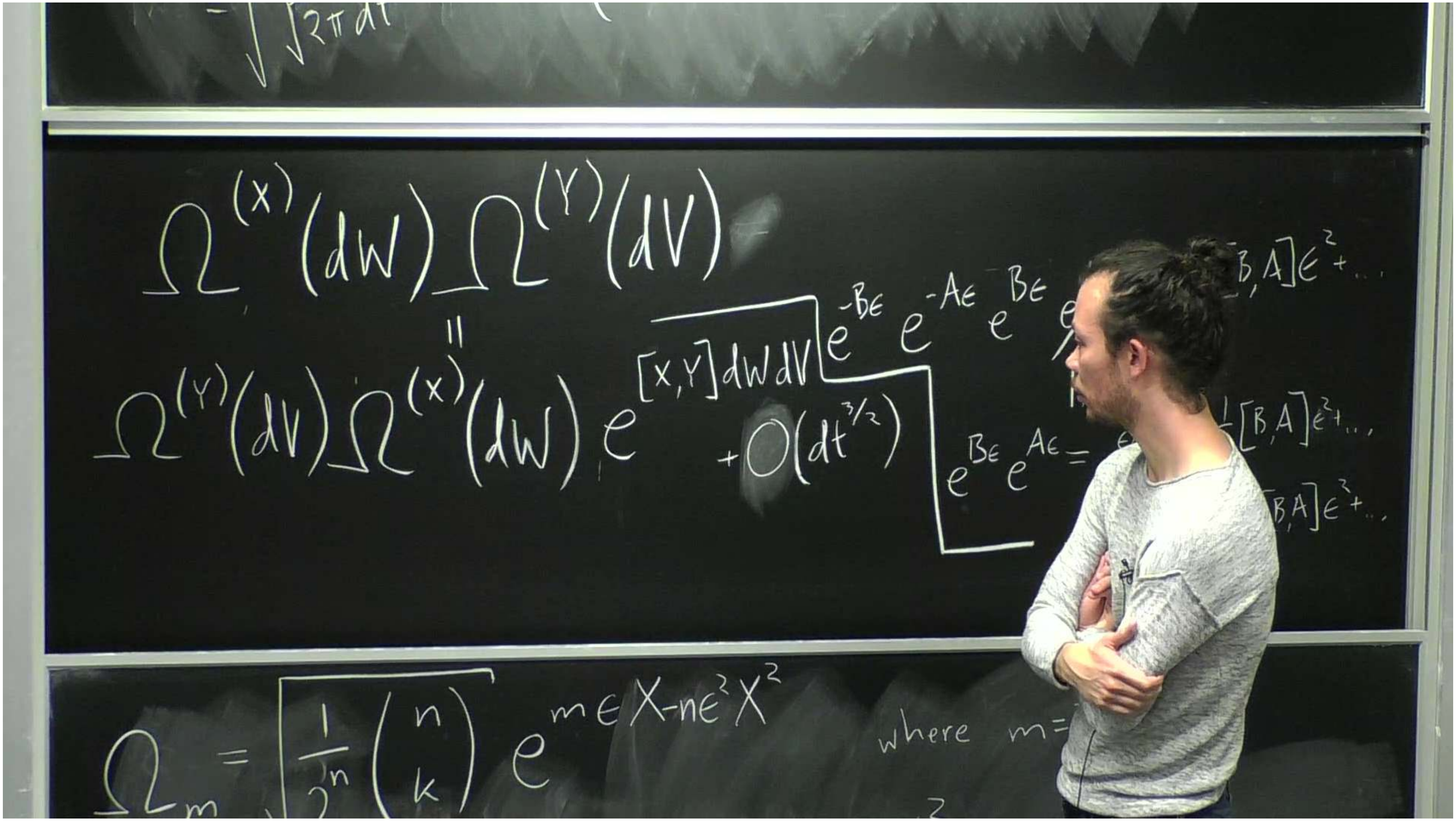
$$e^{-B\epsilon} e^{-A\epsilon} e^{B\epsilon} e^{A\epsilon} = e^{[B,A]\epsilon^2 + \dots}$$

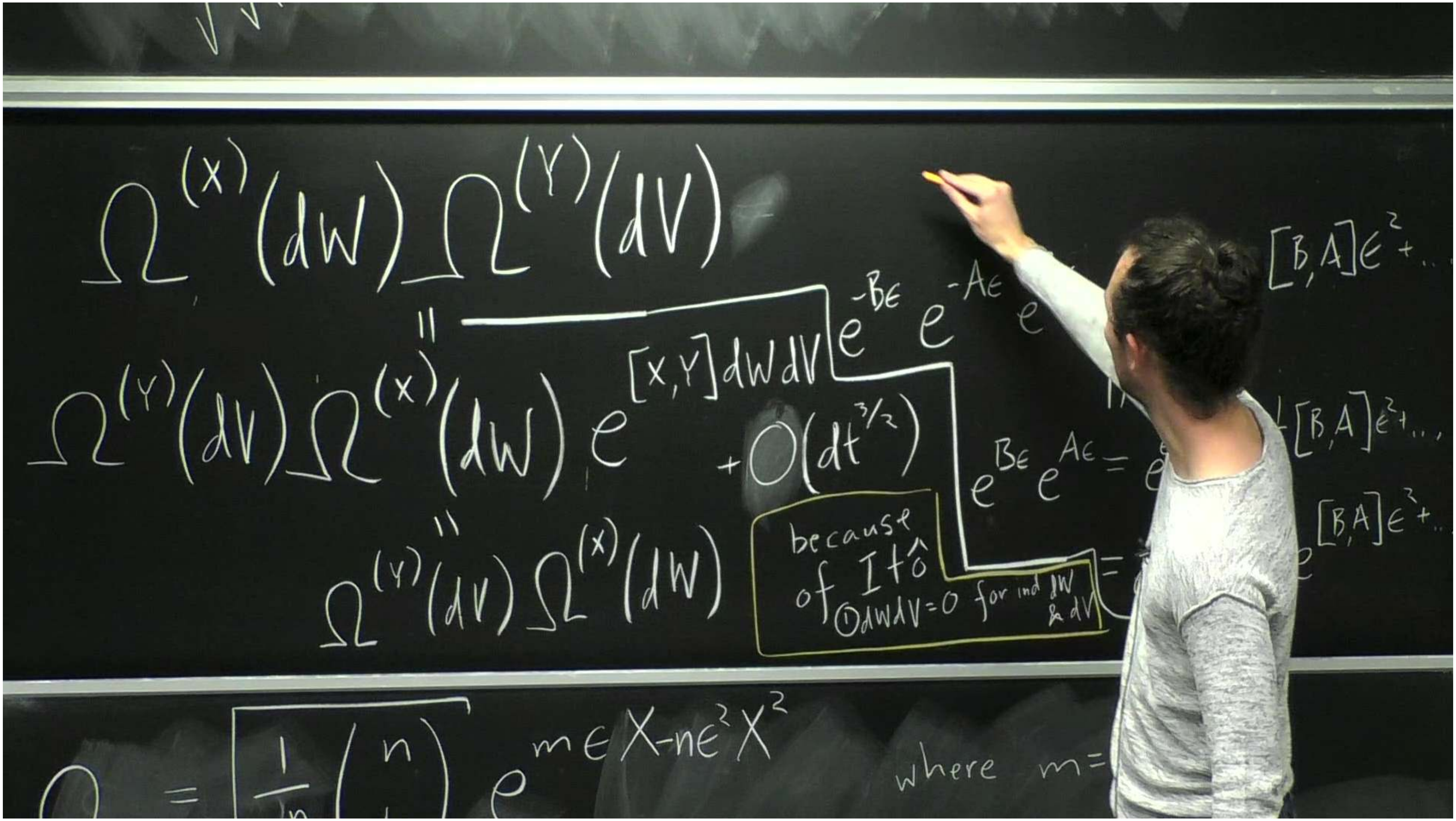
$$\Uparrow$$

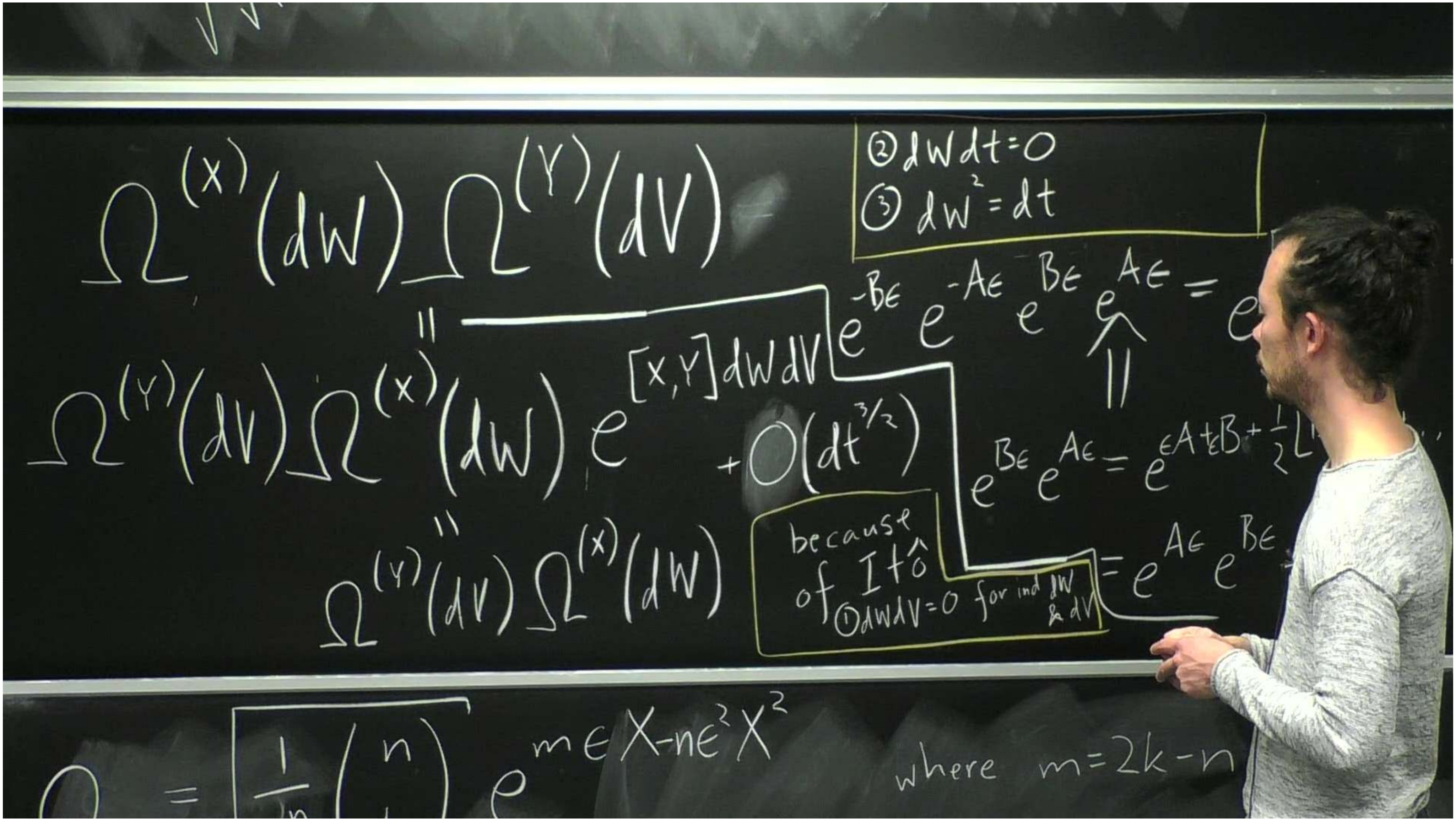
$$e^{B\epsilon} e^{A\epsilon} = e^{\epsilon A + \epsilon B + \frac{1}{2}[B,A]\epsilon^2 + \dots}$$

$$= e^{A\epsilon} e^{B\epsilon} e^{[B,A]\epsilon^2 + \dots}$$

$$\Omega_m = \sqrt{\frac{1}{2^n}\binom{n}{k}}\, e^{m\epsilon X - n\epsilon^2 X^2} \quad \text{where } m = 2k - n$$


$\Omega^{(X)}(dW)\,\Omega^{(Y)}(dV)$
$=$
$\Omega^{(Y)}(dV)\,\Omega^{(X)}(dW)\,e^{[X,Y]dW dV}$
$+O(dt^{3/2})$
$e^{-B\epsilon}\,e^{-A\epsilon}\,e^{B\epsilon}$
$[B,A]\epsilon^2+\dots$
$\frac{1}{2}[B,A]\epsilon^2+\dots$
$e^{B\epsilon}\,e^{A\epsilon}=$
$[B,A]\epsilon^2+\dots$
$\Omega_m=\sqrt{\frac{1}{2^n}\binom{n}{k}}\,e^{m\epsilon X-n\epsilon^2X^2}$
where $m=$


Ω^(X)(dW) Ω^(Y)(dV)
=
Ω^(Y)(dV) Ω^(X)(dW) e^[X,Y]dWdV + O(dt^3/2)
=
Ω^(Y)(dV) Ω^(X)(dW)
because of Itô
① dWdV=0 for ind dW & dV
where m=



② dW dt = 0
③ dW² = dt
Ω^(X)(dW) Ω^(Y)(dV)
Ω^(Y)(dV) Ω^(X)(dW) e^([X,Y] dW dV) + O(dt^(3/2))
Ω^(Y)(dV) Ω^(X)(dW)
because of Itô
① dW dV = 0 for ind dW & dV
e^(-Bε) e^(-Aε) e^(Bε) e^(Aε) = e
e^(Bε) e^(Aε) = e^(εA+εB+½[
= e^(Aε) e^(Bε)
e^(mεX−nε²X²)
where m=2k−n